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General CPT-even dimension-five nonminimal couplings between fermions and photons yielding EDM and MDM

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ABSTRACT

In this letter, we examine a new class of CPT-even nonminimal interactions, between fermions and photons, deprived of higher order derivatives, that yields electric dipole moment (EDM) and magnetic dipole moment (MDM) in the context of the Dirac equation. The couplings are dimension-five CPT-even and Lorentz-violating nonminimal structures, composed of a rank-2 tensor, $T_{\mu\nu}$, the electromagnetic tensor, and gamma matrices, being addressed in its axial and non-axial Hermitian versions, and also comprising general possibilities. We then use the electron's anomalous magnetic dipole moment and electron electric dipole moment measurements to reach upper bounds of 1 part in 10^{20} and 10^{25} $(\mathrm{eV})^{-1}$.

1. Introduction

The Standard Model (SM) structure allows for C, P and T violations (and combinations) as long as the CPT symmetry is kept unharmed. Concerning these symmetries, it is fundamental to test their validity in any possible way. Among the most important tests is the search for the electric dipole moment (EDM). Its interaction term, in a nonrelativistic formulation, has the form $d(\boldsymbol{\sigma}\cdot\mathbf{E})$, in which $\mathbf{E}$ is the electric field, $\boldsymbol{\sigma}$, the spin operator and d, the modulus of the electric dipole moment (EDM). This interaction violates both P and T symmetries, $P(\boldsymbol{\sigma}\cdot\mathbf{E}) \to -(\boldsymbol{\sigma}\cdot\mathbf{E})$, $T(\boldsymbol{\sigma}\cdot\mathbf{E}) \to -(\boldsymbol{\sigma}\cdot\mathbf{E})$, but preserves C, so the CPT symmetry is not lost. The EDM magnitude d, according to the SM [1,2], is $\approx 10^{-38}$ e cm, while the experimental measurements have been improved [3,4], reaching the level $\approx 10^{-31}$ e cm very recently [5]. The gap between the experimental landmark and the theoretical prediction by a factor 10^7 may look discouraging, but it also means that any detection above the SM prediction could indicate New Physics, that is, more sources of CP violation. All this could play an interesting role in explaining the matter-antimatter asymmetry, as the connection of axions and the strong CP problem [6]. By another route, EDM experimental data can be used to set stringent bounds on theories that predict this kind of effect.

The SM electrodynamics can be provided with EDM by introducing the term $id(\bar{\psi}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\psi)$ [7–9], where ψ represents a Dirac spinor. However, there are other mechanisms for introducing EDMs on the SM framework. One of these is to consider background fields, which interact with the spinor and electromagnetic fields via nonminimal couplings. These background fields induce preferred directions in spacetime, violating the Lorentz symmetry but not necessarily harming CPT; a few scenarios involving breaking of Lorentz or CPT symmetries are discussed in Refs. [10,11]. In Ref. [12], the possible generation of EDM by several Lorentz-violating (LV) dimension-5 interaction terms $(c^{\nu}\bar{\psi}\gamma^{\mu}F_{\mu\nu}\psi,\ d^{\nu}\bar{\psi}\gamma^{\mu}\gamma_5 F_{\mu\nu}\psi,\ f^{\nu}\bar{\psi}\gamma^{\mu}\tilde{F}_{\mu\nu}\psi,\ g^{\nu}\bar{\psi}\gamma^{\mu}\gamma_5\tilde{F}_{\mu\nu}\psi)$ was pondered and used as a key factor for constraining their respective magnitudes.

These CPT-odd terms were also studied in Ref. [13], regarding their contribution to the anomalous magnetic moment (MDM). In this work, it was performed an analysis involving the splitting of the g factors of a fermion and an antifermion, and bounds were set. As a matter of fact, investigations concerning the muon's anomalous MDM have shown that its experimental value already deviates from the QED prediction [14–16]. Since muon-related experiments are about 400 times more sensitive to New Physics behavior (and probe mass scales around 20 times higher), these searches motivate the proposal of alternative theories [9], including the ones that allow for Lorentz and CPT violations. In this context, there are investigations about LV effects on the muon MDM [17] and on the neutron EDM [18]. CPT-even and LV one-loop contributions to lepton EDM, induced by the SME fermion term, $d_{\mu\nu}\bar{\psi}\gamma_5\gamma^{\mu}\psi$, were carried out in Ref. [19]. CPT-odd LV effects on lepton EDM were addressed in Ref. [20].

The investigation of Lorentz symmetry violation is indeed a rich line of research, much developed in the framework of the Standard

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Model extension (SME) [21–26], whose developments have scrutinized the Lorentz-violating effects in distinct physical systems and served to state tight upper bounds on the LV coefficients, including photon-fermion interactions [27] and electroweak processes [28,29]. Beyond the minimal SME, there is its nonminimal extension encompassing nonminimal couplings with higher-order derivatives [30]. Other models comprising higher-dimension operators [31] and higher derivatives [32] have also been taken forward.

It is worth to mention that the CPT-odd term $g^{\nu}\bar{\psi}\gamma^{\mu}\gamma_5\tilde{F}_{\mu\nu}\psi$ was first proposed in Ref. [33] by means of the nonminimal derivative, $D_{\mu}=\partial_{\mu}+ieA_{\mu}+i\frac{\lambda}{2}\epsilon_{\mu\lambda\alpha\beta}g^{\lambda}F^{\alpha\beta}$, defined in the context of the Dirac equation, $(i\gamma^{\mu}D_{\mu}-m)\Psi=0$, where g^{μ} can be identified with the Carroll–Field–Jackiw four-vector, $(k_{AF})^{\mu}=(\mathrm{v}_0,\mathbf{v})$, and λ is the coupling constant. This coupling has been studied in numerous aspects [34–36], including the radiative generation of CPT-odd LV terms [37]. See also Ref. [38] and the references therein.

We have recently investigated a dimension-five CPT-even nonminimal coupling [39], and its axial version [40], in the context of the Dirac equation, implemented by means of the extended covariant derivatives,

$$D_{\mu}=\partial_{\mu}+ieA_{\mu}+\frac{\lambda}{2}(K_F)_{\mu\nu\alpha\beta}\gamma^{\nu}F^{\alpha\beta}, \tag{1}$$

$$D_{\mu}=\partial_{\mu}+ieA_{\mu}+i\frac{\lambda_A}{2}(K_F)_{\mu\nu\alpha\beta}\gamma_5\gamma^{\nu}F^{\alpha\beta}, \tag{2}$$

where $(K_F)_{\mu\nu\alpha\beta}$ is the CPT-even tensor of the SME electrodynamics. They are associated with the CPT-even dimension-five couplings deprived of higher derivatives,

$$\begin{aligned}&\lambda\overline{\Psi}(K_F)_{\mu\nu\alpha\beta}\sigma^{\mu\nu}F^{\alpha\beta}\Psi,\\&\lambda_A\overline{\Psi}(K_F)_{\mu\nu\alpha\beta}\gamma_5\sigma^{\mu\nu}F^{\alpha\beta}\Psi,\end{aligned} \tag{3}$$

which are not contained in the broader nonminimal extension of the SME developed in Refs. [30].

