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Higgsed Stueckelberg vector and Higgs quadratic divergence

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A B S T R A C T

Here we show that, a hidden vector field whose gauge invariance is ensured by a Stueckelberg scalar and whose mass is spontaneously generated by the Standard Model Higgs field contributes to quadratic divergences in the Higgs boson mass squared, and even leads to its cancellation at one-loop when Higgs coupling to gauge field is fine-tuned. In contrast to mechanisms based on hidden scalars where a complete cancellation cannot be achieved, stabilization here is complete in that the hidden vector and the accompanying Stueckelberg scalar are both free from quadratic divergences at one-loop. This stability, deriving from hidden exact gauge invariance, can have important implications for modeling dark phenomena like dark matter, dark energy, dark photon and neutrino masses. The hidden fields can be produced at the LHC.

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1. Introduction

With the discovery of a new resonance at the Large Hadron Collider (LHC), having a mass $m_h = 125.9 \pm 0.4$ GeV [1] and couplings well consistent with the Standard Model (SM) predictions [2], the Higgs naturalness problem [3] has become the foremost problem to be tackled. The resolution, if any, brings its own new physics structure. The squared-masses of fundamental scalars, contrary to chiral fermions and gauge bosons whose masses are protected by chiral and gauge invariances, receive additive quantum corrections proportional to Λ^2 – the UV boundary of the SM. In explicit terms, one-loop quantum correction to Higgs squared-mass, originally computed by Veltman [4], reads as

$$\left(\delta m_H^2\right)_{\text{quad}} = \frac{\Lambda^2}{16\pi^2}\left(6\lambda_H + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6g_t^2\right), \qquad (1)$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings of the SM, respectively, and $g_t = m_t/\upsilon_H$ ($\upsilon_H = 246$ GeV is the VEV of the Higgs field) is the top quark Yukawa coupling. The top quark, being the most strongly coupled SM particle to the Higgs field, induces the biggest contribution and ensures a nonvanishing, unremovable

coefficient before Λ^2. The Higgs boson mass is stabilized to electroweak scale if $|\delta m_H^2| < m_H^2 < \Lambda^2$. This is the Veltman condition (VC). The parameters in it have all been measured, and it violates the LHC results for $\Lambda > 500$ GeV [5].

Having no symmetry to prevent the Higgs boson mass from sliding to the higher scales via (1), frequently a cancellation mechanism is implemented via fine-tuning of counter terms in which low and high energy degrees of freedom are mixed. This renders the whole procedure unnatural. It would be more natural, if the cancellation occurred by means of a symmetry principle at higher scales, or if it arose by accidental cancellations of certain terms. In fact, models of new physics constructed to complete the SM beyond Fermi energies have all been motivated by Higgs naturalness problem [5] (see also [6] for studies within supersymmetry). So far, however, in the 7 TeV and 8 TeV LHC searches reaching out beyond the TeV domain, no compelling sign of evidence for new physics has been found [7].

In consequence, having no TeV scale new physics for achieving naturalness, one is forced to understand the electroweak unnaturalness within the SM plus general relativity, albeit with some imperative extensions required by specifics of the approach taken. In 1995, conformal symmetry [8] was proposed as a mechanism for solving the Higgs mass hierarchy problem (the latest studies on the conformal symmetry as a solution to the fine-tuning problem may be found in [9]). Recently, the Higgs coupling to spacetime curvature has been found to stabilize the electroweak scale by a harmless, soft fine-tuning [10]. Furthermore, anti-gravity effects

have been claimed to improve Higgs naturalness [11]. Alternatively, one may view the parameters chosen by nature as the necessity of existence, and this leads to anthropic considerations [12]. In variance with all these approaches, a fine-tuning method based on singlet scalars [13] has also been employed. In this approach, the main idea is to cancel the quadratic divergences in Higgs boson mass with the loops of the singlet scalars that couple to Higgs field [14]. This method, though a fine-tuning operation by itself, nullifies the quadratic divergences and accommodates viable dark matter candidates [15,16]. Nevertheless, for real singlet scalars with vacuum expectation value (VEV), it is not possible to kill the quadratic divergences consistently because there is a mixing between the CP-even component of the Higgs field and the real singlet scalar, and it does not allow for simultaneous cancellation of the quadratic divergences in Higgs boson and singlet scalar masses [17]. There are also studies on two-Higgs doublet models without flavor changing neutral currents, demonstrating that, although the cancellation in the coefficient of the one-loop quadratically divergent terms is possible, the parameter space is severely constrained [18]. An additional complex scalar triplet extension of the SM has also been studied and proven to be a solution to the fine-tuning problem [19].

