Do high-energy neutrinos travel faster than photons in a discrete space–time?

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A B S T R A C T

The recent OPERA measurement of high-energy neutrino velocity, once independently verified, implies new physics in the neutrino sector. We revisit the theoretical inconsistency of the fundamental high-energy cutoff attributing to quantum gravity with the parity-violating gauge symmetry of local quantum field theory describing neutrinos. This inconsistency suggests high-dimension operators of neutrino interactions. Based on these studies, we try to view the OPERA result, high-energy neutrino oscillations and indicate to observe the restoration of parity conservation by measuring the asymmetry of high-energy neutrinos colliding with left- and right-handed polarized electrons.

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1. Introduction

Since their appearance, neutrinos have always been extremely peculiar. Their charge neutrality, near masslessness, flavor mixing, oscillation and parity-violating coupling have been at the center of a conceptual elaboration and an intensive experimental analysis that have played a major role in donating to mankind the beauty of the standard model for particle physics. Recently, the OPERA experiment [1] reports anomaly in flight time of neutrinos ($\langle E_\nu \rangle = 17$ GeV) from CERN to INFN Gran Sasso. This anomaly corresponds to a relative difference of the muon neutrino velocity with respect to the speed of light $(v - c)/c = (2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.})) \times 10^{-5}$. As described in Ref. [1], this value is by far much larger than the relative deviation from the speed of light c of the neutrino velocity due to its finite rest mass, that is expected to be smaller than 10^{-19}. In the past, a high energy ($E_\nu > 30$ GeV) and short baseline experiment has been able to test deviations down to $|v - c|/c < 4 \times 10^{-5}$ [2]. With a baseline analogous to that of OPERA but at lower neutrino energies (E_ν peaking at ~ 3 GeV with a tail extending above 100 GeV), the MINOS experiment reported a measurement of $(v - c)/c = 5.1 \pm 2.9 \times 10^{-5}$ [3]. A larger deviation of the neutrino velocity from c would be a striking result pointing to new physics in the neutrino sector. Given the potential far-reaching consequences of such a result, independent measurements are needed before the effect can either be refuted or firmly established. Nevertheless, in this Letter, we revisit the theoretical inconsistency of the fundamental high-energy cutoff attributing to quantum gravity with the parity-violating gauge symmetry of local quantum field theory describing neutrinos. This inconsistency suggests high-dimension operators of neutrino interactions. Based on

these studies, we try to view the OPERA result, high-energy neutrino oscillations and indicate to observe the restoration of parity conservation by measuring the asymmetry of high-energy neutrinos colliding with left- and right-handed polarized electrons.

In continuum space–time manifold without any length scale at short distances, the Lorentz symmetry group is unbounded at the high boost (or high energy) end. Exact Lorentz symmetry may be broken in the high boost limit [4], if there is a fundamental cutoff, the Planck length (scale) $a_{\text{pl}} \sim 10^{-33}$ cm ($\Lambda_{\text{pl}} = \pi/a_{\text{pl}} \sim 10^{19}$ GeV) at short distances of space–time, intrinsically attributing to quantum gravity. The possibility of Lorentz symmetry breaking has been considered in many sophisticate theories, Loop quantum gravity [5], string theory and non-commutative geometry [6,7], standard model extension with Lorentz symmetry breaking operators [8], and phenomenological quantum gravity model [9,10].

We may conceive that precisely due to the violent fluctuations that the gravitational field *must* exhibit at a_{pl}, space–time "ends" there. Either by the creation of a Wheeler "foam" [11], or by some other mechanism which we need not discuss here, one may conceive that as a result the physical space–time gets endowed with a fundamental Planck length, a_{pl}, and thus the basic arena of physical reality becomes a *random lattice with lattice constant* $a_{\text{min}} \sim a_{\text{pl}}$ [12]. We recently calculate this minimal length $a_{\text{min}} \approx 1.2a_{\text{pl}}$ [13] in studying quantum Einstein–Cartan theory in the framework of Regge calculus and its variant [11,14,15].

