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Nurse–patient assignment models considering patient acuity metrics and nurses' perceived workload

Mustafa Y. Sir [a], Bayram Dundar [b], Linsey M. Barker Steege [c], Kalyan S. Pasupathy [a,*]

[a] Health Systems Engineering, Robert D. and Patricia E. Kern Center for the Science of Health Care Delivery Mayo Clinic, Rochester, MN, United States
[b] Industrial and Manufacturing Systems Engineering University of Missouri, Columbia, MO, United States
[c] School of Nursing University of Wisconsin – Madison, Madison, WI, United States

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ABSTRACT

Patient classification systems (PCSs) are commonly used in nursing units to assess how many nursing care hours are needed to care for patients. These systems then provide staffing and nurse–patient assignment recommendations for a given patient census based on these acuity scores. Our hypothesis is that such systems do not accurately capture workload and we conduct an experiment to test this hypothesis. Specifically, we conducted a survey study to capture nurses' perception of workload in an inpatient unit. Forty five nurses from oncology and surgery units completed the survey and rated the impact of patient acuity indicators on their perceived workload using a six-point Likert scale. These ratings were used to calculate a workload score for an individual nurse given a set of patient acuity indicators. The approach offers optimization models (prescriptive analytics), which use patient acuity indicators from a commercial PCS as well as a survey-based nurse workload score. The models assign patients to nurses in a balanced manner by distributing acuity scores from the PCS and survey-based perceived workload. Numerical results suggest that the proposed nurse–patient assignment models achieve a balanced assignment and lower overall survey-based perceived workload compared to the assignment based solely on acuity scores from the PCS. This results in an improvement of perceived workload that is upwards of five percent.

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1. Introduction and background

The rising costs of healthcare and prevalence of medical errors compel the healthcare industry to more closely scrutinize the cost structure of its operations. To that end, human factors engineering and operations research methods have been widely applied to healthcare to address safety and cost/efficiency problems [1,2]. Approximately 40% of healthcare personnel cost belongs to nursing [3]. Further, according to the Bureau of Health professions, the shortage of full-time equivalent registered nurses is expected to exceed 800,000 nurses by the year of 2020 [4]. Therefore, it is important to focus on the work environment of nurses to improve their job satisfaction and retention, and thereby decrease healthcare cost by increasing efficiency. As reported by Battisto et al. [5], the reasons why nurses leave their current jobs include safety concerns, performing complex job responsibilities such as medication administration, navigating documentation systems, working in an inefficient environment, and musculoskeletal injuries. In a survey conducted [6], 74% of nurses highlighted stress and overwork as a main concern while 62% emphasized musculoskeletal injuries. Ebright et al. [7] claim that 83% of nurses agree that improving nurses' environment and workload promotes nurse retention.

Though lacking a universally accepted definition, workload is generally considered to be a measure of the relationship of the amount of resources demanded by a task situation – the "demands" – to the amount of resources a person has available to complete the task – the "capacity" [8]. Workload can include mental components, which are largely related to a workers' attention capacity and information processing and time demands of a task. Or, workload can be defined primarily as a physical construct relating strength, endurance, and postural demands of a task to energy capacity and biomechanical features of the worker [8].

Current legislative mandates (e.g., California Bill AB 394), that define fixed nurse-patient assignment ratios, are criticized by practitioners and researchers because they fail to account for acuity levels of patients and result in unbalanced distribution of workload

* Corresponding author.
E-mail addresses: Sir.Mustafa@mayo.edu (M.Y. Sir), bd5zc@mail.missouri.edu (B. Dundar), LSteege@wisc.edu (L.M. Barker Steege), PasupathyK@mayo.edu (K.S. Pasupathy).

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among nurses [9]. Translation of our understanding of workload to improve the work environment is much desired. Incorporating information from such analyses into decision support tools and optimization models would result in improved work schedules and conditions and more balanced distribution of workload among nurses.

While assignment is among the classical optimization problems, nurse–patient assignment has not been extensively studied in the operations research literature. On the other hand, nurses scheduling or rostering problems [10–13] and nurse budgeting have been widely studied to reduce costs and improve working conditions and therefore satisfaction of nurses [14]. The ultimate aim of nurse scheduling problems is to assign nurses to certain shifts to decrease healthcare staffing cost, negative patient outcomes, and improve nurse satisfaction [15]. Other work has focused on identifying appropriate nurse–patients ratios with an aim to ameliorate nurses' work condition and improving quality of patient safety and care [16,17,9,18,19].

Staffing is an important tactical decision to ensure sufficient number of nurses are scheduled to care for patients. However, staffing models do not inform nurse manager how to distribute workload among nurses in an equitable manner on a given day. Patient census and associated workload change dynamically. Therefore, nurse–patient assignment models are needed as daily decision support tools. Several approaches have been used to evaluate and improve nurse patient assignments such as simulation [14]; simulation-based optimization [20]; heuristic policies [20]; mixed-integer programming models [21]; stochastic programming [22]; and integer linear programs [3].

Many of these previous studies and approaches aim to equitably distribute, or balance, workload as a function of patient characteristics or acuity measures while assigning patients to nurses. However, workload varies by individual based on a given nurse's capacity for dealing with a specific set of demands. Therefore, for a given patient characteristic or acuity level, the workload will vary depending on nurse characteristics. For example, a nurse may have more experience with surgery patients and be more comfortable handling such patients, whereas another nurse might prefer caring for patients with different indicators. Further, because patient acuity levels are based on a set of indicators representing different types of care demands, two patients having exactly the same classification by PCS, patient acuity classification might result in very different perceived workload for an individual nurse. Existing tools and approaches for supporting patient-assignment decisions fail to account for variations in nurse capacity and response to work demands and are therefore somewhat limited in their application to a specific set of staff. For instance, consider the beginning of a shift, where a charge nurse is responsible for assigning nurses to patients in the current census [23]. Consider the hypothetical assignment situation illustrated in Fig. 1. Both assignments are perfectly balanced in terms of distributing objective acuity scores from the PCS (i.e., each nurse is assigned the same amount of acuity). However, Assignment II significantly lowers the perceived workload for most nurses. In this section we develop optimization models that minimizes average perceived workload while simultaneously ensuring that assignment is balanced both in terms of objective acuity metrics as well as perceived workload. These models can be run at the beginning of a shift to aid a charge nurse to make initial assignments. The proposed methods can also be readily extended to update assignments periodically during a shift to account for dynamic changes in the patient census.