In the present work, as a completely new approach never explored before, we study protection of the Higgs boson mass by a SM-singlet gauge field (not a scalar field as in [14]). In contrast to the attempts based on hidden scalars [14,15,18,19], which are now known to be unable to simultaneously protect the masses of the Higgs boson and the singlet scalar [17], in the present work, we consider a hidden $U(1)$ gauge field V_μ whose invariance is ensured by a Stueckelberg scalar S and whose mass is spontaneously induced by the SM Higgs field. We show that V_μ and S enable cancellation of the quadratic divergence in Higgs boson mass with no quadratic divergence arising in their own masses. It is important that the SM Higgs boson is stabilized at one-loop along with already-stable hidden gauge and Stueckelberg scalar. This phenomenological advantage has important implications not only for stabilizing the Higgs boson mass but also for correlating the SM Higgs field with hidden sectors.

The paper is organized as follows. In Section 2 below, we construct the model starting from the basic Stueckelberg setup. Section 3 is devoted to computation of the quadratic divergences and vanishing of the Higgs mass divergence by fine-tuning. We conclude in Section 4.

2. The model

In this section, we consider a massive Abelian gauge field V_μ accompanied by a real scalar field $S(x)$, introduced to preserve the gauge invariance of the theory. Originally proposed by Stueckelberg [20] and noted afterwards by Pauli [21] that, V_μ satisfies a restricted $U(1)$ gauge invariance, with the gauge function $\Theta(x)$ obeying a massive Klein–Gordon equation. The mechanism provides an alternative to the Higgs mechanism, where the vector boson acquires its mass with the breakdown of the gauge invariance of not the Lagrangian but of the vacuum. These features are encoded in the Stueckelberg model [22]

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}^2 + \frac{1}{2}m^2\left(V^\mu - \frac{1}{m}\partial^\mu S\right)^2 - \frac{1}{2}(\partial_\mu V^\mu + mS)^2, \quad (2)$$

where m is the common mass for V_μ and S. Despite its massive spectrum, this model enjoys a $U(1)_m$ invariance

$$V_\mu(x) \to V'_\mu(x) = V_\mu(x) + \partial_\mu\Theta(x),$$
$$S(x) \to S'(x) = S(x) + m\Theta(x), \quad (3)$$

provided that $(\Box + m^2)\Theta(x) = 0$. Consequently, in spite of its non-vanishing hard mass, V_μ enjoys exact gauge invariance, albeit with a restricted gauge transformation function $\Theta(x)$ [22]. In the massless limit, $m \to 0$, the Stueckelberg Lagrangian (2) reduces to $\mathcal{L}_{m=0} = -\frac{1}{4}V_{\mu\nu}^2 + \frac{1}{2}\partial_\mu S \partial^\mu S$, which is obviously $U(1)_m$ invariant in Lorentz gauge ($\partial_\mu V^\mu = 0$) with an unrestricted $\Theta(x)$. Interestingly, the Stueckelberg scalar S, transforming like the gauge field V_μ in massive case, turns into a gauge-singlet scalar in massless limit.

Inspired from the Stueckelberg model (2), we propose the Higgsed Stueckelberg model

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}^2 + \lambda_1 H^\dagger H\left(V^\mu - \frac{1}{\sqrt{\lambda_1}a_H}\partial^\mu S\right)^2$$
$$- \frac{1}{2}(\partial_\mu V^\mu + \sqrt{\lambda_1}a_H S)^2, \quad (4)$$

where λ_1 is a positive dimensionless constant and a_H is a mass parameter. This model is manifestly gauge-invariant under both the hidden $U(1)_m$ invariance with $m \to \sqrt{\lambda_1}a_H$, and the electroweak gauge group $SU(2)_L \otimes U(1)_Y$. The Higgs potential $V(H) = m_H^2 H^\dagger H + \lambda_H(H^\dagger H)^2$ and hence the total energy is minimized at the Higgs field configuration