This discrete space–time provides a natural regulator for local quantum field theories for particles and gauge interactions. A natural regularized quantum field theory demands the existence of ultra-violet fix points where renormalization group invariance follows so that low-energy observables are independent of high-energy cutoff. Based on low-energy observations of parity violation, the Lagrangian of standard model was built preserving exact chiral-gauge symmetries $SU_L(2) \otimes U_Y(1)$ that are accommodated

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by left-handed lepton doublet $(\nu_i, i)_L$ and right-handed single $(i)_R$ $(i = e, \mu, \tau)$. On the other hand, a profound result obtained 30 years ago, in the form of a no-go theorem [16], tells us that there is no any consistent way to transpose straightforwardly on a discrete space–time the bilinear neutrino Lagrangian of the continuum theory exactly preserving chiral gauge symmetries. This no-go theorem was also demonstrated [17] to be generic and independent of discretization (regularization) scheme of space–time. Either one given up the gauge principle by explicitly breaking chiral gauge symmetries [18] or we were led to consider at least quadrilinear (four) neutrino interactions to preserve chiral-gauge symmetries [20]. This is not the place for a detailed discussion of this important result and its possible relation to quantum gravity. We focus ourselves to discussions that in discrete space–time, Lorentz symmetry is broken, bilinear neutrino kinetic terms do not preserve chiral-gauge symmetries, and quadrilinear (four) neutrino interactions are introduced in connection with recent on going high-energy neutrino experiments. (The natural units $\hbar = c = 1$ are adopted, unless otherwise specified.)

2. Energy–momentum relations

In order to simplify discussions, we start with a hypercube lattice with lattice constant a, instead of a complex discrete space–time like the simplicial manifold discussed for quantum gravity [13]. On a spatial lattice (time continuum), the Klein–Gordon motion for a free massive gauge boson is [19]

$$\ddot{\phi} = \frac{1}{a^2} \Delta \phi(x) - m^2 c^4 \phi(x), \qquad (1)$$

where the differentiating operator

$$\Delta \phi(x) \equiv \sum_{\mu} \big[\phi(x + a\hat{n}_{\mu}) + \phi(x - a\hat{n}_{\mu}) - 2\phi(x) \big], \qquad (2)$$

and the energy–momentum relation is given by

$$E^2 = m^2 + \frac{2}{a^2} w(k), \quad w(k) \equiv 2 \sin^2 \left(\frac{ka}{2} \right) \qquad (3)$$

and

$$E^2 \approx m^2 + k^2 - \frac{1}{12} \big(k^4 a^2 \big) + \cdots, \qquad (4)$$

for $ka \ll 1$. Therefore, the velocity of a massless boson (photon) is given by

$$v_{\gamma} = \frac{dE_{\gamma}}{dk} \approx 1 - \frac{1}{8} (ka)^2 \approx 1 - \frac{1}{8} (E_{\gamma} a)^2 \qquad (5)$$

which is smaller than the speed of light traveling in a continuum space–time.

As discussed in the introductory paragraph, chiral symmetries have to be explicitly broken if Hamiltonian $\mathcal{H}$ is bilinear in left-handed (Weyl) neutrino fields ψ,

$$\mathcal{H} = \sum_x \bigg\{ -\frac{i}{2a} \bar{\psi}(x) \gamma_{\mu} \big[\psi(x + a\hat{n}_{\mu}) - \psi(x - a\hat{n}_{\mu}) \big]$$
$$- \frac{B}{2a} \bar{\psi}(x) \Delta \psi_R(x) + m_{\nu} \bar{\psi}(x) \psi_R(x) \bigg\}, \qquad (6)$$

where ψ_R is a right-handed neutrino, and the second term is an explicit chiral-symmetry-breaking term [18]. The energy–momentum relation is [19]

$$E_{\nu}^2 = m_{\nu}^2 + \frac{\sin^2 ka}{a^2} + B^2 \frac{w^2(k)}{a^2}, \qquad (7)$$

and

$$E_{\nu}^2 \simeq k^2 + m_{\nu}^2 + \frac{1}{4} \left(B^2 - \frac{4}{3} \right) k^4 a^2 + \cdots, \qquad (8)$$