The overall aim of this work was to address this gap through the following objectives: (1) identify the associations between patient acuity indicators from a commercial patient classification system and individual nurses' perceived workload; (2) develop a function to characterize the associations between existing patient acuity measures and individual nurse perceptions; and (3) develop and evaluate a more comprehensive workload balancing nurse–patient assignment optimization model accounting for both objective (patient acuity metrics) and subjective (nurse perceptions of workload) factors. To achieve these objectives, a survey study was conducted with nurses from oncology and surgery nursing units at an

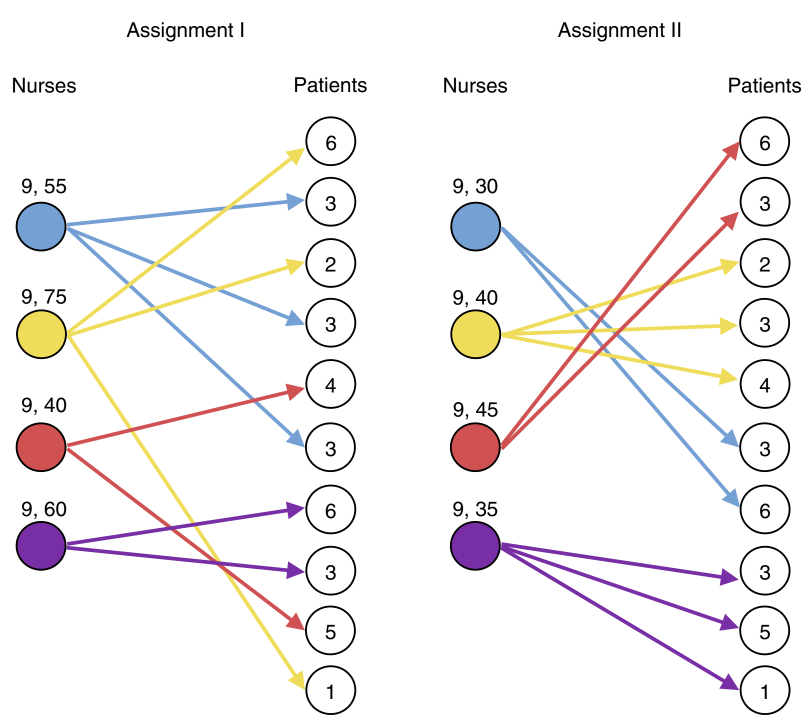


Fig. 1. Two hypothetical nurse–patient assignments for the same patient census. The number inside the circles representing patients are acuity scores. There are two numbers associated with each nurse; the first representing the total acuity score and the second representing the total perceived workload from all assigned patients.

academic medical institution in the Midwestern US. Results of this survey were used to formulate a perceived workload function. This function was used to develop a new workload balancing model, which was then evaluated through comparison to a traditional workload model based solely on acuity scores from a commercial patient classification system.

2. Materials and method

2.1. Patient acuity data

Patient classification systems (PCS) have been widely used to determine how many nursing hours a patient needs for his/her care [24] and help managers to estimate the required number of nursing staff and to accurately determine nurse–patient ratios [25]. PCS are also beneficial for improving patient outcomes, controlling budget, and nurse retention [25]. As such there are a number of commercial PCS available.

AcuityPlus is one such system developed by the QuadraMed Corporation [26]. Methodology used in AcuityPlus has been validated across multiple institutions and over several years [27–30]. AcuityPlus classifies patients into one of six types based on sum of weights associated with each patient acuity indicator (SPAIW). The weights associated with patient acuity indicators and the ranges by which the patient are classified are determined through extensive surveys and data analysis [26,27]. There are several patient acuity indicators available in AcuityPlus. The indicators are proprietary; therefore we only list the general categories of these indicators: activities of daily living; cognitive support; communication support; emotional support; safety management; patient assessment; injury or wound management; observational needs; and medication preparation. For more details on the acuity indicators and QuadraMed's inpatient classification system, the reader is referred to [31,32]. A nurse classifying a patient answers yes or no to each acuity indicator question such as "Does the patient need observation for fall risk?" Detailed information about the definition and examples of appropriate and inappropriate applications of patient acuity indicators can be found in the user manual of the AcuityPlus system [26].

The patient acuity data set was obtained from the oncology and surgery nursing units in an academic medical institution in the Midwestern United States, for all patients from January 1, 2013 until April 9, 2013 (2865 patients who stayed in the oncology nursing unit and 3241 patients who stayed in the surgery nursing unit). The institution has been using the AcuityPlus patient classification system for about twenty years for managing nursing resources. Table 1 lists the available patient information recorded in the AcuityPlus dataset.

Fig. 2 shows that the average SPAIW changes from week to week in the surgery and oncology nursing units. This implies that the mean patient acuity is different between these two nursing units. Fig. 3 shows the percentage of patients having a specific acuity indicator; again, the frequency and distribution of patient acuity indicators are very different for the two nursing units.

Table 1
Patient information recorded in the AcuityPlus dataset obtained from an academic hospital in the Midwestern US.

Field name	Data type
Nursing Unit	Text
Arrival Date	Date & Time
Sum of Patient Acuity Indicator Weights (SPAIW)	Number
Patient Type	Integer
Room	Text
Patient Acuity Indicator 1	Binary
Patient Acuity Indicator 2	Binary
⋮	⋮
Patient Acuity Indicator 26	Binary

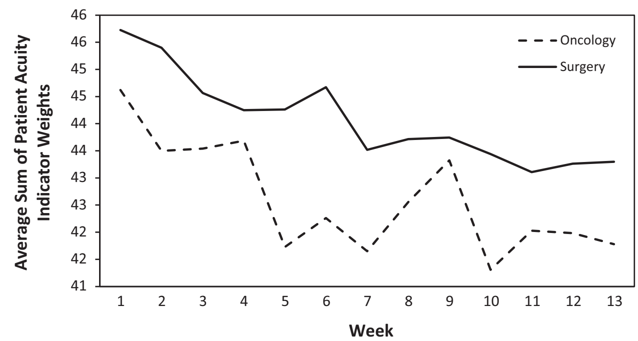


Fig. 2. Weekly average SPAIW for oncology and surgery nursing units.

Table 2 provided initial motivation for this work. As can be seen in the table, patients with the same or close patient types and SPAIW can have different patient acuity indicators. For example, two patients were admitted to the oncology nursing unit on the first day of January (the top two rows in Table 2). Their patient type (3) and SPAIW (42) are exactly the same. However, the first patient has patient acuity indicators 2, 5, 14, and 19 while the second one has indicators 2, 9, 14, and 19. The first patient has indicator 5, which implies that the patient needs assistance of 2–3 caregivers during activities of daily living. The second patient has indicator 9, which indicates that the patient needs behavior/emotional management. These two indicators require very different nursing care and nurses might have very different perception as to how each patient acuity indicator impacts their workload (see Section 2.2). Table 2 shows several similar examples, which clearly show that simply using patient type or SPAIW when assigning nurses to patient may not be ideal.

2.2. Survey-based perceived workload

To identify individual nurse perceptions of workload, a survey study was conducted with all nurses in the selected surgery and oncology units. Approximately 56 nurses were recruited to participate through email. 45 nurses completed the survey (25 from the oncology unit and 20 from the surgery unit) for an 80% completion rate. Nine nurses had partially missing data and were therefore removed from the survey data set, resulting in a final data set based on responses from 36 nurses (23 from the oncology nursing unit, 13 from the surgery nursing unit). Prior to any survey data collection, the study was approved by the local Institutional Review Board.

The survey was comprised of seven questions (see Appendix A). The first five questions captured nurse demographic and work experience, including age, gender, experience level, the highest educational degree, and the nurse's primary work unit. In the remaining questions nurses were asked to rate the impact of patient acuity indicators on their perceived workload. Ratings were provided on a six-point Likert scale, ranging from 1 = no impact on workload to 6 = extreme impact on workload (Table 3).