$$\langle H^\dagger H\rangle = \begin{cases} \frac{\upsilon_H^2}{2} & \text{if } m_H^2 < 0, \\ 0 & \text{if } m_H^2 > 0, \end{cases} \quad (5)$$

where $\upsilon_H = \sqrt{-\frac{m_H^2}{\lambda_H}}$ is the Higgs VEV in the broken phase ($m_H^2 < 0$), to which masses of the SM particles are all proportional. In this phase electroweak gauge group $SU(2)_L \otimes U(1)_Y$ is spontaneously broken down to electromagnetism. In unbroken phase ($m_H^2 > 0$) electroweak group stays exact and all the SM particles but Higgs boson are massless.

From (4) it is clear that, the two phases of the SM directly leave distinguishable effects on the mass of V_μ and kinetic term of S. And the Stueckelberg structure in (2) is achieved properly if the mass parameter a_H can keep track of the two electroweak phases. This feature is implemented into the Higgsed Stueckelberg model (4) by setting

$$a_H = \Re\left(\sqrt{-\frac{m_H^2}{\lambda_H}}\right) = \begin{cases} \upsilon_H & \text{if } m_H^2 < 0, \\ 0 & \text{if } m_H^2 > 0, \end{cases} \quad (6)$$

which obviously dogs the Higgs VEV in (5). It turns out that $\langle H^\dagger H\rangle = a_H^2/2$ in both broken and exact electroweak phases, and $\upsilon_H = a_H$ specifically in the broken phase. This switching ability of a_H ensures that, in the broken phase of electroweak group, there arises, in addition to the massive SM spectrum, a massive vector V_μ with mass $M_V^2 = \lambda_1\upsilon_H^2$ and a massive scalar $m_S^2 = \lambda_1 a_H^2$. In the unbroken phase, however, the Higgs field stands as the only massive field. The rest, including V_μ and S, are all massless. In what follows, we will work in the physical vacuum of the broken electroweak phase and necessarily set $a_H = \upsilon_H$ everywhere.

It is instructive to study the transcription of the Stueckelberg $U(1)_m$ symmetry in (3) into the Higgsed Stueckelberg case. To this end, one notes that the Stueckelberg scalar $S(x)$ facilitates $U(1)_m$ gauge invariance of the hidden sector, and also, helps to keep the Hamiltonian positive definite[1] [20]. In this formalism, Lorentz

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[1] Note that the last term in (4) can also be written as $\mathcal{L}_{gf} = -\frac{1}{2\alpha}(\partial_\mu V^\mu + \alpha\sqrt{\lambda_1}\upsilon_H S)^2$, where α is a real parameter, similar to 't Hooft's parametrization for Abelian Higgs model. The choice of $\alpha = 1$ corresponds to the Stueckelberg–Feynman gauge. When $\alpha \neq 1$, the restriction on the gauge function changes to $(\Box + \alpha\lambda_1\upsilon_H^2)\Lambda(x) = 0$. It is also possible to choose two different parameters α_1

subsidiary condition does not follow from equation of motion. Imposing an operator equation of the form $\partial^\mu V_\mu^{(-)}(x)|\textbf{phys}\rangle = 0$, where $V_\mu^{(-)}(x)$ involves the free field annihilation operators, however, gives rise to conflict between the operator equation and the canonical commutation relations. This puzzle is solved via the introduction of an additional scalar field $S(x)$, replacing the operator equation with $\Phi(x)|\textbf{phys}\rangle \equiv [\partial^\mu V_\mu^{(-)}(x) + mS^{(-)}(x)]|\textbf{phys}\rangle = 0$, where $S^{(-)}(x)$ also involves free field annihilation operators. The operator equation decreases the number of degrees of freedom of the Lagrangian to four. The required constraint to decrease it to three for a massive vector field comes into play with the gauge transformation

$$V_\mu(x) \to V'_\mu(x) = V_\mu(x) + \partial_\mu \Theta(x),$$
$$S(x) \to S'(x) = S(x) + \sqrt{\lambda_1}\upsilon_H \Theta(x), \tag{7}$$

