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Physics Letters B 596 (2004) 116–122

PHYSICS LETTERS B

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Noncompact Kaluza–Klein theory and inflationary cosmology: a complete formalism

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Received 6 May 2004; accepted 28 June 2004

Editor: P.V. Landshoff

Abstract

A formalization of the recently introduced formalism for inflation is developed from a noncompact Kaluza–Klein theory. In particular, the case for a single scalar field inflationary model is studied. We obtain that the scalar potential, which assume different representations in different frames, has a geometrical origin.

PACS: 04.20.Jb; 11.10.Kk; 98.80.Cq

In the last years, many people has worked in extra dimensions [1]. As has been emphasized, Standard Model matter can propagate a large distance in extra dimensions without conflict with observations if Standard Model is confined to a $(3+1)$-dimensional subspace, or “3-brane”, in the higher dimensions [2]. It should be possible if the four familiar dimensions where dependent of coordinate in the extra dimensions. In some works on noncompact extra dimensions the authors studied trapping of matter fields to be effectively four-dimensional (4D) [3] or studied finite-volume but topologically noncompact extra dimensions [4].

A very important question in theoretical physics consists to provide a good geometrical description of matter using only one extra coordinate (say ψ). The explanation of this issue in the framework of the early universe, in particular for inflationary theory [5], should be of great importance in cosmology. In this Letter, we are aimed to study this topic using the Kaluza–Klein formalism where the fifth coordinate is noncompact. In this framework should be interesting to explain the origin of an effective four-dimensional (4D) scalar potential $V(\varphi)$ which could be originated

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doi:10.1016/j.physletb.2004.06.093

from a 5D apparent vacuum. The idea that matter in four dimensions (4D) can be explained from a 5D Ricci-flat ($R_{AB}=0$) Riemannian manifold is a consequence of the Campbell's theorem. It says that any analytic N-dimensional Riemannian manifold can be locally embedded in a $(N+1)$-dimensional Ricci-flat manifold. This is of great importance for establishing the generality of the proposal that 4D field equations with sources can be locally embedded in 5D field equations without sources [6]. For apparent vacuum we understand a 5D flat metric and a 5D Lagrangian for a neutral scalar field, where the 5D dynamics is only kinetic, that is, the 5D potential in the 5D Lagrangian do not exists. In other words, we shall consider an 5D apparent vacuum for scalar fields without sources or interactions.

We consider the 5D metric, recently introduced by Ledesma and Bellini (LB) [7]

$$dS^2 = \psi^2\, dN^2 - \psi^2 e^{2N}\, dr^2 - d\psi^2, \tag{1}$$

where the parameters (N, r) are dimensionless and the fifth coordinate ψ has spatial unities. The metric (1) describes a flat 5D manifold in apparent vacuum ($G_{AB}=0$). To describe neutral matter in a 5D geometrical vacuum (1) we can consider the Lagrangian

$${}^{(5)}L(\varphi, \varphi_{,A}) = -\sqrt{\left|\frac{{}^{(5)}g}{{}^{(5)}g_0}\right|}\, {}^{(5)}\mathcal{L}(\varphi, \varphi_{,A}), \tag{2}$$

where ${}^{(5)}g = \psi^8 e^{6N}$ is the determinant of the 5D metric tensor with components g_{AB} (A, B take the values $0,1,2,3,4$) and ${}^{(5)}g_0 = \psi_0^8 e^{6N_0}$ is a constant of dimensionalization determined by ${}^{(5)}g$ evaluated with the initial conditions of the system: $\psi = \psi_0$ and $N = N_0$. We shall consider $N_0 = 0$, so that ${}^{(5)}g_0 = \psi_0^8$. Since the 5D metric (1) describes a manifold in apparent vacuum, the density Lagrangian $\mathcal{L}$ in (2) must to be

$${}^{(5)}\mathcal{L}(\varphi, \varphi_{,A}) = \frac{1}{2} g^{AB} \varphi_{,A}\varphi_{,B}, \tag{3}$$

which describes a free scalar field because there is no interaction: $V[\varphi(N, r, \psi)] = 0$. Taking into account the metric (1) and the Lagrangian (2), we obtain the equation of motion for φ