Based on the responses of an individual nurse to the survey questions, we calculate how much their perceived workload increases when a patient is assigned by simply adding the scores from the survey for each indicator that the patient has. According to this workload model, which we referred to as survey-based workload or perceived workload in the remainder, the same patient might result in very different workload profiles for different nurses. This is very different from using the acuity metrics

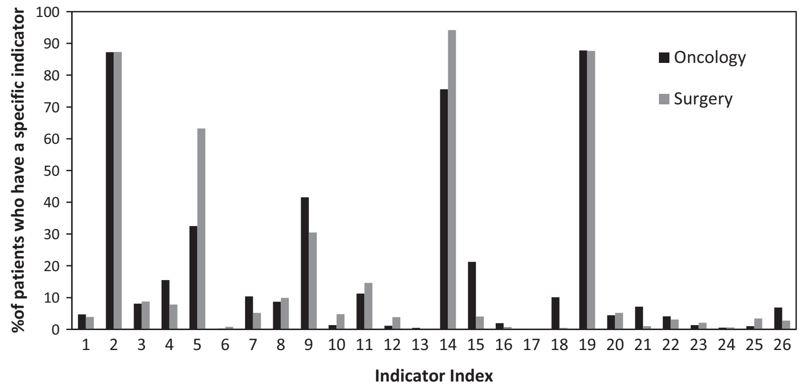


Fig. 3. Percentage of patients who has a specific patient acuity indicator in different nursing units.

Table 2
Example of patients that have the same acuity score but very different acuity indicator distributions.

Unit	Date	Sum of patient acuity indicator weights	Patient type	Patient acuity indicators																
				2	3	4	5	6	7	9	11	12	14	15	18	19	20	23	25	26
Oncology	01/01/13	42	3	✓			✓						✓			✓				
Oncology	01/01/13	42	3	✓						✓			✓			✓				
Surgery	01/29/13	44	3	✓						✓		✓		✓				✓	✓	✓
Surgery	01/30/13	44	3	✓		✓	✓						✓			✓				

Table 3
Six-point Likert scale used in nurse workload survey.

1	No impact on workload
2	Slight impact on workload
3	Some impact on workload
4	Moderate impact on workload
5	High impact on workload
6	Extreme impact on workload

(*e.g.*, patient type or SPAIW) defined in the AcuityPlus PCS during the nurse–patient assignment process because these acuity metrics are fixed for a given patient regardless of which nurse cares for that patient.

2.3. Statistical analysis of survey data

The acuity metrics (*e.g.*, patient type) used in the AcuityPlus PCS are validated periodically through rigorous data analysis at various hospital centers [27–30]. Therefore, one way to validate our survey results is to analyze the relationship between average survey-based workload with acuity metrics (*e.g.*, patient type) used in the AcuityPlus PCS through a regression analysis (see Section 3.2). We also analyze if there are significant differences between individual nurses how they perceive their workload increases for different patient acuity indicators (see Section 3.1).

2.4. Performance measures

The following performance measures are used to analyze the quality of a nurse–patient assignment. For each measure, we give a detailed description and why its minimization is desired.

1. *Difference between Max and Min SPAIWs (MaxMinSPAIW)*: Recall that SPAIW, described in Section 2.1, is a acuity metric used in the AcuityPlus PCS. As mentioned before, this acuity metric is fixed for a given patient regardless of which nurse cares for (i.e., is assigned to) that patient. In other words, it is a population-based measure. Therefore, in most of the nurse–patient assignment models described below, we minimize the difference between the maximum and minimum SPAIWs assigned to nurses so that patients are assigned to nurses in an equitable manner (*i.e.*, total SPAIWs assigned to nurses are as close to each other as possible). We refer to a nurse–patient assignment as "balanced" in terms of patient acuity when MaxMinSPAIW is minimized.
2. *Average Survey-Based Workload (AvgSBW)*: This is the average survey-based (or perceived) workload per nurse for a given nurse–patient assignment. Recall that, according to the survey-based workload model defined in Section 2.2, the same patient might result in a very different workload profiles for different nurses. Therefore, it is possible that AvgSBW changes significantly from one nurse–patient assignment to another.
3. *Difference between Max and Min Survey-Based Workloads (MaxMinSBW)*: Similar to MaxMinSPAIW, this measure is equal to the difference between the maximum and minimum survey-based workload assigned to nurses. We refer to a nurse–patient assignment as "balanced" in terms of perceived workload when MaxMinSBW is minimized.

2.5. Nurse–patient assignment models

Similar to the model by Mullinax et al. [3], the nurse patient assignment policy at the academic medical institution where this study was conducted is based on the acuity metrics (*e.g.*, patient type or SPAIW). Therefore, we adopt their integer linear programming model using acuity metrics from the AcuityPlus PCS, specifically SPAIW, as a benchmark to compare with our proposed nurse–patient assignment models.

The objective functions used in the four nurse–patient assignment models formulated below are based on the performance measures defined in Section 2.4. Table 4 describes how these models differ with regards to performance measures considered.

2.5.1. Nurse–patient assignment model for balancing SPAIWs

We first start with a basic nurse–patient assignment model closely related to formulation proposed by Mullinax et al. [3]. The objective is to balance SPAIWs assigned to nurses (*i.e.*, minimize

Table 4
Difference between nurse–patient assignment models with regards to performance measures considered.

	Balance Patient Acuity	Minimize Perceived Workload	Balance Perceived Workload
Model I	✓		
Model II	✓	✓	
Model III	✓	✓	✓
Model IV		✓	✓

MaxMinSPAIW). This model will be used as a baseline, which represents the current state of practice. We first define notation, which is followed by the formulation.

Sets and indices:

- $\mathcal{N}$ is the set of all nurses, indexed by n.
- $\mathcal{P}$ is the set of all patients, indexed by p.

Parameters:

- l and u are the minimum and maximum number of patients that can be assigned to a nurse, respectively.
- a_p is SPAIW of a patient p obtained from the PCS.

Decision variables:

- $x_{np} = \begin{cases} 1, & \text{ifpatient is assigned to nurse } n, \\ 0, & \text{otherwise.} \end{cases}$
- $z^{\max}$ is the maximum total SPAIW assigned to any nurse.
- $z^{\min}$ is the minimum total SPAIW assigned to any nurse.

The nurse–patient assignment problem for balancing total SPAIWs assigned to nurses can be formulated as follows:

$$\min\ z^{\max} - z^{\min} \tag{1}$$

$$\text{s.t.} \sum_{n \in \mathcal{N}} x_{np} = 1, \quad \forall p \in \mathcal{P} \tag{2}$$

$$l \leqslant \sum_{p \in \mathcal{P}} x_{np} \leqslant u, \quad \forall n \in \mathcal{N} \tag{3}$$

$$\sum_{p \in \mathcal{P}} a_p x_{np} \leqslant z^{\max}, \quad \forall n \in \mathcal{N} \tag{4}$$

$$\sum_{p \in \mathcal{P}} a_p x_{np} \geqslant z^{\min}, \quad \forall n \in \mathcal{N} \tag{5}$$

$$x_{np} \in \{0,1\}, \quad n \in \mathcal{N}, p \in \mathcal{P} \tag{6}$$

$$z^{\max}, z^{\min} \geqslant 0 \tag{7}$$

In the formulation above, the objective function (1) ensures that the difference between the worst-off and best-off nurse (in terms of the total SPAIW assigned to them) is minimized (*i.e.*, MaxMinSPAIW is minimized). This results in the most equitable (*i.e.*, balanced) assignment in terms of distributing overall SPAIW for the entire patient census among nurses. The constraint set (2) assures that each patient is assigned to exactly one nurse. The constraint set (3) limits the number of patients assigned to a nurse to be at least l and at most u. The constraint sets (4) and (5) define variables $z^{\max}$ and $z^{\min}$, respectively. Finally, the constraint set (6) ensures that assignment variables are binary and the constraint set (7) restricts the minimum and maximum total SPAIW to be nonnegative.