which closely follows the Stueckelberg transformation (3). The $U(1)_m$ invariance is ensured if $(\partial^2 + \lambda_1 \upsilon_H^2)\Theta(x) = 0$. This restricted gauge invariance changes to an unrestricted, standard gauge invariance in the unbroken ($m_H^2 > 0$) electroweak phase in which V_μ and S are massless and non-interacting. Moreover, S is a gauge singlet in this phase. The V_μ and its Stueckelberg companion S do possess identical masses in broken and unbroken phases of the electroweak symmetry. In broken phase, Stueckelberg–Feynman gauge, their propagators read as

$$\Delta_{\mu\nu} = -\frac{ig_{\mu\nu}}{q^2 - m^2}, \qquad \Delta = \frac{i}{q^2 - m^2}, \tag{8}$$

where $m^2 = \lambda_1 \upsilon_H^2$ is the common mass for V_μ and S.

3. Phenomenology

In this section we study quantum corrections to masses of the Higgs boson h and Stueckelberg fields S and V_μ. The main constraint on the model is that Higgs boson must weigh $m_h = 125.9$ GeV [1]. As follows from (4), there are three-point and four-point interactions among the vector boson V_μ, the Stueckelberg field S, and the Higgs field h. The vertex factors are summarized in Appendix A. The Higgsed Stueckelberg hidden sector then modifies the Veltman condition (1) as

$$(\delta m_H^2)_{\text{quad}} = \frac{\Lambda^2}{16\pi^2}\left(\frac{11}{3}\lambda_H + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6g_t^2 + \lambda_1\right), \tag{9}$$

wherein λ_1 shows up as a new degree of freedom. In the philosophy of the original attempts in [14], one can suppress $(\delta m_H^2)_{\text{quad}}$ by choosing λ_1 appropriately. In particular, $(\delta m_H^2)_{\text{quad}}$ vanishes for $\lambda_1 = 4.41$. The V_μ and S are degenerate in mass, and for this specific value of λ_1 they weigh $m = \sqrt{\lambda_1}\upsilon_H = 517$ GeV. This value of λ_1 is certainly large but not too large to break perturbativity ($\lambda_1 = 4.41 \ll 4\pi$). In this sense, Higgsed-Stueckelberg model is safe for employing the usual perturbation theory. In spite of this perturbative safety, one may still want to reduce λ_1 for phenomenological and other reasons. In this case, remedy is in enlarging the hidden sector. One possible way would be to consider not just one Stueckelberg pair (V_μ, S) but various such pairs $(V_\mu^{(i)}, S^{(i)})$ ($i = 1, \ldots, N$) each interacting with the SM Higgs field with a coupling $\lambda_{(i)}$ as in (4). These independent Stueckelberg sectors give rise to a Veltman term as in (9) with the replacement
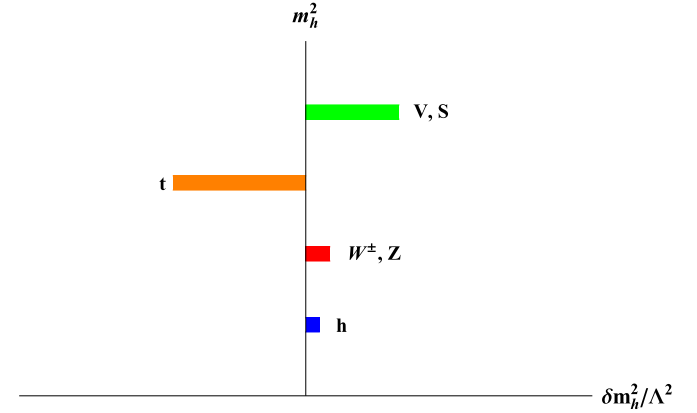
Fig. 1. The schematic representation of the quadratically divergent contributions to Higgs boson mass at one-loop level. Here, h denotes the Higgs boson, $W^\pm$, Z the electroweak bosons, t the top quark, V, S the hidden gauge boson V_μ and the Stueckelberg scalar S, respectively. Higgs mass is protected from destabilizing quantum effects when the hidden gauge sector is included.

$$\lambda_1 \to \sum_{i=1}^{N} \lambda_{(i)} \tag{10}$$

whose fine-tuned value is still 4.41. Now, obviously, a given $\lambda_{(i)}$ does not need to be large. In fact, taking $\lambda_{(i)} = \lambda_1$ for $\forall i$ for simplicity, one finds that $\lambda_1 < 1$ for $N \geq 5$. This means that λ_1 can be sufficiently suppressed by populating hidden sector with some N copies of $(V_\mu^{(i)}, S^{(i)})$. In general, smaller the λ_1 smaller the masses of the Stueckelberg pairs $(V_\mu^{(i)}, S^{(i)})$, and these SM singlets enhance invisible rates at the LHC accordingly. In spite of the possibility of suppression in this way, one should keep in mind that $\lambda_1 = 4.41$ is actually small enough to ensure validity of perturbation theory.