$$\left(2\psi\frac{\partial\psi}{\partial N} + 3\psi^2\right)\frac{\partial\varphi}{\partial N} + \psi^2\frac{\partial^2\varphi}{\partial N^2} - \psi^2 e^{-2N}\nabla_r^2\varphi$$
$$-4\psi^3\frac{\partial\varphi}{\partial\psi} - 3\psi^4\frac{\partial N}{\partial\psi}\frac{\partial\varphi}{\partial\psi} - \psi^4\frac{\partial^2\varphi}{\partial\psi^2} = 0, \tag{4}$$

where $\frac{\partial N}{\partial\psi}$ is zero because the coordinates $(N, \vec{r}, \psi)$ are independents.

Now, as in earlier works [7,8], we can consider the 3D comoving frame $dr = 0$. Taking the metric (1) with $U^r = 0$, the geodesic dynamics $\frac{dU^C}{dS} = -\Gamma^C_{AB} U^A U^B$ with $g_{AB} U^A U^B = 1$, give us the following velocities U^A:

$$U^\psi = -\frac{1}{\sqrt{u^2(N) - 1}}, \qquad U^r = 0,$$
$$U^N = \frac{u(N)}{\psi\sqrt{u^2(N) - 1}}, \tag{5}$$

for $S(N) = -N$ and $u(N) = \coth(N)$. In this representation $\frac{d\psi}{dN} = \psi/u(N)$. Thus the fifth coordinate evolves as

$$\psi(N) = \psi_0 e^{\int dN/u(N)}. \tag{6}$$

Here, ψ_0 is a constant of integration that has spatial unities. From the mathematical point of view, we are taking a foliation of the 5D metric (1) with r constant. Hence, to describe the metric in physical coordinates we can make the following transformations:

$$t = \int \psi(N)\, dN, \qquad R = r\psi,$$
$$L = \psi(N) e^{-\int dN/u(N)}, \tag{7}$$

such that for $\psi(t) = 1/H_c(t)$ (i.e., for $u(N) = -\frac{H_c}{dH_c/dN} > 0$), we obtain the resulting 5D metric

$$dS^2 = dt^2 - e^{2\int H_c(t)\, dt}\, dR^2 - dL^2, \tag{8}$$

where $L = \psi_0$ is a constant and $H_c(t) = \dot{a}/a$ is the classical Hubble parameter. The new variables has physical meaning, because t is the cosmic time and (R, L) are spatial variables. Furthermore $a(t)$ is the scale factor of the universe and describes its 3D Euclidean (spatial) volume. Hence the effective 4D metric is a spatially (3D) flat FRW one

$$dS^2 \to ds^2 = dt^2 - e^{2\int H_c(t)\, dt}\, dR^2, \tag{9}$$

and has a scalar curvature ${}^{(4)}\mathcal{R} = 6(\dot{H}_c + 2H_c^2)$. The metric (9) has a metric tensor with components $g_{\mu\nu}$ (μ, ν take the values $0,1,2,3$). The determinant of this tensor is ${}^{(4)}g = (a/a_0)^6$. Furthermore, the metric (9) describes globally a isotropic and homogeneous universe.

Now we can make the same treatment to the density Lagrangian (3) and the differential equation (4). Using the transformations (7) we obtain

$$
\begin{aligned}
&{}^{(4)}\mathcal{L}\big[\varphi(\vec{R},t),\varphi_{,\mu}(\vec{R},t)\big] \\
&\quad = \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - \frac{1}{2}\left[(RH_c)^2 - \frac{a_0^2}{a^2}\right](\nabla_R\varphi)^2, \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
&\ddot{\varphi} + 3H_c\dot{\varphi} - \frac{a_0^2}{a^2}\nabla_R^2\varphi \\
&\quad + \left[\left(4\frac{H_c^3}{\dot{H}_c} - 3\frac{\dot{H}_c}{H_c} - 3\frac{H_c^5}{\dot{H}_c^2}\right)\dot{\varphi} \right.\\
&\qquad \left. + \left(\frac{a_0^2}{a^2} - H_c^2R^2\right)\nabla_R^2\varphi\right] = 0. \qquad (11)
\end{aligned}
$$