2.5.2. Nurse–patient assignment model for minimizing survey-based workload with balanced SPAIWs

The previous formulation (1)–(7) minimizes the range between maximum and minimum SPAIWs assigned to nurses. As discussed earlier, SPAIW of a particular patient is fixed (*i.e.*, does not change by assigning different nurses to that patient); therefore, the total SPAIW of the entire patient census cannot be influenced through assignment. On the other hand, since how a nurse perceives the impact of a patient acuity indicator on their workload is subjective, it may be possible to reduce the overall "perceived" workload by assigning the "right" patient to the "right" nurse. Below, we describe a new assignment model, which minimizes the mean survey-based workload per nurse (*i.e.*, AvgSBW) while ensuring that SPAIWs assigned to nurses are balanced with respect to the optimal solution obtained from solving the previous assignment model (1)–(7). The formulation for the new model is given below following the definition of additional notation.

Additional sets and parameters:

- $\mathcal{I}$ is the set of acuity indicators, indexed by i.
- $\alpha_{pi} = \begin{cases} 1, & \text{if patient } p \text{ isclassifiedas} \\ & \quad \text{having patient acuity indicator } i, \\ 0, & \text{otherwise.} \end{cases}$
- r^* is the optimal solution of problem (1)–(7) representing the range between the worst-off and best-off nurse (in terms of the total SPAIW assigned to them).
- w_{ni} is the survey-based workload score for nurse n and indicator i (see Section 2.2).

The nurse–patient assignment model for minimizing AvgSBW with balanced SPAIWs is formulated as follows:

$$\min \frac{1}{|\mathcal{N}|} \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{I}} \sum_{n \in \mathcal{N}} w_{ni} \alpha_{pi} x_{np} \tag{8}$$

$$\text{s.t.} \sum_{n \in \mathcal{N}} x_{np} = 1, \quad \forall p \in \mathcal{P} \tag{9}$$

$$l \leqslant \sum_{p \in \mathcal{P}} x_{np} \leqslant u, \quad \forall n \in \mathcal{N} \tag{10}$$

$$\sum_{p \in \mathcal{P}} a_p x_{np} \leqslant z^{\max}, \quad \forall n \in \mathcal{N} \tag{11}$$

$$\sum_{p \in \mathcal{P}} a_p x_{np} \geqslant z^{\min}, \quad \forall n \in \mathcal{N} \tag{12}$$

$$z^{\max} - z^{\min} \leqslant r^* \tag{13}$$

$$x_{np} \in \{0,1\}, \quad n \in \mathcal{N}, p \in \mathcal{P} \tag{14}$$

$$z^{\max}, z^{\min} \geqslant 0 \tag{15}$$

In the formulation above, the objective function (8) minimizes AvgSBW. The constraints are identical to those in problem (1)–(7), except that the constraint (13) ensures that the assignment distributes total SPAIW among nurses in the most equitable (*i.e.*, balanced) way. One can simply choose to neglect constraint (13) and only minimize AvgSBW. From a managerial point of view, this may not be ideal since the survey-based workload for a given patient changes from nurse to nurse. Therefore, balancing SPAIWs (which are objective, *i.e.*, does not change by assigning different nurses to the same patient) while minimizing AvgSBW constitutes a reasonable alternative.

2.5.3. Multi-objective nurse–patient assignment model for simultaneously balancing and minimizing survey-based workload with balanced SPAIWs

In this section, we develop a multi-objective nurse–patient assignment model that simultaneously balances the survey-based workload of nurses (*i.e.*, minimizes MaxMinSBW) and minimizes AvgSBW. As was in the previous models, this model ensures that SPAIWs are balanced with respect to the optimal solution obtained from solving the assignment model (1)–(7). While we use the same notation and variables as previous models, we replace $z^{\max}$ and $z^{\min}$ with the following variables:

- y^{max} is the maximum total survey-based workload assigned to any nurse.
- y^{min} is the minimum total survey-based workload assigned to any nurse.

The multi-objective nurse–patient assignment problem for simultaneously minimizing MaxMinSBW and AvgSBW can be formulated as follows:

$$\min\ y^{max} - y^{min} \tag{16}$$

$$\min \frac{1}{|\mathcal{N}|}\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} w_{ni}\alpha_{pi}x_{np} \tag{17}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} x_{np} = 1, \quad \forall p \in \mathcal{P} \tag{18}$$

$$l \leqslant \sum_{p\in\mathcal{P}} x_{np} \leqslant u, \quad \forall n \in \mathcal{N} \tag{19}$$

$$\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{I}} w_{ni}\alpha_{pi}x_{np} \leqslant y^{max}, \quad \forall n \in \mathcal{N} \tag{20}$$

$$\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{I}} w_{ni}\alpha_{pi}x_{np} \geqslant y^{min}, \quad \forall n \in \mathcal{N} \tag{21}$$

$$\sum_{p\in\mathcal{P}} a_p x_{np} \leqslant z^{max}, \quad \forall n \in \mathcal{N} \tag{22}$$

$$\sum_{p\in\mathcal{P}} a_p x_{np} \geqslant z^{min}, \quad \forall n \in \mathcal{N} \tag{23}$$

$$z^{max} - z^{min} \leqslant r^* \tag{24}$$

$$x_{np} \in \{0,1\}, \quad n \in \mathcal{N}, p \in \mathcal{P} \tag{25}$$

$$z^{max}, z^{min} \geqslant 0 \tag{26}$$

$$y^{max}, y^{min} \geqslant 0 \tag{27}$$

In the formulation above, the objective function (16) ensures that the difference between the worst-off and best-off nurses (in terms of the total survey-based workload assigned to them) is minimized (*i.e.*, MaxMinSBW is minimized). The other objective function (17) minimizes AvgSBW. The constraints are identical to those in problem (8)–(15), except that the constraint sets (20), (21), and (27) define variables y^{max} and y^{min}.

Table 5
Experience distribution of nurses who responded to the survey.