In Fig. 1, a schematic representation of the one-loop quantum corrections to Higgs mass is shown in our extended scenario. As it is apparent from this figure, a hidden Abelian gauge sector splendidly cancels the quadratically divergent contributions to Higgs mass from the SM fields.

It is clear that suppressing $(\delta m_H^2)_{\text{quad}}$ requires λ_1 to be finely tuned. The fine-tuning here is of the same size as the fine-tunings required for hidden scalar sectors [14,15,17–19]. There is one big difference, however. Indeed, the models based on hidden scalars suffer from the fact that masses of the hidden scalars and of the SM Higgs boson cannot be protected simultaneously [17]. The hidden scalar continues to have a mass $\mathcal{O}(\Lambda)$ after suppressing the radiative contribution to the Higgs boson mass. In the Higgs–Stueckelberg model this impasse is overcome. To see this, one notes that mass of the Stueckelberg field does actually receive quadratically divergent radiative corrections from two self energy diagrams (one with Higgs boson in the loop and another with both Higgs and the Stueckelberg field S in the loop). The self energy diagram with a Higgs boson and vector boson V_μ in the loop diverges logarithmically. The spruceness of this scenario emerges at this point in that the quadratically-divergent contributions to the mass of the Stueckelberg field from the two loop diagrams cancel out to give

$$(\delta m_S^2)_{\text{quad}} = 0. \tag{11}$$

In the same manner, the mass of V_μ is protected against quadratically-divergent quantum corrections

$$(\delta m_V^2)_{\text{quad}} = 0. \tag{12}$$

Leaving aside the logarithmic corrections, masses of $V\mu$ and S are found to be UV-insensitive. This is actually expected by gauge invariance because there exists an unbroken $U(1)_m$ invariance in both broken and unbroken electroweak phases. This invariance protects the mass of V_μ. Interestingly, the same invariance also protects the mass of S because S by itself acts like a gauge field when V_μ is massive and becomes a non-interacting $U(1)_m$ singlet when V_μ is massless. Clearly, the radiative stability of the hidden sector can have important implications for modeling 'dark phenomena' like dark matter, dark energy, dark photon and neutrino masses.

4. Conclusion and outlook

The discovery of a new scalar [1] at the LHC, consistent with the SM Higgs boson, has accelerated studies on the UV-sensitivity of the Higgs boson. As opposed to the physical masses of chiral fermions and gauge bosons, which are protected by chiral and gauge symmetries, there is no symmetry principle to protect the Higgs boson mass against quadratically divergent quantum corrections. In the very absence of TeV-scale new physics, one is left with a finely-tuned Higgs sector where nature and degree of fine-tuning vary with the modeling details. In the presence of hidden scalars, despite the protection of the Higgs boson mass the hidden sector itself is UV-unstable. In case the hidden sector is formed by the spacetime curvature scalar, the fine-tuning is severe yet harmless because the SM fields and couplings are immune to its presence. The fine-tuning is as severe as hidden scalars in other field-theoretic approaches.

In this Letter we have shown that a hidden sector spanned by an Abelian vector field whose mass is induced by electroweak breaking and whose gauge invariance is sustained by a Stueckelberg scalar can lead to stabilization of the Higgs boson mass by finely tuning its coupling to the SM Higgs field. In spite of this unavoidable fine-tuning, the Higgsed Stueckelberg model possesses the striking property that the hidden sector is insensitive to the UV scale. This stability, deriving from unbroken hidden gauge invariance, can have important collider, astrophysical and cosmological implications. Indeed, a stable hidden sector can be utilized in constructing viable models of dark matter, dark energy, dark photon and neutrino masses. The model can be tested at the LHC (and its successor FCC) via direct productions of V_μ and S fields.