In the Dirac equation, these couplings provide a nonrelativistic Hamiltonian endowed with contributions to the EDM and to the MDM, which rendered upper bounds on the LV parameters at the level of 1 part in 10^{20} $(\mathrm{eV})^{-1}$ and 1 part in 10^{24} $(\mathrm{eV})^{-1}$, respectively. Related studies arguing the generation of topological phases have been reported as well [41].

In this letter, we propose some general dimension-five CPT-even nonminimal couplings, composed of a rank-2 LV tensor, $T_{\mu\nu}$, in the context of the Dirac equation, and also not contained in the nonminimal SME extension proposed in Ref. [30]. Initially, the following extensions in the covariant derivative are conceived,

$$D_{\mu}=\partial_{\mu}+ieA_{\mu}+i\lambda_1T_{\mu\nu}F^{\nu\beta}\Gamma_{\beta}, \tag{4}$$

$$D_{\mu}=\partial_{\mu}+ieA_{\mu}+i\lambda_1T_{\mu\nu}F_{\alpha\beta}\gamma^{\nu}\Gamma^{\alpha\beta}, \tag{5}$$

where $\Gamma_{\beta}=\gamma_{\beta},\gamma_{\beta}\gamma_5$ stand for the axial and non-axial forms, and $\Gamma^{\alpha\beta}=\sigma^{\alpha\beta}$. These terms engender contributions to the Dirac Lagrangian, which are reorganized in such a way to be Hermitian. Table 1 contains four types of possible extensions, followed by their respective Hermitian versions.

After presenting each case and accessing the nonrelativistic regime of the resulting Hermitian Dirac equation, we use the electron's MDM and EDM data to impose limits on the magnitude of the nonminimal LV terms at the level of until 1 part in 10^{25} $(\mathrm{eV})^{-1}$ or 1 part in 10^{16} $(\mathrm{GeV})^{-1}$.

2. An axial and non-axial CPT-even LV nonminimal coupling in the Dirac equation

In the context of the Dirac equation, we examine general CPT-even nonminimal couplings not composed of higher derivative orders, representing scenarios where an unusual electromagnetic interaction between fermions and photons is induced by a fixed rank-2 tensor, without any symmetry (in principle).

Table 1
EDM and MDM contributions arisen from possible LV extensions and their respective Hermitian versions. Here, "*yes*" means the conditional possibility of measuring EDM or MDM in a non-conventional set up.

Coupling	Hermitian	*EDM*	*MDM*
$\lambda_1\bar{\Psi}T_{\mu\nu}F^{\nu\beta}\gamma^{\mu}\gamma_{\beta}\gamma^5\Psi$	*no*	–	–
$i\lambda_1\bar{\Psi}T_{\mu\nu}F^{\nu}{}_{\beta}\sigma^{\mu\beta}\gamma^5\Psi$	*yes*	*yes*	"*yes*"
$\lambda_1'\bar{\Psi}T_{\mu\nu}F^{\nu}_{\ \beta}\gamma^{\mu}\gamma^{\beta}\Psi$	*no*	–	–
$\lambda_1'\bar{\Psi}T_{\mu\nu}F^{\nu}{}_{\beta}\sigma^{\mu\beta}\Psi$	*yes*	"*yes*"	*yes*
$\lambda_3\bar{\Psi}T_{\alpha\nu}F_{\mu\beta}\gamma^{\mu}\gamma^{\beta}\gamma^{\alpha}\gamma^{\nu}\Psi$	*no*	–	–
$\lambda_3\bar{\Psi}\left(T_{\alpha\nu}F_{\mu\beta}+T_{\mu\beta}F_{\alpha\nu}\right)\sigma^{\mu\beta}\sigma^{\alpha\nu}\Psi$	*yes*	*no*	*no*
$\lambda_4\bar{\Psi}\left(T_{\alpha\nu}F_{\mu\beta}-T_{\mu\beta}F_{\alpha\nu}\right)\sigma^{\mu\beta}\sigma^{\alpha\nu}\Psi$	*yes*	"*yes*"	"*yes*"

2.1. The axial nonminimal coupling

The starting point is the proposal of a general CPT-even nonminimal term in the form of an extension in the covariant derivative,

$$D_{\mu}=\partial_{\mu}+ieA_{\mu}+i\lambda_1T_{\mu\nu}F^{\nu\beta}\gamma^5\gamma_{\beta}, \tag{6}$$

with $T_{\mu\nu}$ being a rank-2 arbitrary, dimensionless, LV tensor, in principle without any symmetry. It generates a dimension-five operator, coupling the fields in this modified quantum electrodynamics, i.e., $\lambda_1\overline{\Psi}T_{\mu\nu}F^{\nu\beta}\gamma^{\mu}\gamma^5\gamma_{\beta}\Psi$. The coupling constant λ_1 is a real parameter with mass-dimension equals to -1. In order to yield a Hermitian contribution, one should consider the new nonminimal covariant derivative as

$$D_{\mu}=\partial_{\mu}+ieA_{\mu}-\frac{i}{2}\lambda_1\left(T_{\mu\nu}F^{\nu\beta}-T_{\beta\nu}F^{\nu}{}_{\mu}\right)\gamma_{\beta}\gamma^5. \tag{7}$$

Accordingly, the modified Dirac equation reads,

$$\left[i\gamma^{\mu}\partial_{\mu}-e\gamma^{\mu}A_{\mu}-i\lambda_1T_{\mu\nu}F^{\nu}{}_{\beta}\sigma^{\mu\beta}\gamma^5-m\right]\Psi=0, \tag{8}$$

with $i\lambda_1\overline{\Psi}T_{\mu\nu}F^{\nu}{}_{\beta}\sigma^{\mu\beta}\gamma^5\Psi$ representing the corresponding LV interaction between spinors and the electromagnetic field.