Years	Percentage of nurses (%)
1–5	66.67
6–10	4.44
11–15	6.67
16–20	2.22
21–25	15.56
26–	4.44

2.5.4. Multi-objective nurse–patient assignment model for simultaneously balancing and minimizing survey-based workload

The last model is identical to the one in (16)–(27), with the difference that the constraint sets (22), (23), and (24) are removed. This means that balancing SPAIWs assigned nurses is not taken into account in this model. This results in a larger feasible set of possible nurse–patient assignments, and therefore it becomes possible to further reduce AvgSBW and find a more balanced survey-based workload distribution. The formulation is given as follows:

$$\min\ y^{max} - y^{min} \tag{28}$$

$$\min \frac{1}{|\mathcal{N}|}\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} w_{ni}\alpha_{pi}x_{np} \tag{29}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} x_{np} = 1, \quad \forall p \in \mathcal{P} \tag{30}$$

$$l \leqslant \sum_{p\in\mathcal{P}} x_{np} \leqslant u, \quad \forall n \in \mathcal{N} \tag{31}$$

$$\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{I}} w_{ni}\alpha_{pi}x_{np} \leqslant y^{max}, \quad \forall n \in \mathcal{N} \tag{32}$$

$$\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{I}} w_{ni}\alpha_{pi}x_{np} \geqslant y^{min}, \quad \forall n \in \mathcal{N} \tag{33}$$

$$x_{np} \in \{0,1\}, \quad n \in \mathcal{N}, p \in \mathcal{P} \tag{34}$$

$$y^{max}, y^{min} \geqslant 0 \tag{35}$$

2.5.5. Design of numerical experiments

Using nursing survey and AcuityPlus data obtained from the oncology and surgery nursing units at an academic medical institution in the Midwestern US, we present numerical results to show that the nurse–patient assignment models described in Section 2.5 achieve balanced assignment while effectively reducing survey-based workload.

We optimize 200 randomly generated nurse-assignment problems (100 for the oncology unit and 100 for the surgery unit) to compare the performance of various nurse–patient assignment models described in Section 2.5. In the each experiment, 30 patients are randomly selected either from 2865 or 3241 patients from the oncology and surgery nursing units, respectively. The number of patients used in the experiments (*i.e.*, 30) are close to the average number of admitted patients per day for the two nursing units. Moreover, in each experiment, 5 nurses are randomly

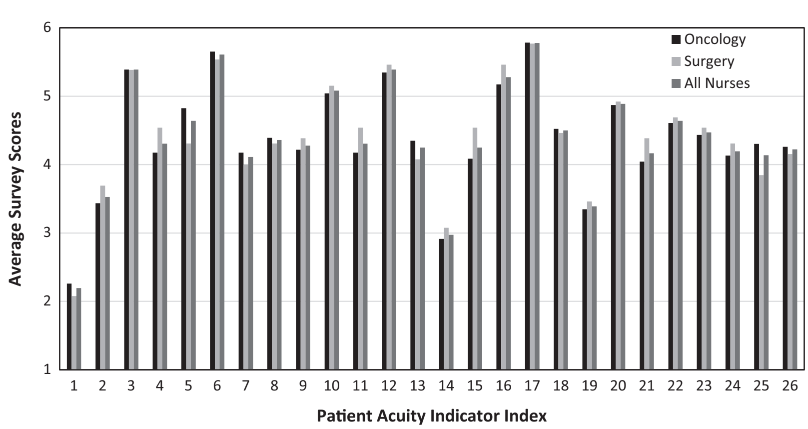


Fig. 4. Average survey scores for each patient acuity indicators. The numbers on the x-axis are the indices of the patient acuity indicators.

selected among either 23 or 13 nurses, who have participated in the survey from the oncology and surgery nursing units, respectively.

The nurse–patient assignment models, listed in Table 4, are implemented using Python (version 2.7) [33] and solved by the Gurobi solver [34]. Running time for most experiments is only a few minutes on a personal computer with an Intel Core i7 (2.80 GHz) processor and 4 GB RAM.

We use a weighted-sum approach described in Lin et al. [35] to solve multi-objective Models III and IV. More specifically, we combine the two objectives (minimize AvgSBW and minimize MaxMinSBW) in these models into a single objective using positive weights that sum up to one. Among the solutions generated by changing the objective weights, we choose the one, which has the smallest sum of normalized values of the two objectives.

To compare various nurse–patient assignment models described above, we use ANOVA to check if there is a statistically significant difference between mean values of various performance measures in different nurse–patient assignment models. Next, we use Tukey's Honestly Significant Differences (HSD) test to determine for which pair of models, these mean values are statistically different. We provide box plots to graphically verify the results of these statistical analyses.

3. Results

3.1. Descriptive statistics for survey

All respondents to the survey were registered nurses and a significant majority (80.56%) were female. The experience distribution of the nurses is given in Table 5, which shows that about two thirds of the respondents have less than five years of experience. The mean age of nurses across the two units is approximately 39 years.

Fig. 4 shows average survey scores (*i.e.*, the average of the weights of the six-point Likert scale selected by nurses in the survey) for each of the patient acuity indicators. Table 6 lists the mean, standard deviation, range, minimum, and maximum survey scores in the two nursing units for each patient acuity indicator. As indicated by relatively large standard deviations and range values, the perceived impact of different patient acuity indicators on workload can be quite different between individual nurses. These significant differences are also apparent in box plots of survey scores for each acuity indicator shown Fig. 5. These results implies that two patients having exactly the same classification by PCS might result in very different perceived workload for an individual nurse. Therefore, using an individualized nurse workload measure can help to make better assignment decisions.

3.2. Survey validation

Fig. 6 show that AcuityPlus patient type and average survey-based workload have a positively correlated linear relation. Simple linear regression analysis also confirms that there exist a statistically significant linear relationship between the AcuityPlus patient type and mean survey-based workload (P-value = 0.001 for the oncology nursing and P-value = 0.003 for the surgery nursing unit). These analyses indicate that the proposed survey-based workload is able to predict the AcuityPlus patient type, which is a well-documented and validated acuity metric.

3.3. Numerical experiments

The design of numerical experiments is described in Section 2.5.5. Through these experiments, we compare the

Table 6
Basic statistics of survey scores for each patient acuity indicator.

Statistics	Patient acuity indicators												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Oncology													
Mean	2.26	3.43	5.39	4.17	4.83	5.65	4.17	4.39	4.22	5.04	4.17	5.35	4.35
Std	1.01	0.99	0.66	1.07	0.78	0.57	0.98	0.89	0.74	0.82	1.03	1.11	0.83
Range	4	3	2	5	3	2	4	3	2	3	4	5	3
Min	1	2	4	1	3	4	2	3	3	3	2	1	3
Max	5	5	6	6	6	6	6	6	5	6	6	6	6
Surgery													
Mean	2.08	3.69	5.38	4.54	4.31	5.54	4.00	4.31	4.38	5.15	4.54	5.46	4.08
Std	0.86	0.63	0.77	1.13	0.95	0.78	1.15	1.32	1.04	0.69	0.78	0.78	1.32
Range	2	2	2	3	3	2	4	4	4	2	3	2	4
Min	1	3	4	3	3	4	2	2	2	4	3	4	2
Max	3	5	6	6	6	6	6	6	6	6	6	6	6