for $ka \ll 1$. Analogously, the velocity of neutrinos is

$$v_{\nu} = \frac{dE_{\nu}}{dk} \approx 1 - \frac{1}{2} \frac{m_{\nu}^2}{E_{\nu}^2} + \frac{3}{8} \left(B^2 - \frac{4}{3} \right) (E_{\nu} a)^2, \qquad (9)$$

where $k^2 \approx E_{\nu}^2 \gg m_{\nu}^2$. For the Wilson parameter $B^2 > 4/3$ and large neutrino energy E_{ν}, Eq. (9) implies the possibility that high-energy neutrinos can travel faster than photons in discrete space–time. In this low-energy region, photon and neutrino energies $E_{\gamma, \nu}$ are much smaller than the cutoff scale $\Lambda = \pi/a$ of discrete space–time ($E_{\gamma, \nu} \ll \Lambda$), the Lorentz symmetry is restored, energy–momentum relations $E_{\gamma} = k$ and $E_{\nu} \approx k \gg m_{\nu}$, velocities $v_{\gamma} = 1$ and $v_{\nu} < 1$.

Assuming a more complex discrete space–time, inspired by Eqs. (4), (8) one may parameterize energy–momentum relations as follow

$$E_{\gamma}^2 = k^2 \big[1 - (k/\Lambda_{\gamma})^{\alpha_{\gamma}} \big], \qquad (10)$$

$$E_{\nu}^2 = m_{\nu}^2 + k^2 \big[1 + (k/\Lambda_{\nu})^{\alpha_{\nu}} \big] \qquad (11)$$

where parameters are indexes $\alpha_{\gamma, \nu} = 1, 2, \ldots$, and the characteristic scales $\Lambda_{\gamma, \nu}$ of Lorentz symmetry breaking. Eqs. (10) and (11) are reminiscent of phenomenological energy–momentum relations adopted for studying arrival time variations of high-energy cosmic particles due to Lorentz symmetry breaking [10]. Suppose that $\alpha_{\nu} = 2$, $\Lambda_{\nu, \gamma} \sim \Lambda_{\text{pl}} \simeq 10^{19}$ GeV the Planck scale due to quantum gravity and neutrino energy (mass) $E_{\nu} \approx 17$ GeV ($m_{\nu} \sim 1$ eV), neutrino velocity-variation is negligibly small, as given by Eq. (9), because of Planck-scale suppression. The second term due to neutrino mass $-(m_{\nu}/E_{\nu})^2 \sim -10^{-20}$ and last term due to discrete space–time gives $+10^{-36}$, as a result the neutrino velocity-variation $(v_{\nu} - c)/c \approx -10^{-20} + 10^{-36} \lesssim 0$, and the neutrino velocity is smaller than the speed of light ($v_{\nu} \lesssim c$). Analogously, Eq. (5) gives very small velocity-variation of photons $(v_{\gamma} - c)/c \approx -10^{-36} \lesssim 0$ due to discrete space–time. In order to observe such small effect of Lorentz symmetry breaking due to quantum gravity, one is bound to detect variations of arrival times of high-energy gamma ray and neutrinos from astrophysical sources at cosmological distances.

Instead of small-ka expansions (8) and (9), using Eqs. (3) and (7), we calculate photon velocity dE_{γ}/dk and neutrino velocity dE_{ν}/dk, which are plotted in Fig. 1. We find that (i) the photon velocity $v_{\gamma} < 1$ for any value of ka; (ii) the neutrino velocity $v_{\nu} > 1$ for $k \in [k_{\min}, k_{\max}]$; (iii) $v_{\nu} < 1$ for $k \notin [k_{\min}, k_{\max}]$. We approximately obtain

$$k_{\min} a \approx (am_{\nu})^{1/2} \big(2^{3/4}/B^{1/2} \big);$$
$$k_{\max} a \approx 2 \arcsin(1 + 2B/10)^{1/2}. \qquad (12)$$