Here, $F_{0j}=E^j$, $F_{mn}=\epsilon_{mnp}B_p$, while $\sigma^{0j}=i\alpha^j$, $\sigma^{ij}=\epsilon_{ijk}\Sigma^k$, with the Dirac matrices given as

$$\alpha^i=\begin{pmatrix}0&\sigma^i\\\sigma^i&0\end{pmatrix},\quad\Sigma^k=\begin{pmatrix}\sigma^k&0\\0&\sigma^k\end{pmatrix},\quad\gamma^5=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$

in which σ^i stands for the Pauli matrices. In the momentum coordinates, $i\partial_{\mu}\to p_{\mu}$, the corresponding Dirac equation can be written as

$$i\partial_t\Psi=\left[\boldsymbol{\alpha}\cdot\boldsymbol{\pi}+eA_0+m\gamma^0+H_{\mathrm{LV}}\right]\Psi, \tag{9}$$

where $\boldsymbol{\pi}=(\mathbf{p}-\mathbf{eA})$ is the canonical momentum and

$$H_{\mathrm{LV}}=i\lambda_1T_{\mu\nu}F^{\nu}{}_{\beta}\gamma^0\sigma^{\mu\beta}\gamma^5, \tag{10}$$

is the LV piece, which, inserted in the Dirac equation (9), implies

$$\begin{aligned}i\partial_t\Psi=\big[&\boldsymbol{\alpha}\cdot\boldsymbol{\pi}+eA_0+m\gamma^0\\&-\lambda_1T_{00}E^j\gamma^0\Sigma^j+\lambda_1T_{ij}E^i\gamma^0\Sigma^j\\&-\lambda_1\epsilon_{ijk}T_{0i}B^k\gamma^0\Sigma^j+i\lambda_1\epsilon_{jki}T_{j0}E^k\gamma^i\\&+i\lambda_1TB^k\gamma^k-i\lambda_1T_{jk}B^j\gamma^k)\big]\Psi,\end{aligned} \tag{11}$$

where $T=\mathrm{Tr}(T_{ij})$. The presence of the matrix, γ^0, in the terms $T_{00}\gamma^0E^i\Sigma^i$, $T_{ij}\gamma^0E^j\Sigma^i$, indicates effective induction of EDM by electromagnetic interaction, once it will contribute to the energy of the system, circumventing the Schiff's theorem (see Refs. [9,40]). As a consequence, one can use the electron EDM data to constrain the magnitude of the coefficients T_{00}, T_{ij}.

In order to investigate the role played by this nonminimal coupling, we should evaluate the nonrelativistic limit of the Dirac equation. We begin by writing the LV Hamiltonian of Eq. (9) in a matrix form,

$$H_{\rm LV}=\begin{pmatrix} H_{11} & -H_{12} \\ H_{12} & -H_{11}\end{pmatrix}, \tag{12}$$

with

$$H_{11}=\lambda_1(-T_{00}\,(\boldsymbol{\sigma}\cdot\boldsymbol{E})-T_{0i}(\boldsymbol{\sigma}\times\boldsymbol{B})^i+T_{ij}E^j\sigma^i), \tag{13}$$

$$H_{12}=i\lambda_1(-\epsilon_{ijk}T_{i0}\sigma^kE^j-T\,(\boldsymbol{\sigma}\cdot\boldsymbol{B})+T_{ij}\sigma^jB^i). \tag{14}$$

Labeling the small (χ) and large (ψ) components of the spinor Ψ, the Dirac equation (9) provides two 2-component equations,

$$[E-eA_0-m-H_{11}]\psi-[\boldsymbol{\sigma}\cdot\boldsymbol{\pi}-H_{12}]\chi=0, \tag{15}$$

$$[\boldsymbol{\sigma}\cdot\boldsymbol{\pi}+H_{12}]\psi-[E-eA_0+m+H_{11}]\chi=0. \tag{16}$$

After some algebraic evaluations, at first order in the Lorentz violating parameters, the nonrelativistic Hamiltonian (for uniform fields) is

$$\begin{aligned}H_{\rm NR}&=\frac{1}{2m}\left[\boldsymbol{\pi}^2-e\boldsymbol{\sigma}\cdot\boldsymbol{B}\right]+eA_0\\&\quad-\lambda_1\left(T_{00}\boldsymbol{\sigma}\cdot\boldsymbol{E}+T_{0i}(\boldsymbol{\sigma}\times\boldsymbol{B})^i-T_{ij}E^j\sigma^i\right)\\&\quad+\frac{1}{2m}\left[(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})H_{12}-H_{12}(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\right].\end{aligned} \tag{17}$$

Examining this Hamiltonian, we notice that it is able to generate electric dipole moment for the electron at tree level, by the terms $\lambda_1T_{00}\,(\boldsymbol{\sigma}\cdot\boldsymbol{E})$, $\lambda_1T_{ij}E^j\sigma^i$, but no anomalous magnetic moment. The EDM terms could also be seen as generator of a kind of electric Zeeman effect.

It is interesting to mention that only the symmetric part of the tensor $T_{\mu\nu}$ is able to induce EDM, once the antisymmetric part has null main diagonal, avoiding the appearance of a term containing $(\boldsymbol{\sigma}\cdot\boldsymbol{E})$ in $H_{\rm NR}$.

We also discuss the behavior of the modified Dirac equation (8) and (9) under the discrete symmetries C, P, T. Table 1 displays the response of the elements λT_{00}, λT_{0i}, λT_{ij} under the C, P, T operators. We can also observe that λT_{00}, λT_{ij} are P-odd and T-odd, compatible with the EDM character.

The Hamiltonian (17) contains LV EDM terms, $\lambda T_{00}\,(\boldsymbol{\sigma}\cdot\boldsymbol{E})$, $\lambda T_{ij}E^j\sigma^i$, on which one can set stringent upper bounds using experimental data. We begin with

$$\lambda_1T_{00}\,(\boldsymbol{\sigma}\cdot\boldsymbol{E})\,, \tag{18}$$

where we notice that λT_{00} plays the role of the electron EDM magnitude, $|\mathbf{d}_e|$. The magnitude of $\mathbf{d}_e$ has been constrained with increasing precision [3,4,7], recently reaching the level $|\mathbf{d}_e|\leq 8.7\times10^{-31}$ e m or $|\mathbf{d}_e|\leq 3.8\times10^{-25}$ (eV)$^{-1}$ [5]. Considering this experimental measure, we attain the following upper bound:

$$|\lambda_1T_{00}|\leq 3.8\times10^{-16}\ (\mathrm{GeV})^{-1}. \tag{19}$$

As for the term, $\lambda_1T_{ij}E^j\sigma^i$, it can be written as $\lambda(T/3)\,(\boldsymbol{\sigma}\cdot\mathbf{E})$, where $T=\mathrm{Tr}(T_{ij})$. Similarly, we state on the trace:

$$|\lambda_1T|\leq 1.1\times10^{-15}\ (\mathrm{GeV})^{-1}, \tag{20}$$

or for a specific component λ_1T_{ii},

$$|\lambda_1T_{ii}|\leq 3.8\times10^{-16}\ (\mathrm{GeV})^{-1}. \tag{21}$$

These upper limits are among the best ones to be stated over dimension five CPT-even LV nonminimal coefficients (not involving higher derivatives). Considering a situation in which there is impossibility of identifying the LV effects stemming from T_{00} and T_{ii}, the EDM interaction is taken as

$$\lambda_1\left(T_{00}\boldsymbol{\sigma}\cdot\boldsymbol{E}-T_{ij}E^j\sigma^i\right)=\lambda_1\,(T_{00}-T_{ii})\,(\boldsymbol{\sigma}\cdot\boldsymbol{E})\,, \tag{22}$$

reflecting the case when the diagonal elements T_{ii} are equal. It yields the following upper bound:

$$\lambda_1\,|T_{00}-T_{ii}|\leq 3.8\times10^{-16}\ (\mathrm{GeV})^{-1}. \tag{23}$$