Acknowledgements

Appendix A

Here we list the vertex factors:

$$\lambda_{hhVV} = 2i\lambda_1 g^{\mu\nu},$$
$$\lambda_{hhSS} = -\frac{2i}{\upsilon_H^2} k_\mu q_\nu g^{\mu\nu},$$
$$\lambda_{hVV} = 2i\lambda_1 \upsilon_H g^{\mu\nu},$$
$$\lambda_{hSS} = -\frac{2i}{\upsilon_H} k_\mu q_\nu g^{\mu\nu},$$
$$\lambda_{hVS} = 2\sqrt{\lambda_1} k_\mu g^{\mu\nu}, \tag{13}$$

where k_μ is the momentum of S. We used $a_H = \upsilon_H$ in a_H dependent vertices.

References

[1] G. Aad, et al., ATLAS Collaboration, Phys. Lett. B 716 (2012) 1, arXiv:1207.7214 [hep-ex];
S. Chatrchyan, et al., CMS Collaboration, Phys. Lett. B 716 (2012) 30, arXiv:1207.7235 [hep-ex].

[2] J. Ellis, T. You, J. High Energy Phys. 1306 (2013) 103, arXiv:1303.3879 [hep-ph];
A. Djouadi, G. Moreau, arXiv:1303.6591 [hep-ph].

[3] V.F. Weisskopf, Phys. Rev. 56 (1939) 72;
K.G. Wilson, Phys. Rev. D 3 (1971) 1818;
L. Susskind, Phys. Rev. D 20 (1979) 2619.

[4] M.J.G. Veltman, Acta Phys. Pol. B 12 (1981) 437.

[5] J.L. Feng, arXiv:1302.6587 [hep-ph];
J.D. Wells, arXiv:1305.3434 [hep-ph];
G.F. Giudice, arXiv:1307.7879 [hep-ph];
G. Altarelli, Phys. Scr. T 158 (2013) 014011, arXiv:1308.0545 [hep-ph].

[6] M. Liu, P. Nath, Phys. Rev. D 87 (9) (2013) 095012, arXiv:1303.7472 [hep-ph];
I. Masina, M. Quiros, Phys. Rev. D 88 (2013) 093003, arXiv:1308.1242 [hep-ph];
X. Lu, H. Murayama, J.T. Ruderman, K. Tobioka, arXiv:1308.0792 [hep-ph];
A. Fowlie, arXiv:1403.3407 [hep-ph].

[7] M. Flechl, CMS Collaboration, ATLAS Collaboration, arXiv:1307.4589 [hep-ex];
J.L. Feng, J.-F. Grivaz, J. Nachtman, Rev. Mod. Phys. 82 (2010) 699, arXiv:0903.0046 [hep-ex].

[8] W.A. Bardeen, FERMILAB-CONF-95-391-T.

[9] D.A. Demir, arXiv:1207.4584 [hep-ph];
M. Heikinheimo, A. Racioppi, M. Raidal, C. Spethmann, K. Tuominen, arXiv:1304.7006 [hep-ph];
G.M. Tavares, M. Schmaltz, W. Skiba, Phys. Rev. D 89 (2014) 015009, arXiv:1308.0025 [hep-ph];
Y. Kawamura, PTEP, Proces. Teh. Energ. Poljopr. 2013 (11) (2013) 113B04, arXiv:1308.5069 [hep-ph];
M. Holthausen, J. Kubo, K.S. Lim, M. Lindner, J. High Energy Phys. 1312 (2013) 076, arXiv:1310.4423 [hep-ph];
O. Antipin, M. Mojaza, F. Sannino, Phys. Rev. D 89 (2014) 085015, arXiv:1310.0957 [hep-ph];
J. Guo, Z. Kang, arXiv:1401.5609 [hep-ph].

[10] D.A. Demir, arXiv:1405.0300 [hep-ph].

[11] A. Salvio, A. Strumia, J. High Energy Phys. 1406 (2014) 080, arXiv:1403.4226 [hep-ph].

[12] V. Agrawal, S.M. Barr, J.F. Donoghue, D. Seckel, Phys. Rev. D 57 (1998) 5480, arXiv:hep-ph/9707380.

[13] R.S. Chivukula, M. Golden, Phys. Lett. B 267 (1991) 233;
R.S. Chivukula, M. Golden, M.V. Ramana, Phys. Lett. B 293 (1992) 400, arXiv:hep-ph/9206255;
J.D. Bjorken, Int. J. Mod. Phys. A 7 (1992) 4189.