Hence, with this representation the effective scalar 4D potential $V(\varphi)$ and its derivative with respect to $\varphi(\vec{R},t)$ are

$$
V(\varphi) \equiv \frac{1}{2}\left[(RH_c)^2 - \left(\frac{a_0}{a}\right)^2\right](\nabla_R\varphi)^2, \qquad (12)
$$

$$
\begin{aligned}
V'(\varphi) &\equiv \left(4\frac{H_c^3}{\dot{H}_c} - 3\frac{\dot{H}_c}{H_c} - 3\frac{H_c^5}{\dot{H}_c^2}\right)\dot{\varphi} \\
&\quad + \left(\frac{a_0^2}{a^2} - H_c^2R^2\right)\nabla_R^2\varphi. \qquad (13)
\end{aligned}
$$

Eqs. (10) and (11) describe the dynamics of the inflaton field $\varphi(\vec{R},t)$ in a metric (9) with a Lagrangian

$$
\begin{aligned}
&{}^{(4)}\mathcal{L}\big[\varphi(\vec{R},t),\varphi_{,A}(\vec{R},t)\big] \\
&\quad = -\sqrt{\left|\frac{{}^{(4)}g}{{}^{(4)}g_0}\right|}\left[\frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + V(\varphi)\right], \qquad (14)
\end{aligned}
$$

where $|{}^{(4)}g_0| = 1$. In the new representation (R,t,L), we obtain the following new velocities $\hat{U}^A = \frac{\partial\hat{x}^A}{\partial x^B}U^B$

$$
U^t = \frac{2u(t)}{\sqrt{u^2(t)-1}}, \qquad U^R = -\frac{2r}{\sqrt{u^2(t)-1}},
$$
$$
U^L = 0, \qquad (15)
$$

where the old velocities U^B are U^N, $U^r = 0$ and U^ψ. Furthermore, the velocities $\hat{U}^B$ complies with the constraint condition

$$
\hat{g}_{AB}\hat{U}^A\hat{U}^B = 1. \qquad (16)
$$

The important fact here is that the new frame give us an effective spatially flat FRW metric embedded in a 5D manifold where the initial value of the fifth coordinate $L_0 = \psi_0 = 1/H_0$ is the primordial Hubble horizon, which emerges naturally as a constant in this representation.

The solution $N = \operatorname{arctanh}[1/u(t)]$ corresponds to a power-law expanding universe with time dependent power $p(t)$ for a scale factor $a \sim t^{p(t)}$. Since $H_c(t) = \dot{a}/a$, the resulting Hubble parameter is

$$
H_c(t) = \dot{p}\ln(t/t_0) + p(t)/t, \qquad (17)
$$

where t_0 is the time for which inflation ends. The function u written as a function of time is

$$
u(t) = -\frac{H_c^2}{\dot{H}_c}, \qquad (18)
$$

where the overdot represents the derivative with respect to the time. In this frame, the 4D energy density ρ and the pressure p are [7]

$$
8\pi G\rho = 3H_c^2, \qquad (19)
$$
$$
8\pi G\mathrm{p} = -\big(3H_c^2 + 2\dot{H}_c\big). \qquad (20)
$$

Furthermore, note that the condition (16) implies that $|u(t)| = \sqrt{\frac{4r^2(a/a_0)^2-1}{3}} > 1$, where r is a constant. Moreover, the function $u(t)$ can be related to the deceleration parameter $q(t) = -\ddot{a}a/\dot{a}^2$: $u(t) = 1/[1 + q(t)]$, such that for inflationary models the required condition $|q(t)| \simeq 1$ (but with negative q), is fulfilled for $r = R(t)H_c(t) \gg 1$. In other words, it means that the effective 4D background metric (9) is only valid on super Hubble scales: $R \gg 1/H_c$, in agreement with the expected for a background metric. Note that the function $1/u(t) = -\dot{H}/H^2 \ll 1$ give us the slow-roll parameter [9] during inflation [10]. So, the feasible values for the constant r during inflation being given only from geometrical arguments. This is an important prediction of the model here developed.