For a specific value of i, it holds $\left(\mathrm{tr}T_{\mu\nu}\right)=(T_{00}-T)=(T_{00}-3T_{ii})$ and $\left(\mathrm{tr}T_{\mu\nu}\right)=(T_{00}-T_{ii})-2T_{ii}$. Considering that the blocks $2T_{ii}$ and $(T_{00}-T_{ii})$ are limited as stated in Eqs. (21), (23), the trace element can be restrained as

$$\left|\lambda_1\left(\mathrm{tr}T_{\mu\nu}\right)\right|\leq 7.6\times10^{-16}\ (\mathrm{GeV})^{-1}. \tag{24}$$

We still comment on a nonconventional interpretation that allows to constrain the component $\lambda_1T_{0i}(\sigma\times B)^i=\lambda_1(\epsilon_{ijk}T_{0i}\sigma^j)B^k$ of Hamiltonian (17), which can be read as $\lambda_1\tilde{\boldsymbol{\sigma}}\cdot\mathbf{B}=\lambda_1\tilde{\sigma}^kB^k$, where $\tilde{\sigma}^k=\epsilon_{ijk}T_{0i}\sigma^j$ is a kind of "rotated" spin operator yielding a "rotated" magnetic moment generated by the Lorentz violating background. This term can only be constrained with MDM data if one conceives a non-conventional experimental set up able to measure a non-null spin expectation value, $\langle S_i\rangle$, in a direction orthogonal to the applied magnetic field (B). In this specific context, the same procedure developed in Eqs. (39) and (40) could imply the upper bound

$$|\lambda_1T_{0i}|\leq 5.5\times10^{-11}\ (\mathrm{GeV})^{-1}. \tag{25}$$

Moreover, the bounds (21) and (25) are subject to sidereal variations, since the LV background field is approximately constant only on the Sun's reference frame (RF). It is necessary, therefore, to bring these bounds to the Earth-located Lab's RF, at the colatitude χ, rotating around the Earth's axis with angular velocity Ω. For experiments up to a few weeks long, the transformation law for a rank-2 tensor, $A_{\mu\nu}$, according to Refs. [40,42] is merely a spatial rotation,

$$A^{(\rm Lab)}_{\mu\nu}=\mathcal{R}_{\mu\alpha}\mathcal{R}_{\nu\beta}A^{(\rm Sun)}_{\alpha\beta}, \tag{26}$$

where

$$R_{\mu\nu}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\chi\cos\Omega t & \cos\chi\sin\Omega t & -\sin\chi\\ 0 & -\sin\Omega t & \cos\Omega t & 0\\ 0 & \sin\chi\cos\Omega t & \sin\chi\sin\Omega t & \cos\chi\end{pmatrix}, \tag{27}$$

in which the first line and column were included. The T_{00} (as its time average) does not change, just as the spatial trace, $T^{(\rm Lab)}=\mathrm{Tr}(T_{ij})^{(\rm Lab)}$. However, any specific main diagonal component T_{ii} (for a particular i value) in (21) is modified, having time average given as

$$\begin{aligned}\langle T^{(\rm Lab)}_{ii}\rangle&=\langle(\mathcal{R}_{i1})^2\rangle T^{(\rm Sun)}_{11}+\langle(\mathcal{R}_{i2})^2\rangle T^{(\rm Sun)}_{22}\\&\quad+\langle(\mathcal{R}_{i3})^2\rangle T^{(\rm Sun)}_{33},\end{aligned} \tag{28}$$

because $\langle R_{i1}R_{i2}\rangle=\langle R_{i1}R_{i3}\rangle=\langle R_{i2}R_{i3}\rangle=0$, for any i, due to their dependence on $\sin\Omega t$ and $\cos\Omega t$. As for the bound (25), it transforms as a regular vector, and its time average becomes

$$\langle T^{(\rm Lab)}_{0i}\rangle=(-\delta_{i1}\sin\chi+\delta_{i3}\cos\chi)T^{(\rm Sun)}_{03}, \tag{29}$$

since $\langle R_{i1}\rangle=\langle R_{i2}\rangle=0$. Hence, the bounds (20), (21) and (25) on the Lab's RF are, respectively,

$$|\lambda_1 \langle T^{(\mathrm{Sun})} \rangle| \leq 1.1 \times 10^{-15}\ (\mathrm{GeV})^{-1}, \tag{30}$$

$$|\lambda_1 \langle (\mathcal{R}_{i1})^2 \rangle T_{11}^{(\mathrm{Sun})} + \lambda_1 \langle (\mathcal{R}_{i2})^2 \rangle T_{22}^{(\mathrm{Sun})} + \lambda_1 \langle (\mathcal{R}_{i3})^2 \rangle T_{33}^{(\mathrm{Sun})}| \leq 3.8 \times 10^{-16}\ (\mathrm{GeV})^{-1}, \tag{31}$$

$$\left| \lambda_1 (-\delta_{i1} \sin\chi + \delta_{i3} \cos\chi) T_{03}^{(\mathrm{Sun})} \right| \leq 5.5 \times 10^{-11}\ (\mathrm{GeV})^{-1}. \tag{32}$$

2.2. Non-axial nonminimal coupling

Other possibility of coupling the fermion and electromagnetic fields by means of a CPT-even nonminimal coupling involving a rank-2 tensor to be mentioned is the Hermitian and non-axial version of the coupling (7), that is,

$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda_1'}{2} \left(T_{\mu\nu} F^\nu{}_\beta - T_{\beta\nu} F^\nu{}_\mu \right) \gamma^\beta, \tag{33}$$

leading to the following Dirac equation,

$$\left[i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu + \lambda_1 T_{\mu\nu} F^\nu{}_\beta \sigma^{\mu\beta} - m \right] \Psi = 0. \tag{34}$$