Statistics	Patient acuity indicators												
	14	15	16	17	18	19	20	21	22	23	24	25	26
Oncology													
Mean	2.91	4.09	5.17	5.78	4.52	3.35	4.87	4.04	4.61	4.43	4.13	4.30	4.26
Std	1.12	1.08	0.83	0.52	0.73	0.78	0.69	1.02	1.20	1.08	1.14	1.15	1.18
Range	4	4	3	2	3	3	2	4	5	5	5	5	5
Min	1	2	3	4	3	2	4	2	1	1	1	1	1
Max	5	6	6	6	6	5	6	6	6	6	6	6	6
Surgery													
Mean	3.08	4.54	5.46	5.77	4.46	3.46	4.92	4.38	4.69	4.54	4.31	3.85	4.15
Std	1.26	1.05	0.88	0.83	1.20	1.45	0.95	1.19	1.03	0.88	1.03	1.07	0.99
Range	4	3	3	3	4	5	3	3	3	3	4	3	3
Min	1	3	3	3	2	1	3	3	3	3	2	2	3
Max	5	6	6	6	6	6	6	6	6	6	6	5	6

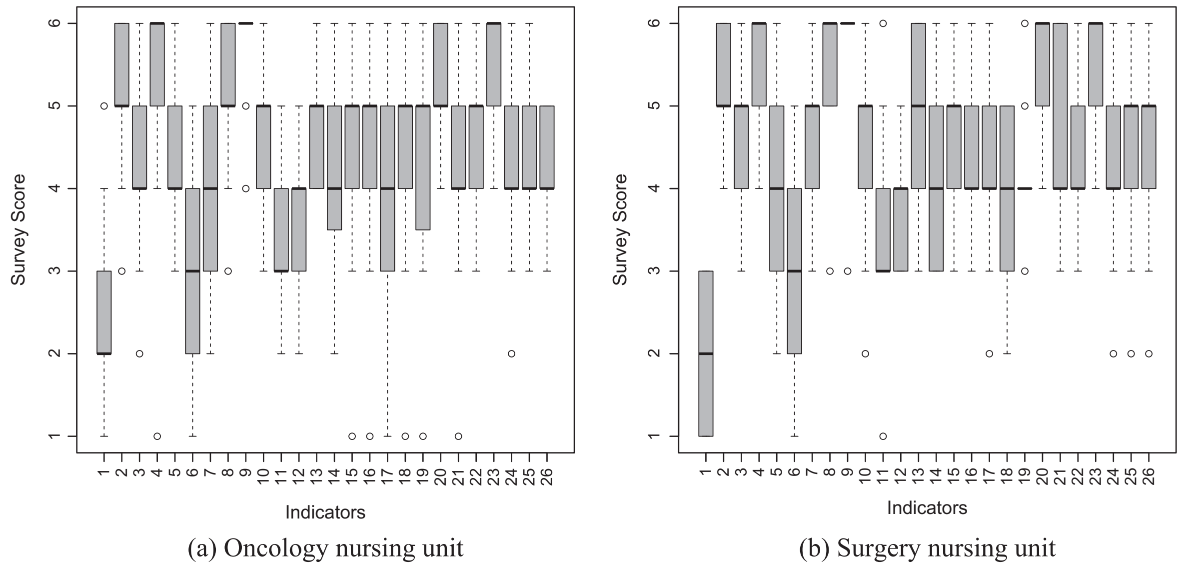


Fig. 5. Box plots of survey scores in the two nursing units for each patient acuity indicator.

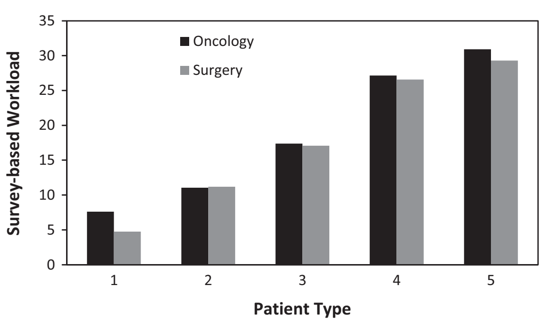


Fig. 6. The relationship between average survey-based workload and patient type acuity metric from AcuityPlus. There is only one patient (among 6106 patients) in the AcuityPlus dataset who has a patient type of 6; this patient is not included in this analysis.

performance of various nurse–patient assignments models defined in Section 2.5 in terms performance measures MaxMinSPAIW, AvgSBW, and MaxMinSBW. Recall that Models I–III ensure that MaxMinSPAIW is at the lowest possible level. In other words, in these models, the difference between the worst-off and best-off nurse (in terms of the total SPAIW assigned to them) is minimized. This results in the most equitable (*i.e.*, balanced) assignment in terms of distributing SPAIW among nurses. In Model IV, on the other hand, the distribution of SPAIW among nurses is not taken into account. Fig. 7 shows the histograms of how much MaxMinSPAIW increase in Model IV with respect to the optimal MaxMinSPAIW level in other models. As can be seen in the figure,

Table 7
Summary statistics for AvgSBW by model and nursing unit.

	Oncology		Surgery	
	Mean	Std. dev.	Mean	Std. dev.
Model I	80.9	6.6	86.1	6.8
Model II	76.6	6.0	82.6	6.5
Model III	77.7	6.1	83.3	6.5
Model IV	74.7	5.9	79.3	6.6

Table 8
Summary statistics for MaxMinSBW by model and nursing unit.

	Oncology		Surgery	
	Mean	Std. dev.	Mean	Std. dev.
Model I	39.8	13.0	53.6	15.2
Model II	32.8	11.8	41.3	11.4
Model III	10.4	7.2	27.2	10.5
Model IV	2.7	1.9	3.5	1.9

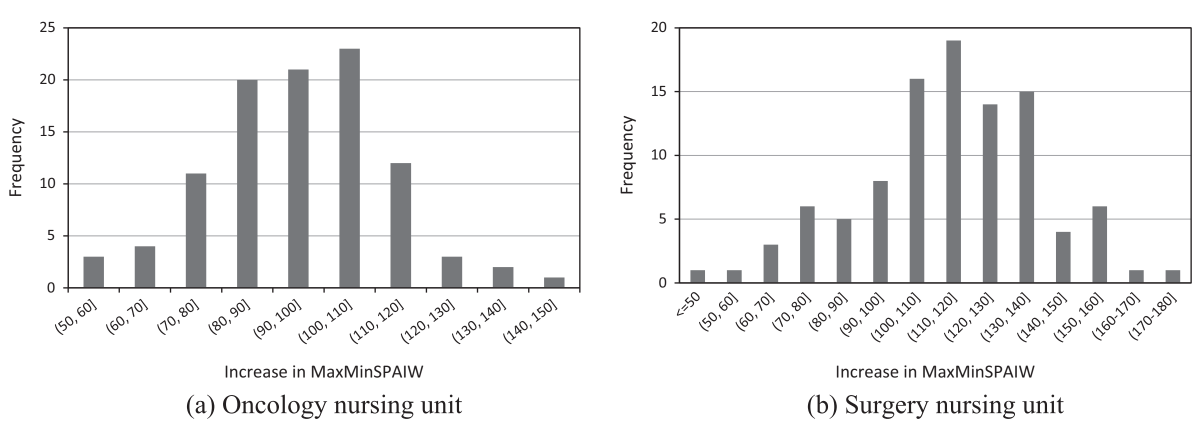


Fig. 7. Model IV vs. other models in terms of balancing SPAIWs. The histograms show how much MaxMinSPAIW increase in Model IV with respect to the optimal MaxMinSPAIW level in other models. Recall that in Models I–III, MaxMinSPAIW is optimized.