The recent OPERA result shows $(v_{\nu} - c)/c \approx 2 \times 10^{-5}$ for average neutrino energy $\langle E_{\nu} \rangle \approx 17$ GeV [1], and this neutrino velocity anomaly roughly occurs in the energy range (15 GeV–45 GeV). Using Eq. (11), we find $\Lambda_{\nu} \sim 10^3$ GeV, which is much smaller than the Planck scale Λ_{pl}. This is indeed surprising, because we do not have any compelling reason to introduce at this energy scale either an explicit chiral-symmetry breaking or a spontaneous chiral-symmetry breaking with new Goldstone bosons for violations of chiral gauge and Lorentz symmetries. This implies that the OPERA result would be very important to indicate new physics and this is also one of reasons to call further confirmations of the OPERA result. Nevertheless, we start to revisit a theoretical scenario, 15 years old, to view the OPERA result, high-energy neutrino oscillations and indicate to observe the restoration of parity conservation
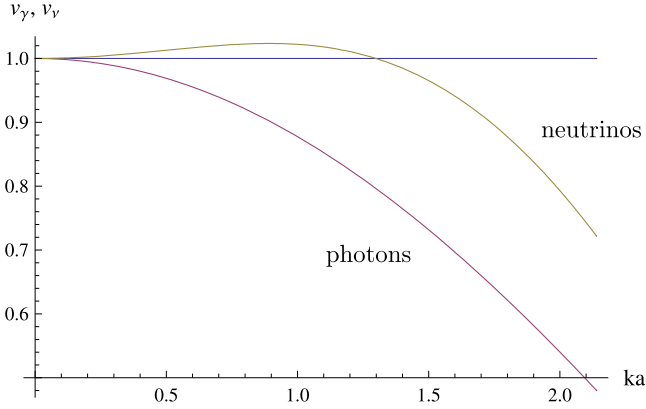
Fig. 1. The photon and neutrino velocities in a discrete space–time are plotted as functions of ka, and $k_{max}a = 1.3$ for $B = (3/2)^{1/2} = 1.225$.

by measuring the asymmetry of high-energy neutrinos colliding with left- and right-handed polarized electrons.

3. Four-neutrino interactions

As discussed in the introductory paragraph, in order to preserve chiral symmetries of Lagrangian in a discrete space–time, we introduced a chiral-symmetric interaction of four neutrino fields [20]

$$\frac{g}{4a^2} \sum_{x,i} \bar{\psi}_L^i(x) \Delta \psi_R(x) \Delta \bar{\psi}_R(x) \psi_L^i(x),$$ (13)

where $SU_L(2)$ doublets $\psi_L^i(x) = (\nu_i, i)$ carry lepton flavors indicated by "$i = e, \mu, \tau$", $\psi_R(x)$ a unique gauge singlet, and strong four-neutrino coupling $g \gg 1$. The Ward identity of the shift-symmetry $\psi_R(x) \to \psi_R(x) + \text{const.}$ leads to the one-particle irreducible vertex of four-neutrino interacting,

$$\Gamma^{(4)} = gw(p + q/2)w(p' + q/2),$$ (14)

where $p + q/2$ and $p' + q/2$ are the momenta of the ψ_R field; $p - q/2$ and $p' - q/2$ are the momenta of the ψ_L^i field (q is the momentum transfer). In high energies, namely large neutrino energy and energy transfer, $\Gamma^{(4)}$ is so large that bound states of neutrinos, $\Psi_R^i \sim (\bar{\psi}_R \psi_L^i)\psi_R$, are form and carry opposite chirality of left-handed neutrinos ψ_L^i. Namely, Ψ_R^i is a right-handed $SU_L(2)$ doublet. Left-handed neutrino ψ_L^i and right-handed composite neutrino Ψ_R^i couple to intermediate gauge bosons in the same way, as a consequence the parity symmetry is restored. In addition, ψ_L^i combines with Ψ_R^i to form a Dirac neutrino $\Psi_D^i = (\psi_L^i, \Psi_R^i)$, whose inverse propagator obtained by the method of strong coupling expansion in terms of $(1/g)$ [20,21]

$$S_{ij}^{-1}(k) = \delta_{ij} \frac{i}{a} \gamma_\mu \sin(k_\mu a) + \delta_{ij} M(k),$$ (15)

where we neglect small neutrino masses m_ν and

$$M(k) = \frac{g}{a} w(k),$$ (16)

is chiral-gauge-symmetric masses of composite Dirac neutrinos.[1] In the light of Occam's razor, we assume that four-neutrino coupling g is independent of lepton flavors so that the spectrum (15) is

diagonal in the flavor space. Instead, normal neutrino mass (m_ν) matrix is not diagonal.