The corresponding LV Hamiltonian, $H'_{\mathrm{LV}} = -\lambda_1' T_{\mu\nu} F^\nu{}_\beta \gamma^0 \sigma^{\mu\beta}$, can be expressed as

$$H'_{\mathrm{LV}} = -i\lambda_1' T_{00} E^i \gamma^i - i\lambda_1' T_{0i} \epsilon_{ijb} B^b \gamma^j - \lambda_1' T_{i0} \epsilon_{ija} E^j \gamma^0 \Sigma^a + i\lambda_1' T_{ij} E^j \gamma^i - \lambda_1' T B^a \gamma^0 \Sigma^a + \lambda_1' T_{ia} B^i \gamma^0 \Sigma^a, \tag{35}$$

or in the matrix form (12), with components:

$$H'_{11} = -\lambda_1' (T_{i0} E^j \epsilon_{ijk} \sigma^k + T\sigma^p B^p - T_{ij} B^i \sigma^j), \tag{36}$$

$$H'_{12} = i\lambda_1' (T_{00} E^i \sigma^i - T_{ij} E^j \sigma^i + T_{0i} \epsilon_{ijk} \sigma^j B^k). \tag{37}$$

The nonrelativistic Hamiltonian is

$$H_{\mathrm{NR}} = \frac{1}{2m} \left[(\boldsymbol{p} - e\boldsymbol{A})^2 - e\boldsymbol{\sigma} \cdot \boldsymbol{B} \right] + eA_0 + \lambda_1' T_{i0} (\boldsymbol{\sigma} \times \boldsymbol{E})^i - \lambda_1' T (\boldsymbol{\sigma} \cdot \boldsymbol{B}) + \lambda_1' T_{ij} \sigma^j B^i + \frac{1}{2m} [(\boldsymbol{\sigma} \cdot \boldsymbol{\pi}) H'_{12} - H'_{12} (\boldsymbol{\sigma} \cdot \boldsymbol{\pi})]. \tag{38}$$

Clearly, the terms $T(\boldsymbol{\sigma} \cdot \boldsymbol{B})$ and $T_{ij} \sigma^j B^i$ are MDM contributions, associated only with the symmetric part of the tensor $T_{\mu\nu}$, whose magnitude can be limited by the known experimental error in the MDM.

The electron's magnetic moment is $\boldsymbol{\mu} = -g\mu_B \boldsymbol{S}$, with $\mu_B = e/2m$ being the Bohr magneton and $g = 2(1+a)$ being the gyromagnetic factor, with $a = \alpha/2\pi \simeq 0.00116$ representing the deviation from the usual case, $g = 2$. The magnetic interaction is $H' = -\mu_B g (\boldsymbol{S} \cdot \mathbf{B})$. The most precise calculation up to date of a is found in Ref. [43]. Experimentally, precise measurements [44] reveal that the error on the electron MDM is at the level of 2.8 parts in 10^{13}, that is, $\Delta a \leq 2.8 \times 10^{-13}$. Hamiltonian (38) possesses tree-level LV contributions to the usual $g = 2$ gyromagnetic factor, that is

$$\lambda_1' T (\boldsymbol{\sigma} \cdot \boldsymbol{B}) - \lambda_1' \frac{T}{3} (\boldsymbol{\sigma} \cdot \boldsymbol{B}) = \frac{2}{3} \lambda_1' T (\boldsymbol{\sigma} \cdot \boldsymbol{B}), \tag{39}$$

which cannot be larger than Δa. The total magnetic interaction in Eq. (38) is $\mu_B (\boldsymbol{\sigma} \cdot \boldsymbol{B}) + \frac{2}{3} \lambda_1' T (\boldsymbol{\sigma} \cdot \boldsymbol{B})$, so that this interaction assumes the form

$$\mu_B \left[1 + \frac{2}{3} \frac{2m}{e} \lambda_1' T \right] (\boldsymbol{\sigma} \cdot \boldsymbol{B}), \tag{40}$$

where $\frac{4m}{3e} \lambda_1' T_{ii}$ stands for the tree-level LV correction that should be smaller than Δa. This leads to the following upper bound for the trace:

Table 2
Classification under C, P, T for the coefficients of the axial and non-axial CPT-even nonminimal couplings.

	$\lambda_1 T_{00}$	$\lambda_1 T_{0i}$	$\lambda_1 T_{ij}$	$\lambda_1' T_{00}$	$\lambda_1' T_{0i}$	$\lambda_1' T_{ij}$	$\lambda_4 T_{0i}$	$\lambda_4 T_{ij}$
C	+	+	+	+	+	+	+	+
P	−	+	−	+	−	+	−	+
T	−	+	−	+	−	+	−	+
CPT	+	+	+	+	+	+	+	+

$$\left| \lambda_1' T \right| \leq 3.5 \times 10^{-11}\ (\mathrm{GeV})^{-1}, \tag{41}$$

being less restrictive by a factor $\simeq 10^5$ than the previous ones derived by EDM.

On the Sun's RF, this bound is equally written as

$$\left| \lambda_1' T^{(\mathrm{Sun})} \right| \leq 3.5 \times 10^{-11}\ (\mathrm{GeV})^{-1}, \tag{42}$$

once the trace element is invariant under the rotation (27).

The behavior of the term $\lambda_1' T_{i0}$ of Hamiltonian (38), under the discrete operations C, P and T (see Table 2), is compatible with the EDM properties. Accordingly, the term $\lambda_1' T_{i0} (\boldsymbol{\sigma} \times \boldsymbol{E})^i = \lambda_1' T_{i0} \epsilon_{ijk} \sigma^j E^k$ could be considered as EDM if we take

$$\lambda_1' T_{i0} (\boldsymbol{\sigma} \times \boldsymbol{E})^i = \lambda_1' \tilde{\sigma}^k E^k, \tag{43}$$

with $\tilde{\sigma}^k = \epsilon_{ijk} T_{0i} \sigma^j$ implying a kind of "rotated" EDM. Analogously to the magnetic case, it can only be limited with EDM data if there is a non-conventional experimental device able to measure a non-null spin expectation value, $\langle S_i \rangle$, in a direction orthogonal to the applied electric field (E). In this particular situation, the procedure of Eq. (19) could engender the upper bound

$$\left| \lambda_1' T_{i0} \right| \leq 3.8 \times 10^{-16}\ (\mathrm{GeV})^{-1}, \tag{44}$$

which is also subject to sidereal variations, that is

$$\left| \lambda_1' (-\delta_{i1} \sin\chi + \delta_{i3} \cos\chi) T_{30}^{(\mathrm{Sun})} \right| \leq 3.8 \times 10^{-16}\ (\mathrm{GeV})^{-1}. \tag{45}$$

2.3. Comments and correspondences

The classification of the LV coefficients of the axial and non-axial couplings, including some other of the couplings to be examined in the next section, under C, P and T, is enclosed in Table 2.