On the other hand, $V(\varphi)$ and $V'(\varphi)$ can be written as a function of the old coordinates (N,r,ψ) in the comoving frame $U^r = 0$

$$
V(\varphi) \equiv \frac{1}{2}\big[r^2 - e^{-2N}\big]\frac{1}{r^2}\left(\frac{1}{\overset{\star}{\psi}}\overset{\star}{\varphi}\right)^2, \qquad (21)
$$

$$V'(\varphi) \equiv \left(3\frac{\overset{\star}{\psi}}{\psi^3} - \frac{4}{\psi\overset{\star}{\psi}} - \frac{3}{\overset{\star}{\psi}{}^2}\right)\overset{\star}{\varphi} + \left[\left(\frac{a_0}{a}\frac{1}{r}\right)^2 - 1\right]\frac{\partial^2\varphi}{\partial\psi^2}. \qquad (22)$$

Here, the overstar denotes the derivative with respect to N. Note that ΔN is the number of e-folds of the universe. To inflation solves the horizon/flatness problems it is required that $\Delta N \geqslant 60$ at the end of inflation.

At this point we can introduce the 4D Hamiltonian $\mathcal{H} = \pi^0\dot{\varphi} - {}^{(4)}L$, where the 4D Lagrangian is ${}^{(4)}L(\varphi, \varphi_{,\mu}) = \sqrt{\frac{|{}^{(4)}g|}{|{}^{(4)}g_0|}}\,{}^{(4)}\mathcal{L}(\varphi, \varphi_{,\mu})$ (see Eq. (14)):

$$\mathcal{H} = \frac{1}{2}\frac{a^3}{a_0^3}\left[\dot{\varphi}^2 + \frac{a_0^2}{a^2}(\nabla\varphi)^2 + 2V(\varphi)\right]. \qquad (23)$$

Hence, we can define the energy density operator ρ such that $\mathcal{H} = \frac{a^3}{a_0^3}\rho$. Hence

$$\rho = \frac{1}{2}\left[\dot{\varphi}^2 + \frac{a_0^2}{a^2}(\nabla\varphi)^2 + 2V(\varphi)\right]. \qquad (24)$$

The 4D expectation value of the Einstein equation $H^2 = \frac{8\pi G}{3}\rho$ on the 4D FRW metric (9), will be

$$\langle H^2\rangle = \frac{4\pi G}{3}\left\langle \dot{\varphi}^2 + \frac{a_0^2}{a^2}(\nabla\varphi)^2 + 2V(\varphi)\right\rangle, \qquad (25)$$

where G is the gravitational constant. We can make a semiclassical treatment [10,11] for the quantum field φ, such that $\langle\varphi(\vec{R}, t)\rangle = \phi_c(t)$:

$$\varphi(\vec{R}, t) = \phi_c(t) + \phi(\vec{R}, t), \qquad (26)$$

where $\langle\phi\rangle = 0$. Furthermore, we impose that $\langle\dot{\phi}\rangle = 0$. With this approach the classical dynamics on the background 4D FRW metric (9) is well described by the equations

$$\ddot{\phi}_c + 3\frac{\dot{a}}{a}\dot{\phi}_c + V'(\phi_c) = 0, \qquad (27)$$

$$H_c^2 = \frac{8\pi G}{3}\left(\frac{\dot{\phi}_c^2}{2} + V(\phi_c)\right). \qquad (28)$$

Since $\dot{\phi}_c = -\frac{H_c'}{4\pi G}$, from Eq. (28) we obtain the classical scalar potential $V(\phi_c)$ as a function of the classical Hubble parameter H_c

$$V(\phi_c) = \frac{3M_p^2}{8\pi}\left[H_c^2 - \frac{M_p^2}{12\pi}(H_c')^2\right],$$

where $M_p = G^{-1/2}$ is the Planckian mass. The quantum dynamics is described by

$$\ddot{\phi} + 3\frac{\dot{a}}{a} - \frac{a_0^2}{a^2}\nabla^2\phi + \sum_{n=1}\frac{1}{n!}V^{(n+1)}(\phi_c)\phi^n = 0, \qquad (29)$$

$$\langle H^2\rangle = H_c^2 + \frac{8\pi G}{3}\left\langle \frac{\dot{\phi}}{2} + \frac{a_0^2}{2a^2}(\nabla\phi)^2 + \sum_{n=1}\frac{1}{n!}V^{(n)}(\phi_c)\phi^n\right\rangle. \qquad (30)$$