In low energies, namely small neutrino energy and energy transfer, $\Gamma^{(4)}$ becomes so small that the binding energies $E_{bind}(g, a)$ of bound states of three neutrinos vanishes, as a result bound states dissolve into their constituents[2] and the mass term $M(k)$ (16) vanishes, that we call dissolving phenomenon. Therefore, neutrino family contains three left-handed neutrinos ν_e, ν_μ, ν_τ and the parity symmetry is violated as described by the standard model,[3] in addition to a right-handed neutrino $\psi_R = \nu_R$.[4]

However, to quantitatively show such dissolving phenomenon in low-energies by using Eqs. (15) and (16) obtained in high-energies, one is bound to find a ultra-violet fix point and renormalization group equation in the neighborhood of ultra-violet fix point, where the dimension-10 interaction (13) receives anomalous dimensions and becomes renormalizable dimension-4 operator. This is a complicate and difficult issue and needs non-perturbative calculations. Nevertheless, in Refs. [20,22], we assumed a characteristic energy threshold "$\mathcal{E}$" at which the dissolving phenomenon occurs and postulated that the energy threshold $\mathcal{E}$ be much larger than electroweak symmetry breaking scale $\Lambda_{ew} \sim 250$ GeV,

$$\Lambda_{ew} \ll \mathcal{E} < \Lambda_{pl}.$$ (17)

The energy threshold $\mathcal{E} = \mathcal{E}(g, a)$ also characterizes the energy scale of Lorentz symmetry breaking due to non-vanishing mass term $M(k)$ (16). In this Letter, in order to obtain an effective energy–momentum relation in low-energy, we further assume that as the four-neutrino coupling $g(a)$ is running with a the lattice constant, an effective four-neutrino coupling $G_X(g, a) \equiv a\mathcal{E}g(a)$ follows renormalization group equation in the neighborhood of ultra-violet fix point. In consequence, Eq. (15) gives the energy–momentum relation

$$E_\nu^2 = m_\nu^2 + \frac{\sin^2 ka}{a^2} + \frac{(2G_X)^2}{a^2} \sin^4(ka/2),$$ (18)

and for $ka \ll 1$

$$E_\nu^2 \simeq k^2 + m_\nu^2 + \left[(2G_X)^2 - \frac{1}{3} \right] k^4 a^2 + \cdots,$$ (19)

where we replace the four-neutrino coupling g by the effective one G_X. From Eq. (19) and $(2G_X)^2 \gg 1/3$, the velocity of high-energy neutrinos is given by

$$v_\nu = \frac{dE_\nu}{dk} \approx 1 - \frac{1}{2}\left(\frac{m_\nu}{E_\nu}\right)^2 + (2G_X)^2(ka)^2 + \cdots$$

$$\approx 1 - \frac{1}{2}\left(\frac{m_\nu}{E_\nu}\right)^2 + \left(\frac{E_\nu}{\bar{\mathcal{E}}}\right)^2 + \cdots,$$ (20)

where $\bar{\mathcal{E}} = \mathcal{E}/g$, $k^2 \gg m_\nu^2$ and $k \approx E_\nu$. Eq. (20) implies that high-energy neutrino velocity $v_\nu > 1$, provided the composite Dirac neutrino (15) is formed, the energy-threshold and neutrino energy obey $2^{1/2}E_\nu/\bar{\mathcal{E}} > m_\nu/E_\nu$. Low-energy neutrino velocity $v_\nu < 1$, for

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[1] In Refs. [20,21], we also obtained neutral composite Dirac neutrinos $\Psi_D^n = (\Psi_L^n, \psi_R)$ (gauge singlet), where $\Psi_L^n \sim (\bar{\psi}_L^i \psi_R)\psi_L^i$. These states can be candidates for cold dark matter, for being very massive and very weak interacting [22].