Table 9
P-Values from ANOVA analyses comparing AvgSBW and MaxMinSBW of the four nurse–patient assignment models described in Section 2.5. A small *P*-value (typically $\leqslant 0.05$) implies that there is a statistically significant difference between the mean values of the corresponding measure. Such *P*-values are marked with a '*'.

	Oncology	Surgery
AvgSBW	6.666×10^{-11}*	4.845×10^{-11}*
MaxMinSBW	2.2×10^{-16}*	2.2×10^{-16}*

Table 10
Result of the Tukey's Honestly Significant Differences test for pairwise comparison of AvgSBW for the four assignment models with $\alpha = 0.05$. The difference in the mean AvgSBW for the models under the same group is not statistically significant.

Models	Means	Groups
Oncology nursing unit		
Model I	80.9	A
Model II	76.3	B, C
Model III	77.7	B
Models IV	74.7	C
Surgery nursing unit		
Model I	86.1	A
Model II	82.6	B
Model III	83.3	B
Model IV	79.3	C

Table 11
Result of the Tukey's Honestly Significant Differences test for pairwise comparison of MaxMinSBW for the four assignment models with $\alpha = 0.05$. The difference in the mean MaxMinSBW for the models under the same group is not statistically significant.

Models	Means	Groups
Oncology nursing unit		
Model I	39.8	A
Model II	32.8	B
Model III	10.4	C
Models IV	2.7	D
Surgery nursing unit		
Model I	53.6	A
Model II	41.3	B
Model III	27.2	C
Model IV	3.5	D

Model IV results in unbalanced nurse–patient assignments in terms of distribution of SPAIW, which is a population-based acuity metric. In the analysis below, we show that removal of the constraint that ensures MaxMinSPAIW is at the optimal level in Model IV results in better performance in terms of survey-based workload measures (*i.e.*, AvgSBW and MaxMinSBW).

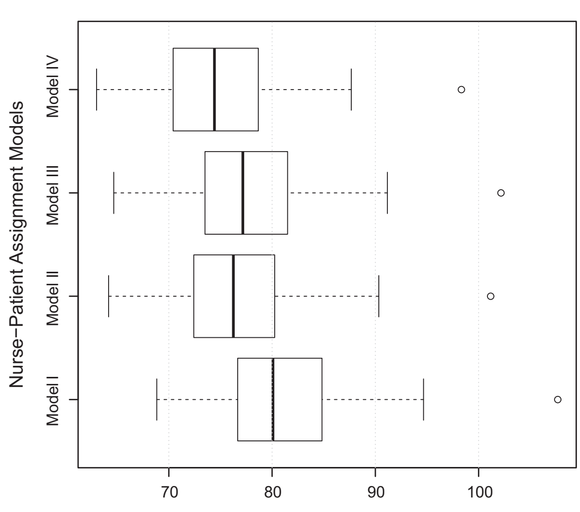
Tables 7 and 8 list summary statistics for AvgSBW and MaxMinSBW performance measures by model and nursing unit, respectively. Mean values of both measures are lower for Models II–IV compared to Model I. This is expected since Model I, adopted from Mullinax et al. [3], does not take survey-based workload into account and is based on population-based acuity metrics. Therefore, it serves as a baseline model.

Table 9 gives *P*-values from the ANOVA analyses comparing AvgSBW and MaxMinSBW of the four nurse–patient assignment models. According to the results, the mean values of both AvgSBW and MaxMinSBW in both nursing units are statistically different between different models.

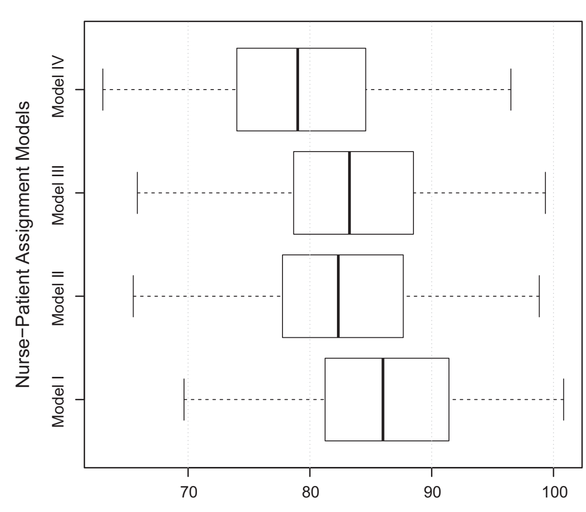
Table 10 gives the Tukey's HSD test results with $\alpha = 0.05$ for AvgSBW. For the oncology nursing unit, Model I, Model III and Model IV are in different groups, indicating that their AvgSBW mean values are statistically different. The mean value of AvgSBW in Model II is in between those in Model III and Model IV. For the surgery nursing unit, Model I, Models II and III, and Model IV form different groups.

Table 11 gives the Tukey's HSD test results for MaxMinSBW. The difference between models in terms of MaxMinSBW is more apparent compared to AvgSBW. All models for both nursing units fall into different groups, indicating that the mean values of MaxMinSBW are statistically different for all models.

Having established that the nurse–patient assignment models are statistically different in terms of the mean values of AvgSBW and MaxMinSBW, we conduct further analysis to determine how the models compare with each other directionally with respect to these performance measures. Fig. 8(a) and (b) and are box plots of AvgSBW resulting from all experiments for the oncology and surgery nursing units, respectively. For both nursing units, Model I has the largest AvgSBW values and while Model IV has the smallest AvgSBW values. While MaxMinSPAIW is set at its optimal level in Models I–III, AvgSBW level is substantially lower in Models II and III. This implies that survey-based workload can be reduced without affecting the balanced distribution of SPAIW among