On cosmological scales, the quantum fluctuations are small, so that a linear approximation (i.e., $n = 1$) is sufficient to make a realistic description for the evolution of ϕ. Furthermore, the second term in (30) is negligible when ϕ is considered spatially very homogeneous. However, such that term could be very important on sub Hubble scales [12]. With the aim to make a description of the dynamics on cosmological scales, we shall consider this term as null. For this reason we shall take $\frac{\dot{a}^2}{a^2} = \langle H^2\rangle \simeq H_c^2$. Once done the linear approximation for the semiclassical treatment we can make the identification of the squared mass for the inflaton field $m^2 = V''(\phi_c)$. Hence, after make a linear expansion for $V'(\varphi)$ in Eq. (22), we obtain

$$V'(\phi_c) = \left(4\frac{H_c^3}{\dot{H}_c} - 3\frac{\dot{H}_c}{H_c} - 3\frac{H_c^5}{\dot{H}_c^2}\right)\dot{\phi}_c, \qquad (31)$$

$$m^2\phi \equiv \left(4\frac{H_c^3}{\dot{H}_c} - 3\frac{\dot{H}_c}{H_c} - 3\frac{H_c^5}{\dot{H}_c^2}\right)\frac{\partial\phi}{\partial t} + \left(\frac{a_0^2}{a^2} - H_c^2R^2\right)\nabla_R^2\phi. \qquad (32)$$

Taking into account the expressions (27) with (31) and (29) with (32), we obtain the dynamics for ϕ_c and ϕ. Hence, the equations $\ddot{\phi}_c + 3H_c\dot{\phi}_c + V'(\phi_c) = 0$ and $\ddot{\phi} + 3H_c\dot{\phi}_c - (a/a_0)^2\nabla_R^2\phi + V''(\phi_c)\phi = 0$ now take the form

$$\ddot{\phi}_c + [3H_c + f(t)]\dot{\phi}_c = 0, \qquad (33)$$

$$\ddot{\phi} + [3H_c(t) + f(t)]\dot{\phi} - H_c^2R^2\nabla_R^2\phi = 0,. \qquad (34)$$

where

$$f(t) = \left(4\frac{H_c^3}{\dot{H}_c} - 3\frac{\dot{H}_c}{H_c} - 3\frac{H_c^5}{\dot{H}_c^2}\right). \qquad (35)$$

Moreover, $V'(\phi_c)$ and $V''(\phi_c)\phi$ can be written in terms of the metric (1), for a comoving observer (with

$U^r = 0$)

$$V'(\phi_c) = \left[3\frac{\overset{\star}{\psi}}{\psi^3} - \frac{4}{\psi\,\overset{\star}{\psi}} - \frac{3}{\overset{\star}{\psi}{}^2}\right]\overset{\star}{\phi}_c, \tag{36}$$

$$m^2\phi \equiv \left[3\frac{\overset{\star}{\psi}}{\psi^3} - \frac{4}{\psi\,\overset{\star}{\psi}} - \frac{3}{\overset{\star}{\psi}{}^2}\right]\overset{\star}{\phi} + \left[\left(\frac{a_0}{a}\frac{1}{r}\right)^2 - 1\right]\frac{\partial^2\phi}{\partial\psi^2}. \tag{37}$$

Note that in this representation (a) the squared mass is a differential operator that acts on the quantum fluctuations ϕ, and (b) the 4D potential and its derivatives with respect to $\varphi(\vec{R},t)$ are consequence of the evolution of $\psi(N)$. In other words the nonzero curvature of the 4D potential is induced by the geodesic evolution of the fifth coordinate for an observer in a comoving frame with $U^r = 0$. So, the inflationary dynamics, which is described by the inflaton field, should be determined by the evolution of the space-like fifth coordinate on a foliation of the 5D metric (1) where r is a constant. This is the main result of this Letter.