We should comment on a partial equivalence: in the case the tensor $T_{\mu\nu}$ is symmetric and traceless, the nonminimal couplings (7) and (33) recover part of the nonminimal coupling first analyzed in Refs. [40] and [39], respectively. Indeed, we begin by taking a symmetric and traceless tensor, $\kappa_{\nu\beta}$ ($\kappa^\nu{}_\nu = 0 \rightarrow \kappa_{00} = \kappa_{jj}$), linked to the (K_F) tensor by $\kappa_{\nu\beta} = (K_F)^\alpha{}_{\nu\alpha\beta}$, and

$$(K_F)_{\mu\nu\alpha\beta} = \frac{1}{2} \left(g_{\mu\alpha} \kappa_{\nu\beta} - g_{\mu\beta} \kappa_{\nu\alpha} + g_{\nu\beta} \kappa_{\mu\alpha} - g_{\nu\alpha} \kappa_{\mu\beta} \right). \tag{46}$$

Replacing this latter expression in the couplings $\overline{\Psi} \lambda_A (K_F)_{\mu\nu\alpha\beta} \times \sigma^{\mu\nu} \gamma^5 F^{\alpha\beta} \Psi$ and $\overline{\Psi} \lambda (K_F)_{\mu\nu\alpha\beta} \sigma^{\mu\nu} F^{\alpha\beta} \Psi$, yields $\overline{\Psi} \lambda \kappa_{\mu\beta} F^\beta_\nu \sigma^{\mu\nu} \times \gamma^5 \Psi$ and $\overline{\Psi} \lambda \kappa_{\mu\beta} F^\beta_\nu \sigma^{\mu\nu} \Psi$, respectively, which have the same form of the operators appearing in the Dirac equations (8) and (34). This equivalence holds only if the tensor $T_{\mu\nu}$ is symmetric and traceless, however. We point out that the present development is broader, since the tensor $T_{\mu\nu}$ has not a definite symmetry (in principle). We also comment on the possible correspondence between the previous upper bounds on the couplings $\lambda_A (K_F)$ and $\lambda (K_F)$, achieved in Refs. [40] and [39], and the present ones, as shown in Table 3.

In Table 3, we notice that the first and fourth bound find a correspondence with the present nonminimal couplings. The first

Table 3
Previous upper bounds for the axial and non-axial CPT-even nonminimal couplings. All bounds expressed in terms of $(\text{eV})^{-1}$.

	$\lambda(K_F)$	Counterpart
MDM	$\lambda(\kappa_{HB})_{33} \leq 2.3 \times 10^{-20}$	$\lvert\lambda_1' T\rvert \leq 3.5 \times 10^{-20}$
EDM	$\lambda(\kappa_{HE})_{11} \leq 3.8 \times 10^{-25}$	no
	$\lambda_A(K_F)$	
MDM	$\lvert\lambda_A(\kappa_{DB})_{33}\rvert \leq 2.3 \times 10^{-20}$	no
EDM	$\lvert\lambda_A(\kappa_{DE})_{ii}\rvert \leq 1.1 \times 10^{-24}$	$\lvert\lambda_1(\mathrm{tr}T_{\mu\nu})\rvert \leq 3.8 \times 10^{-25}$

one involves the component T_{00}, which can be justified by the relation

$$(\kappa_{DE})^{jk} = \delta^{jk}\kappa_{00} - \kappa^{jk}, \tag{47}$$

with $(\kappa_{DE})^{jj} = 2\kappa_{00}/3$, for a specific value of j, and $\mathrm{tr}(\kappa_{DE}) = 2\kappa_{00}$. Concerning the first bound of Table 3, the relation

$$(\kappa_{HB})^{jk} = -\delta^{jk}\kappa^{ll} + \kappa^{kj}, \tag{48}$$

leads to $(\kappa_{HB})^{jj} = -2\kappa_{00}/3$ for a specific value of j, and $\mathrm{tr}(\kappa_{HB}) = -2\kappa_{ii}$. In this sense, the first bound on $\lambda(\kappa_{HB})_{33}$ in Table 3 can be related to the bound (41) on $\lambda_1' T$.

A possible correspondence to the terms involving (κ_{DB}), (κ_{HE}) in the models of Ref. [40] takes place by the appearance of the pieces $\lambda_1 T_{0i}(\sigma \times B)^i$ and $\lambda_1' T_{i0}(\sigma \times E)^i$ in Hamiltonians (17) and (38), which does not reflect a direct correspondence for the second and the third bounds of Table 3, once they express constraints over birefringent components of the tensor (K_F) which are not embraced in the symmetric tensor $\kappa_{\mu\nu}$. Indeed, starting from the relation $(\kappa_{DB})^{ij} = -(\kappa_{HE})^{ij} = -\epsilon^{ijq}\kappa^{0q}$, one notes that $(\kappa_{DB})^{ii} = (\kappa_{HE})^{ii} = 0$ (for any i value), preventing the association of these bounds with the symmetric $T_{\mu\nu}$ components.

In spite of such equivalence, the tensor $T_{\mu\nu}$, as proposed in the couplings of this letter, is not necessarily defined as symmetric and traceless. The present approach allows to constrain pieces of the background, as the trace of $T_{\mu\nu}$, see Eq. (24), that would not be restrained starting from the previous models [39,40], where this piece is null.

3. New possibilities for general nonminimal couplings

A matter of interest consists in inquiring about the possibility of other couplings, sharing the general CPT-even structure of the ones already proposed, but physically different. It is easy to notice that the couplings (7) and (33) enclose the axial and non-axial options involving two gamma matrices in the Dirac equation. A new possibility would be associated with the following nonminimal derivative:

$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda_2}{4}T^{\alpha\beta}F_{\alpha\beta}\gamma_\mu\gamma_5, \tag{49}$$

and with the LV Hamiltonian contribution $H_{LV} = -\lambda_2 T^{\alpha\beta}F_{\alpha\beta}\gamma^0\gamma_5$, which does not provide any kind of spin interaction (neither magnetic, nor electric). Therefore, this is not an interesting case for our purposes. The same holds for the non-axial version. New possibilities arise when the tensors $T_{\mu\nu}$ and $F^{\alpha\beta}$ have no mutually contracted indices, leaving three free indices to be contracted with gamma matrices, such as

$$D_\mu = \partial_\mu + ieA_\mu + i\lambda_3 T_{\alpha\nu}F_{\mu\beta}\gamma^\beta\gamma^\alpha\gamma^\nu, \tag{50}$$

which implies the Lagrangian piece as

$$\mathcal{L} = \bar{\Psi}(\lambda T_{\alpha\nu}F_{\mu\beta}\gamma^\mu\gamma^\beta\gamma^\alpha\gamma^\nu)\Psi. \tag{51}$$