[14] M. Ruiz-Altaba, B. González, M. Vargas, CERN-TH.5558/89, 1989;
M. Capdequi Peyranere, J.C. Montero, G. Moultaka, Phys. Lett. B 260 (1991) 138;
A.A. Andrianov, R. Rodenberg, N.V. Romanenko, Nuovo Cimento A 108 (1995) 577, arXiv:hep-ph/9408301;
A. Kundu, S. Raychaudhuri, Phys. Rev. D 53 (1996) 4042, arXiv:hep-ph/9410291;
F. Bazzocchi, M. Fabbrichesi, P. Ullio, Phys. Rev. D 75 (2007) 056004, arXiv:hep-ph/0612280.

[15] I. Chakraborty, A. Kundu, Phys. Rev. D 87 (5) (2013) 055015, arXiv:1212.0394 [hep-ph].

[16] J. McDonald, Phys. Rev. D 50 (1994) 3637, arXiv:hep-ph/0702143;
D.A. Demir, Phys. Lett. B 450 (1999) 215, arXiv:hep-ph/9810453;
C.P. Burgess, M. Pospelov, T. ter Veldhuis, Nucl. Phys. B 619 (2001) 709, arXiv:hep-ph/0011335;
V. Barger, P. Langacker, M. McCaskey, M.J. Ramsey-Musolf, G. Shaughnessy, Phys. Rev. D 77 (2008) 035005, arXiv:0706.4311 [hep-ph];
V. Barger, P. Langacker, M. McCaskey, M. Ramsey-Musolf, G. Shaughnessy, Phys. Rev. D 79 (2009) 015018, arXiv:0811.0393 [hep-ph];
W.-L. Guo, Y.-L. Wu, J. High Energy Phys. 1010 (2010) 083, arXiv:1006.2518 [hep-ph];
A. Djouadi, O. Lebedev, Y. Mambrini, J. Quevillon, Phys. Lett. B 709 (2012) 65, arXiv:1112.3299 [hep-ph];
M. Gonderinger, H. Lim, M.J. Ramsey-Musolf, Phys. Rev. D 86 (2012) 043511, arXiv:1202.1316 [hep-ph];
B. Batell, D. McKeen, M. Pospelov, J. High Energy Phys. 1210 (2012) 104, arXiv:1207.6252 [hep-ph];
S. Baek, P. Ko, W.-I. Park, E. Senaha, J. High Energy Phys. 1211 (2012) 116, arXiv:1209.4163 [hep-ph];

A. Biswas, D. Majumdar, Pramana 80 (2013) 539, arXiv:1102.3024 [hep-ph];

G. Belanger, K. Kannike, A. Pukhov, M. Raidal, J. Cosmol. Astropart. Phys. 1301 (2013) 022, arXiv:1211.1014 [hep-ph];

J.M. Cline, K. Kainulainen, J. Cosmol. Astropart. Phys. 1301 (2013) 012, arXiv:1210.4196 [hep-ph];

D.A. Demir, M. Frank, B. Korutlu, Phys. Lett. B 728 (2014) 393, arXiv:1308.1203 [hep-ph];

N. Haba, K. Kaneta, R. Takahashi, J. High Energy Phys. 1404 (2014) 029, arXiv:1312.2089 [hep-ph].

[17] C.N. Karahan, B. Korutlu, Phys. Lett. B 732 (2014) 320, arXiv:1404.0175 [hep-ph].
[18] I. Chakraborty, A. Kundu, arXiv:1404.3038 [hep-ph].
[19] I. Chakraborty, A. Kundu, arXiv:1404.1723 [hep-ph].
[20] E.C.G. Stueckelberg, Helv. Phys. Acta 11 (1938) 299;
E.C.G. Stueckelberg, Helv. Phys. Acta 11 (1938) 225.
[21] W. Pauli, Rev. Mod. Phys. 13 (1941) 203.
[22] B. Kors, P. Nath, Phys. Lett. B 586 (2004) 366, arXiv:hep-ph/0402047;
B. Kors, P. Nath, J. High Energy Phys. 0507 (2005) 069, arXiv:hep-ph/0503208.