This formalism could take important consequences in the early universe. During the inflationary epoch, the slow-roll condition $\gamma(t) = -\dot{H}_c/H_c^2 \ll 1$ holds. Since $u(t) = 1/\gamma(t)$, we obtain that $u \gg 1$. This assures that all the velocities in U^A in (5) and $\hat{U}^A$ in (15) to be real, and imposes the condition $r \gg 1$ [8]. Furthermore the equation of state can be written in terms of the function $u(t)$

$$\langle \mathrm{p}\rangle = -\left[1 - \frac{2}{3u(t)}\right]\langle\rho\rangle,$$

which, since $u \gg 1$ during inflation, complies with the required condition for this stage: $\langle \mathrm{p}\rangle \simeq -\langle\rho\rangle$. Moreover, speaking in terms of the effective 4D FRW metric (9), the geodesic evolution of the fifth coordinate give us the Hubble horizon $\psi(t) = 1/H(t)$ and the resulting fifth (constant) coordinate $L = \psi_0$ is given by the primordial Hubble horizon: $L = 1/H_c(t_0)$.

We can define the *redefined quantum fluctuations* $\chi(\vec{R},t) = e^{1/2\int[3H_c(t)+f(t)]\,dt}\phi$, so that the equation of motion for χ yields

$$\ddot{\chi} - \left[H_c^2R^2\nabla_R^2 + \frac{1}{4}\big(3H_c + f(t)\big)^2 + \frac{1}{2}\big(3\dot{H}_c + \dot{f}(t)\big)\right]\chi = 0, \tag{38}$$

so that the modes $\chi_k(\vec{R},t)$ of the field χ complies the differential equation

$$\ddot{\chi}_k + H_c^2R^2\big(k^2 - k_0^2(t)\big)\chi_k = 0, \tag{39}$$

with

$$k_0^2(t) = \frac{1}{R^2H_c^2}\left[\frac{1}{4}\big(3H_c + f(t)\big)^2 + \frac{1}{2}\big(3\dot{H}_c + \dot{f}(t)\big)\right], \tag{40}$$

where $f(t)$ is a function of the classical Hubble parameter (see Eq. (35)). Hence, all the dynamics of the quantum fluctuations being described only by the classical Hubble parameter $H_c = \dot{a}/a$.

To illustrate the formalism we can study an example where $\psi(N) = -1/(\alpha N)$, so that $H_c(N) = -\alpha N$. This implies that the classical Hubble parameter (written as a function of time) is given by $H_c(t) = H_0e^{\alpha\Delta t}$. At the end of inflation $\alpha\Delta t \ll 1$, so that $H_c(t) \simeq H_0(1 + \alpha\Delta t)$ and $3H_c(t) + f(t) \simeq 3H_0(1 + \alpha\Delta t) + 3\alpha - (4H_0^2/\alpha)(1 + 2\alpha\Delta t) - (3H_0^3/\alpha^2)(1 + 3\alpha\Delta t)$, where $\Delta t = t_0 - t$ and t_0 is the time for which inflation ends. At the end of inflation it is sufficient to make a Δt-first order expansion for k_0^2, so that it can be approximated to

$$k_0^2(t) = \frac{1}{r^2}(A - Bt). \tag{41}$$

With this approximation, the general solution for the modes $\chi_k(t)$ is

$$\chi_k(t) = C_1\,\mathrm{Ai}\big[x(t)\big] + C_2\,\mathrm{Bi}\big[x(t)\big], \tag{42}$$

where $\mathrm{Ai}[x(t)]$ and $\mathrm{Bi}[x(t)]$ are the Airy functions with argument $x(t)$. Furthermore, (C_1, C_2) are some constants and

$$A = \frac{1}{4}\left(3H_0 - 3\frac{H_0^3}{\alpha^2} + \alpha - 3\frac{H_0^2}{\alpha}\right)^2 + \frac{1}{2}\left(8H_0^2 - 9\frac{H_0^3}{\alpha} - \alpha H_0\right) - \frac{1}{2}\left(3H_0 - 3\frac{H_0^3}{\alpha^2} + 3\alpha - 8\frac{H_0^2}{\alpha}\right) \times \left(8H_0^2 + 9\frac{H_0^3}{\alpha} - 3H_0\alpha\right)t_0, \tag{43}$$

$$B = \frac{1}{2}\left(3H_0 - 3\frac{H_0^3}{\alpha^2} + 3\alpha - 8\frac{H_0^2}{\alpha}\right) \times \left(3H_0\alpha - 8H_0^2 - 9\frac{H_0^3}{\alpha}\right), \quad (44)$$

$$x(t) = \frac{[(A - k^2) - Bt]}{b}\left(\frac{b}{r^2}\right)^{1/3}. \quad (45)$$