The derivative (53) is one of the combinations comprised in the general expression

$$D_\mu = \partial_\mu + ieA_\mu + i\lambda_3 T_{\{\alpha\nu}F_{\mu\beta\}}\gamma^{\{\beta}\gamma^\alpha\gamma^{\nu\}}, \tag{52}$$

with the symbol {} denoting possible permutation of the indexes μ, ν, α, β. Among them, we start from the nonminimal covariant derivative

$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda_3}{8}\left(T_{\alpha\nu}F_{\mu\beta} + T_{\mu\beta}F_{\alpha\nu}\right)\gamma^\beta\sigma^{\alpha\nu}, \tag{53}$$

with real λ_3. This coupling is distinct from the all previous ones analyzed, yielding the following Hermitian piece for the Dirac equation:

$$-\frac{\lambda_3}{8}\left(T_{\alpha\nu}F_{\mu\beta} + T_{\mu\beta}F_{\alpha\nu}\right)\sigma^{\mu\beta}\sigma^{\alpha\nu}. \tag{54}$$

Here only the antisymmetric part of $T_{\mu\nu}$ can contribute with a new interaction, due to the antisymmetry of $\sigma^{\mu\beta}$. In fact, the symmetric part of $T_{\mu\nu}$ is associated to the Lagrangian piece, $\lambda_3 T^\nu{}_\nu F_{\mu\beta}\bar{\Psi}\sigma^{\mu\beta}\Psi$, which provides the usual MDM interaction. The modified Dirac equation assumes the form

$$i\partial_t\Psi = \left[\boldsymbol{\alpha}\cdot\boldsymbol{\pi} + eA_0 + m\gamma^0 + H_{\mathrm{LV3}}\right]\Psi, \tag{55}$$

where the LV Hamiltonian is

$$H_{\mathrm{LV3}} = -\frac{\lambda_3}{8}\left[T_{\alpha\nu}F^0_{\mu\beta} + T_{\mu\beta}F_{\alpha\nu}\right]\gamma^0\sigma^{\mu\beta}\sigma^{\alpha\nu}, \tag{56}$$

whose nonrelativistic form is

$$\begin{aligned} H_{NR} = {} & \frac{1}{2m}\left[\boldsymbol{\pi}^2 - e\boldsymbol{\sigma}\cdot\boldsymbol{B}\right] + eA_0 \\ & + \lambda_3 T_{0i}E^i + \lambda_3 T_{ab}\epsilon_{abc}B^c \\ & + \frac{1}{2m}\left[(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})H_{12}^{(3)} - H_{12}^{(3)}(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\right], \end{aligned} \tag{57}$$

where $H_{12}^{(3)} = i\lambda_3(T_{0i}B^i - T_{ab}\epsilon_{abj}E^j)$. This coupling is not of interest for our purposes, since it does not generate any spin interaction (at the dominant level).

Another possibility is

$$D_\mu = \partial_\mu + ieA_\mu + i\frac{\lambda_4}{8}\left(T_{\alpha\nu}F_{\mu\beta} - T_{\mu\beta}F_{\alpha\nu}\right)\gamma^\beta\sigma^{\alpha\nu}, \tag{58}$$

for a real λ_4. This coupling is also distinct from the previous ones, leading to the Dirac equation contribution:

$$\frac{\lambda_4}{8}T^\nu{}_\nu F_{\mu\beta}\sigma^{\mu\beta} + i\frac{\lambda_4}{8}\left(T_{\alpha\nu}F_{\mu\beta} - T_{\mu\beta}F_{\alpha\nu}\right)\sigma^{\mu\beta}\sigma^{\alpha\nu}. \tag{59}$$

In this case, the symmetric part of $T_{\mu\nu}$ is associated to the Lagrangian piece, $\lambda_4 T^\nu{}_\nu F_{\mu\beta}\bar{\Psi}\sigma^{\mu\beta}\Psi$, which provides the usual MDM interaction. The modified Dirac equation assumes the form

$$i\partial_t\Psi = \left[\boldsymbol{\alpha}\cdot\boldsymbol{\pi} + eA_0 + m\gamma^0 + H_{\mathrm{LV4}}\right]\Psi, \tag{60}$$

where the LV piece is

$$H_{\mathrm{LV4}} = i\frac{\lambda_3}{8}\left[T_{\alpha\nu}F_{\mu\beta} - T_{\mu\beta}F_{\alpha\nu}\right]\gamma^0\sigma^{\mu\beta}\sigma^{\alpha\nu}, \tag{61}$$

and its nonrelativistic version,

$$\begin{aligned} H_{\mathrm{NR}(4)} = {} & \frac{1}{2m}\left[(\boldsymbol{p} - e\boldsymbol{A})^2 - e\boldsymbol{\sigma}\cdot\boldsymbol{B}\right] + eA_0 \\ & + \lambda_4 T_{0i}E^j\epsilon_{ijk}\sigma^k + \lambda_4 T_{ad}B^a\sigma^d \\ & + \frac{1}{2m}\left[(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\,H_{12}^{(4)} - H_{12}^{(4)}(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})\right], \end{aligned} \tag{62}$$

with $H_{12}^{(4)} = i\lambda_4(T_{0i}\epsilon_{ikj}B^k\sigma^j - T_{jb}E^j\sigma^b)$. As shown in Table 2, the coefficient T_{0i} is P-odd and T-odd, being able to generate EDM, in principle. This kind of coupling could be restrained by EDM data, using the approach of Eqs. (43), (44), $\lambda_4 \left(T_{0i}\epsilon_{ijk}\sigma^k\right) E^j = -\lambda_4\tilde{\sigma}^j E^j$, which yields

$$|\lambda_4 T_{0i}| \leq 3.8 \times 10^{-16}\ (\text{GeV})^{-1}. \tag{63}$$

A similar analysis holds for the term $\lambda_4 T_{ad}\sigma^d B^a = \lambda_4\hat{\sigma}^a B^a$, where $\hat{\sigma}^a = T_{ad}\sigma^d$, which is compatible with a non-usual MDM generation. Using MDM data, the implied upper bound is

$$|\lambda_4 T_{ad}| \leq 5.5 \times 10^{-11}\ (\text{GeV})^{-1}. \tag{64}$$

The time-averaged sidereal variations of the bounds (63) and (64), according to the transformation law (26) are, respectively

$$\left|\lambda_4(-\delta_{i1}\sin\chi + \delta_{i3}\cos\chi)T_{03}^{(\text{Sun})}\right| \leq 3.8 \times 10^{-16}\ (\text{GeV})^{-1}, \tag{65}$$

$$\left|\lambda_4\langle\mathcal{R}_{ap}\mathcal{R}_{dq}\rangle T_{ad}^{\text{Sun}}\right| \leq 5.5 \times 10^{-11}\ (\text{GeV})^{-1}. \tag{66}$$