[2] Bound states of three neutrinos are presented as poles of four-neutrino interacting S-matrix, locating at $E_{bind}(g, a) < 0$ in the energy–momentum plane. When the coupling g becomes smaller, $E_{bind} \to 0^-$, these poles of the four-neutrino interacting S-matrix become cuts, representing three elementary neutrino states.

[3] In the phase transition of neutrino bound states $\Psi_R \sim (\bar{\psi}_R \psi)\psi_R$ formation (dissolution) from (into) their constitutes, chiral symmetries are exactly preserved, and realized by different chiral fermion (neutrino) contents [20]. It should be pointed out that this phenomenon is not a spontaneous symmetry breaking and no Goldstone bosons are produced [28].

[4] In Refs. [20,22], it can be found that the Ward identity of the shift-symmetry $\nu_{R,L}(x) \to \nu_{R,L}(x) + \text{const.}$ requires the right(left)-handed neutrino $\nu_{R,L}$ to be free particles that could be candidates of warm dark matter.
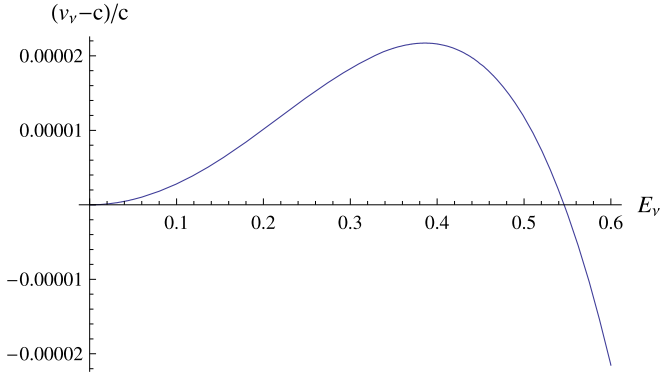
Fig. 2. Based on Eq. (21), the neutrino velocity $v_\nu = dE_\nu/dk$ is numerically calculated for $(2G_X)^2 = 5.35$ and the neutrino velocity variation $(v_\nu - c)/c$ is plotted as a function of the neutrino energy $E_\nu \approx k_\nu$ in unit of the energy-scale $\bar{\mathcal{E}} = \mathcal{E}/g$.

the reason that the composite Dirac neutrino (15) is not formed, the mass term $M(k)$ (16) vanishes and the third term in Eq. (20) is absent. As well known, at much lower energy, in the 10 MeV range, a stringent limit of $|v - c|/c < 2 \times 10^{-9}$ was set by the observation of (anti)neutrinos emitted by the SN1987A supernova [23].

In unit of the energy-scale $\bar{\mathcal{E}}$, we rewrite the energy–momentum relation (18),

$$E_\nu^2 = m_\nu^2 + (2G_X)^2 \sin^2(k/2G_X) + (2G_X)^4 \sin^4(k/4G_X), \qquad (21)$$

and the chiral symmetric masses (16)

$$M(k) = (2G_X)^2 \sin^2(k/4G_X). \qquad (22)$$

Using Eq. (21), we numerically calculate the neutrino velocity $v_\nu = dE_\nu/dk$ and plot the neutrino velocity-variation $(v_\nu - c)/c$ in Fig. 2. Numerical calculations show a critical value $G_X^{\text{crit}} \approx 1.15$ $[(2G_X^{\text{crit}})^2 = 5.335]$, above which $G_X > G_X^{\text{crit}}$, it appears an intermediate energy-range $[E_\nu^{\min}, E_\nu^{\max}]$ where neutrino velocity-variation $(v_\nu - c)/c > 0$. This energy-range and velocity-variation depend on three parameters G_X, $\mathcal{E}$ and g. Current high-energy neutrino experiments might start to gain an insight into this theoretical scenario.