Note that in this example H_0 denotes the value of the Hubble parameter at the end of inflation. On cosmological scales (i.e., for $k^2 \ll A - Bt$), the solution for χ_k is unstable. However in the UV sector (i.e., for $k^2 \gg A - Bt$), the modes oscillate. This behavior is well described by the function $\mathrm{Bi}[x(t)]$, so that we shall take $C_1 = 0$. Hence, at the end of inflation the modes χ_k will be

$$\chi_k(t) = C_2\,\mathrm{Bi}\big[x(t)\big]. \quad (46)$$

Since the modes of the quantum fluctuations ϕ are $\phi_k = e^{-1/2\int[3H_c+f(t)]\,dt}\chi_k$, the squared fluctuations are given by

$$\langle\phi^2\rangle \simeq \frac{1}{2\pi^2}e^{-[3(H_0+\alpha)-4\frac{H_0^2}{\alpha}-3\frac{H_0^3}{\alpha^2}]t}\int dk\,k^2|\chi_k^2|, \quad (47)$$

where the modes χ_k are given by Eq. (46). Furthermore the density fluctuations at the end of inflation can be estimated by the expression

$$\frac{\delta\rho}{\rho} \sim \frac{H_0^2}{\dot\phi_c} \sim 2\pi^{1/2}\frac{H_0^{3/2}}{M_p\alpha^{1/2}}, \quad (48)$$

which are of the order of 10^{-5} for $H_0 \sim 10^{-5}\,M_p$ and $\alpha \sim 10^{-5}\,M_p$. In our case, the spectral index n_s being given by $n_s - 1 = -\frac{6}{u(t)}$. During inflation $u \gg 1$, so that $|n_s - 1| \ll 1$. Hence, during inflation the spectrum approaches very well with a Harrison–Zeldovich one. A more exhaustive treatment for density fluctuations go beyond the scope of this Letter.

In this Letter we have studied a single scalar field inflationary model which emerges from a 5D apparent vacuum described by a flat 5D metric with coordinates (N, r, ψ) and a Lagrangian for a free scalar field. The interesting is that the scalar potential $V(\varphi)$ appears in the 3D comoving frame characterized by $U^r = 0$ (see Eq. (21)). A further transformation to physical coordinates $t = \int\psi(N)\,dN$, $R = r\psi$ and $L = \psi e^{-\int dN/u(N)}$ give us the possibility to describe the system in a effective 4D (but 3D spatially flat) FRW metric. Such that metric is view as a particular frame (characterized with $U^L = 0$) of the 5D metric (8), where the potential $V(\varphi)$ is represented as the differential operator (12). In other words, the potential, which assume different representations in different frames, has a geometrical origin. Moreover, the mass of the inflaton field appears in the frame $U^L = 0$ as a differential operator applied to que quantum fluctuations $\phi(\vec{R}, t)$. Hence, for the semiclassical (and linear on ϕ) treatment here developed, $m^2\phi(\vec{R}, t)$ is a local operator with nonzero expectation value. At this point we must to exalt this result, because a particular frame in physics is intrinsically related to an observer (or experimental result). Note that in the potential (12) the KK modes are excluded. These modes should be related to a spin-2 graviton that appears in the KK theory when the electromagnetic effects are included. This is not our case. In this Letter we are excluded the electromagnetics fields in the metric because the electromagnetics effects should be nonimportant on cosmological scales.

To conclude, this formalism could be generalized to other inflationary models with many scalar fields [13]. Deflationary models could be examined using a line element $S(N) = N$ (we emphasize that here we have used $S(N) = -N$ which describes expanding universes). Moreover, models with nonzero cosmological parameters could be studied. The formalism also could be successfully extended to the study of topological defects [14] by using a line element $S \equiv S(r)$ on some frame $U^N = 0$, rather the here used $S \equiv S(N)$.

Acknowledgements

J.E.M.A. acknowledges CONACyT and IFM of UMSNH (México) for financial support. M.B. acknowledges CONICET, AGENCIA and UNMdP (Argentina) for financial support.

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