One could still suppose other possibilities of coupling, similar but different from the form (51). Obviously, the forms $\lambda T_{\alpha\nu}F_{\mu\beta}\bar{\Psi}\gamma^\beta\gamma^\mu\gamma^\alpha\gamma^\nu\Psi$ or $\lambda T_{\alpha\nu}F_{\mu\beta}\bar{\Psi}\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\alpha\Psi$ are already considered in the Dirac term (54), due to the antisymmetry of $T_{\alpha\nu}$, $F_{\mu\beta}$. In principle, a distinct possibility could be

$$\mathcal{L} = \lambda T_{\alpha\nu}F_{\mu\beta}\bar{\Psi}\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\nu\Psi. \tag{67}$$

Yet, using $\gamma^\alpha\gamma^\beta = -\gamma^\beta\gamma^\alpha + 2g^{\alpha\beta}$, one shows that

$$\mathcal{L} = -\lambda\bar{\Psi}T_{\alpha\nu}F_{\mu\beta}\gamma^\mu\gamma^\beta\gamma^\alpha\gamma^\nu\Psi + 2\lambda\bar{\Psi}T_{\alpha\nu}F_\mu{}^\alpha\gamma^\mu\gamma^\nu\Psi, \tag{68}$$

achieving the couplings of Eq. (53) and Eq. (33), with the observation that the tensor $T_{\alpha\nu}$ now is antisymmetric. The same holds for the combination $\lambda T_{\alpha\nu}F_{\mu\beta}\bar{\Psi}\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\Psi$. Hence, the Hermitian coupling corresponding to Eq. (67), $\lambda T_{\alpha\nu}F_{\mu\beta}(\sigma^{\mu\alpha}\sigma^{\beta\nu} - \sigma^{\beta\nu}\sigma^{\mu\alpha})$, and others involving 4 gamma matrices, are already contained in the previous cases.

4. Conclusion and final remarks

We have analyzed a new class of dimension-five, CPT-even and Lorentz-violating nonminimal couplings between fermions and photons, composed of a general tensor, $T_{\mu\nu}$, in the context of the Dirac equation, addressing its axial and non-axial versions. The nonrelativistic axial Hamiltonian was carried out, revealing tree-level electron effective EDM and a non-conventional MDM contribution, whereas the nonrelativistic non-axial Hamiltonian implied effective electron MDM and a non-usual EDM contribution. The CPT-even nonminimal coupling proposed in Eq. (8) evades the Schiff's theorem, once it yields physical effects for the energy of the system, $\Delta U = -d_e \cdot E$, as explained in Ref. [9], allowing to directly use the electron EDM data to set upper bounds on the nonminimal LV parameters, which does not happen for CPT-odd nonminimal couplings discussed in Ref. [12]. Recent experimental data about the electron EDM and MDM were used to establish upper bounds as tight as $|\lambda_1 T_{00}|$, $|\lambda_1 T_{ii}|$, $\left|\lambda_1' T_{0i}\right|$, $|\lambda_4 T_{0i}| \leq 3.8 \times 10^{-16}$ $(\text{GeV})^{-1}$ and $|\lambda_1 T_{0i}|$, $|\lambda_4 T_{ad}| \leq 5.5 \times 10^{-16}$ $(\text{GeV})^{-1}$, $\left|\lambda_1' T_{ii}\right| \leq 3.5 \times 10^{-11}$ $(\text{GeV})^{-1}$, respectively, where the non-diagonal components bounds involve non-conventional experiments, as explained. The sidereal analysis were also performed. The bounds found on this axial and non-axial couplings are at the same level of the ones obtained in Ref. [40], standing among the best ones in the literature, if compared to bounds over dimension-five and higher derivatives-free nonminimal LV couplings.

We point out that we have examined all the main dimension-five nonminimal couplings involving fermions and photons, composed of a 2-rank background tensor, the electromagnetic tensor, and gamma matrices. In accordance with our analysis, there exist still other forms of couplings, such as

$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda_4'}{2}\left(T_{\mu\nu}F^\nu_{\ \beta} - T_{\beta\nu}F^\nu_{\ \mu}\right)\gamma_\alpha\gamma^\beta\gamma^\alpha, \tag{69}$$

$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda_5'}{2}\left(T_{\mu\nu}F^\nu_{\ \beta} - T_{\beta\nu}F^\nu_{\ \mu}\right)\gamma_\alpha\gamma^\lambda\gamma^\beta\gamma^\alpha\gamma_\lambda, \tag{70}$$

which are, however, redundant to the ones here analyzed in physical content, being subjected to the same upper bounds [see Eqs. (20), (21), (41)].

The "rotated" MDM and EDM non-conventional interpretation could allow us to constrain non-diagonal components of the LV tensor, as shown is Sects. 2.1, 2.2 and 3. In order to accomplish this, it would be necessary to control the direction of the electric or magnetic field, as well as the particle's spin state. As an example, consider the fragment extracted from the Hamiltonian (17)

$$H_{\text{EDM-LV}} = \lambda_1(T_{00}\boldsymbol{\sigma}\cdot\boldsymbol{E} - T_{ij}E^j\sigma^i). \tag{71}$$

If the electric field points, say, in the $\hat{\boldsymbol{x}}$ direction, the Hamiltonian would becomes

$$H_{\text{EDM-LV}} = \lambda_1[T_{00}\sigma_x E^1 - (T_{11}E^1\sigma_x + T_{21}E^1\sigma_y + T_{31}E^1\sigma_z)], \tag{72}$$

whose expected value $\langle S|H_{\text{EDM-LV}}|S\rangle$ depends on the particle's spin state. Considering a spin polarized in the $|S_y\pm\rangle$ state, the T_{11} and T_{31} terms average to zero, remaining only $\langle S_y\rangle \neq 0$, so that following bound can be imposed

$$|\lambda_1 T_{21}| \leq 3.8 \times 10^{-16}\ (\text{GeV})^{-1}. \tag{73}$$

A polarized beam in the $\hat{\boldsymbol{z}}$ direction would yield the same bound for the T_{31} term. Hence, an experimental procedure, capable of measuring spin expectation values orthogonally to the fields, could allow to constrain off-diagonal $T_{\mu\nu}$ elements using EDM and MDM data. The obtained bounds are also subject to sidereal variations, whose transformation laws have been discussed in Sect. 2.1.

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