We turn to the discussions of high-energy neutrino oscillations in this scenario. For a two-flavor system, considering Eq. (15) and normal neutrino masses m_ν, the Hamiltonian in the base of flavor eigenstates is given by

$$\mathcal{H}_{\text{flavor}} = E_\nu + \frac{m_1^2 + m_2^2}{4E_\nu} + \left(\frac{\Delta m_{12}^2}{4E_\nu}\right) \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$
$$+ \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}, \qquad (23)$$

where $\Delta m_{12}^2 = m_2^2 - m_1^2$, the first part is a unitary transformation of the Hamiltonian in the base of mass eigenstates

$$\mathcal{H}_{\text{mass}} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \simeq E_\nu + \begin{pmatrix} m_1^2/2E_\nu & 0 \\ 0 & m_2^2/2E_\nu \end{pmatrix}, \qquad (24)$$

and the leading contribution to neutrino energy $E_{1,2}$ is obtained by assuming $k_1 \approx k_2 \approx E_\nu$, and $k_{1,2}a \ll 0$; the unitary transformation U and mixing angle θ are given by

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad \tan 2\theta = \frac{2\hat{\mathcal{H}}_{12}}{\hat{\mathcal{H}}_{22} - \hat{\mathcal{H}}_{11}}. \qquad (25)$$

Note that four-neutrino coupling g and chiral symmetric mass M (16) do not depends on lepton flavors, otherwise the missing angle

θ would depend on neutrino energy [24]. As a result, the energy-difference between neutrino flavors $E_\nu^i - E_\nu^j = \Delta m_{12}^2/E_\nu$ and the probability of two flavor neutrino oscillation,

$$P_{i \leftrightarrow j}(L) \approx \sin^2 2\theta \sin^2[(E_\nu^i - E_\nu^j)L], \qquad (26)$$

where L distance neutrino travels.

In addition, the parity symmetry should be restored when the composite Dirac neutrino (15) is formed above the energy-threshold $(\mathcal{E})$, as already mentioned. For example, the effective one-particle irreducible interacting vertex between neutrino (p), electron (p') and $W^\pm$ gauge boson $(q = p' - p)$ [22],

$$\Gamma_\mu^{ij}(p, p') = i\frac{g_2}{2\sqrt{2}}U_{ij}\gamma_\mu[P_L + f(p, p')] \qquad (27)$$

where g_2 is the $SU_L(2)$ coupling, U_{ij} the CKM matrix, and left-handed projector $P_L = (1 - \gamma_5)/2$. The vector-like (parity conserving) form factor $f(p, p')$ is related to the chiral-symmetric mass (16) by the Ward identity of chiral gauge symmetries. In high-energies, $f(p, p') \neq 0$ for $p, p' \geqslant \mathcal{E}$, and chiral-gauge coupling becomes vector-like. Similar discussions and calculations for chiral-gauge coupling to the neutral gauge boson Z^0 can be found in Ref. [22]. In consequence, the parity symmetry is restored. This could be experimentally checked by measuring the left–right asymmetry

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \qquad (28)$$

where σ_L (σ_R) is the cross-section of high-energy neutrino colliding with left-handed (right-handed) polarized electron. We expect the restoration of parity symmetry, $A_{LR} \to 0$, for high-energy neutrinos and electrons. In low-energies, bound states of neutrinos dissolve into their constituents, the vector-like form factor $f(p, p')$ in Eq. (27),

$$f(p, p')\big|_{p, p' \to \mathcal{E} + 0^+} \to 0, \qquad f(p, p')\big|_{p, p' < \mathcal{E}} = 0, \qquad (29)$$

which leads to the parity-violating gauge-coupling (P_L-term in Eq. (27)), and $A_{LR} = 1$ as measured in low-energy experiments. All these features could possibly be verified or falsified by current high-energy neutrinos experiments.

4. Some remarks

We end this Letter by making some remarks. Due to quantum gravity, the very-small-scale structure of space–time and high-dimensional operators of fermion-interactions must be much more complex.[5] Because it is difficult to have a complete theory at the Planck scale and carry out non-perturbative calculations leading to observables at low-energy scale. In our theoretical scenario, we adopt the simplest space–time discretization and four-neutrino interaction (13), leading to a phenomenological model in low-energies. The recent experiment for high-energy neutrino [1], if it is further confirmed, might begin to shed light on this most elusive and fascinating arena of fundamental particle physics.

In a recent paper [25], based on the radiative correction to an electron propagator via exchanging a W-boson within the standard model, authors show

$$\delta v_e = g_{\text{su}2}^2 \int \frac{d^4k}{(4\pi)^4} \frac{\delta v_\nu(k)}{(k^2)[(k+p)^2 - M_W^2]}$$
$$\gtrsim \left(\frac{E_{\text{OPERA}}}{(4\pi)\Lambda_{\text{ew}}}\right)^2 \delta c_\nu(E_{\text{OPERA}}) \sim 10^{-9}, \qquad (30)$$

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[5] In the Einstein–Cartan theory, four-fermion interactions are generated by integrating over static torsion fields, see for example, Refs. [21,29].

for the OPERA result $\delta v_\nu(E_{OPERA}) \sim 10^{-5}$ at $E_{OPERA} \sim 20$ GeV. Eq. (30) is clearly inconsistent with the experimental bound on electron velocity-variation $\delta v_e < 10^{-14}$. This is indeed a problem that implies two possibilities either the OPERA result is wrong or the standard model needs some modifications. In the scenario we present, high-energy neutrino acquires a large chiral-symmetric mass M (16) and gauge-coupling to W-bosons is no longer purely left-handed (27). Therefore, (i) the radiative correction to an electron propagator contains not only kinetic term, but also mass term, (ii) neutrino propagator $\sim 1/(k^2 + M^2)$, this can give a suppression factor $(k/M)^2$ in the right-handed side of Eq. (30). It looks a possibility to get around this problem. In future work, one needs to do some detailed calculations of radiative corrections to the propagators of neutrinos and charged leptons in the scenario presented in this Letter.

In another recent paper [26], based on the standard model, authors discuss the Cherenkov radiation of superluminal neutrinos via the process $\nu \to \nu + e^+ + e^-$ with the energy-threshold $E_0 = 2m_e/(v_\nu^2 - v_e^2)^{1/2}$. The OPERA result $\delta v_\nu \approx 10^{-5}$ and $\delta v_e < 10^{-14}$ lead to $E_0 = 140$ MeV. This indicates that it is impossible to observe 17–45 GeV superluminal neutrinos, because of energy-lost via Cherenkov radiation. This again implies that either the OPERA result is wrong or the standard model needs some modifications. In this scenario, in addition to the four-fermion interaction (13) for the set of left-handed $SU_L(2)$-doublets, there is an analogous four-fermion interaction for right-handed $SU_L(2)$-singlets [20,22],

$$\frac{g}{4a^2} \sum_{x,i} \bar{\psi}_R^i(x) \Delta \nu_L(x) \Delta \bar{\nu}_L(x) \psi_R^i(x), \tag{31}$$

where singlet $\psi_R^i = e_R, \mu_R, \tau_R$ and ν_L is the left-handed counterpart of ν_R (see footnote 4). Analogously, strong coupling ($g \gg 1$) leads to form left-handed bound states (singlets), for example $(\bar{\nu}_L \cdot e_R)\nu_L$, which combines with e_R to form a composite Dirac particle $[e_R, (\bar{\nu}_L \cdot e_R)\nu_L]$ with a large chiral-gauge invariant mass M (16). In this case, electron mass m_e should be replaced by the mass M in the energy-threshold $E_0 = 2M/(v_\nu^2 - v_e^2)^{1/2}$, which becomes much larger than 140 MeV for $M \gg m_e$. This may provide a possibility that the OPERA result is not in contradiction to energy-lost via Cherenkov radiation. In future work, some detailed analysis of Cherenkov radiation of superluminal neutrinos should be carried out in the scenario presented in this Letter. We would like to mention some early papers on superluminal neutrinos as tachyons [27]. At the present situation, we are facing two possibilities: (i) the OPERA result is confirmed to be wrong; (ii) the OPERA result is confirmed to be correct and further neutrino experiments should be proposed to indicate what modifications standard model needs at high-energies.

Acknowledgements

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