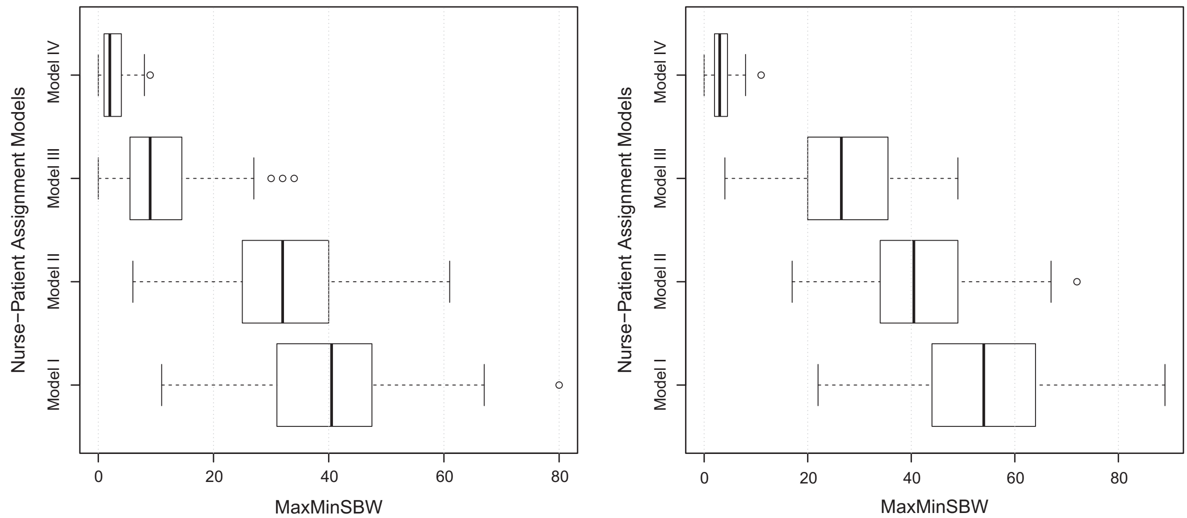


(a) Oncology nursing unit



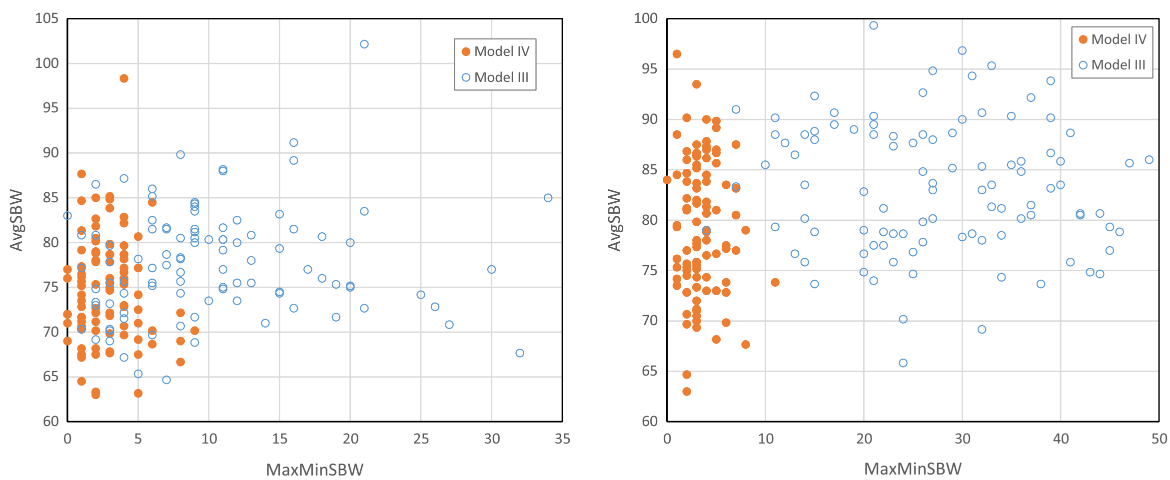
(b) Surgery nursing unit

Fig. 8. Comparison of models with respect to AvgSBW. Each figure is a box plot of AvgSBW from 100 experiments.



(a) Oncology nursing unit (b) Surgery nursing unit

Fig. 9. Comparison of models with respect to MaxMinSBW. Each figure is a box plot of MaxMinSBW from 100 experiments.



(a) Oncology nursing unit (b) Surgery nursing unit

Fig. 10. Multi-objective comparison of Models III and IV with respect to AvgSBW and MaxMinSBW.

nurses. Survey-based workload can be further reduced by simply removing the constraint that sets MaxMinSPAIW at the optimal level as in Model IV (see box plots corresponding to Model IV in Fig. 8). As discussed above, this results in unbalanced nurse–patient assignments in terms of distribution of SPAIW, which is a population-based acuity metric (see Fig. 7).

While reducing the overall survey-based workload (*i.e.*, minimizing AvgSBW) is important, an equally (if not more) important criterion is balancing the survey-based workload of nurses (*i.e.*, minimizing MaxMinSBW). Fig. 9(a) and (b) are box plots of MaxMinSBW resulting from all experiments for the oncology and surgery nursing units, respectively. As can be seen in the figures, the difference between models with respect to MaxMinSBW performance measure is very apparent. Model III and IV are multi-objective models that simultaneously balance the survey-based workload of nurses (*i.e.*, minimizes MaxMinSBW) and reduce the overall survey-based workload (*i.e.*, minimizes AvgSBW). Therefore, there is a large difference between the location and range of MaxMinSBW box plots for these models compared to those for Models I and II, which do not take MaxMinSBW into account. Comparison of Models II and III in terms of resulting AvgSBW and MaxMinSBW values provides insights into the rationale of using a multi-objective model. While box plots in Fig. 8 shows that Model III gives slightly higher values of AvgSBW compared to Model II, it, in return, results in a substantial reduction in MaxMinSBW values as can be seen in Fig. 9.

Models III and IV are identical except that Model III includes a constraint that sets MaxMinSPAIW at the optimal level. Removal of this constraint in Model IV results in a larger feasible region and therefore shifts the Pareto-optimal frontier with respect to performance measure AvgSBW (lower is better) and MaxMinSBW (lower is better) towards lower left corner as shown in Fig. 10. In fact, with respect to AvgSBW and MaxMinSBW, Model IV strictly dominates Model III (*i.e.*, Model IV gives strictly lower values for both AvgSBW and MaxMinSBW than Model III) in 88% and 100% of the experiments in Oncology and Surgery nursing units, respectively.

4. Discussion and conclusion

We develop several nurse–patient assignment (NPA) models to achieve a balanced assignment in terms of population-based acuity metrics and reduce overall survey-based perceived workload. We describe three main performance measures, namely MaxMinSPAIW, AvgSBW, and MaxMinSBW, which are used to evaluate the quality of a nurse–patient assignment. MaxMinSPAIW is based on SPAIW, which is population-based acuity metric defined in the commercial AcuityPlus PCS. AvgSBW and MaxMinSBW are based on our proposed survey-based nurse workload model described in Section 2.2. Our workload model is individualized for each nurse given their responses to the survey questions, which ask how they perceive each patient acuity indicator to impact their workload. The workload of an individual nurse is calculated by simply adding the scores from the survey for each acuity indicator that the patients assigned to that nurse have.

Four nurse–patient assignment models, listed in Table 4, are proposed by considering different combinations of performance measures MaxMinSPAIW, AvgSBW, and MaxMinSBW in the objectives and constraints. AvgSBW is used to minimize overall survey-based workload assigned to all nurses, while MaxMinSPAIW and MaxMinSBW are used to balance the nurse–patient assignment in terms of SPAIW and survey-based workload distributed among nurses. Model I is adopted from Mullinax et al. [3], which only considers balancing SPAIWs assigned to nurses (*i.e.*, minimize MaxMinSPAIW). Since this model does not consider how much nurses perceive a particular patient acuity indicator increases their workload, we incorporate AvgSBW into Model II. In addition, we incorporate MaxMinSBW as a second objective function in multi-objective Models III and IV. While Models I–III ensure that the difference between the worst-off and best-off nurse in terms of the total SPAIW assigned to them is minimized (*i.e.*, MaxMinSPAIW is at the lowest possible level), in Model IV, the distribution of SPAIW among nurses is not taken into account.

Numerical results show that the proposed nurse–patient assignment model can help improving nurses' work conditions and retain them in nursing by reducing and balancing their workload. According to statistical analysis in Section 3, it is possible to reduce the overall survey-based workload (upwards of five percent) and balance its distribution among nurses while keeping MaxMinSPAIW at its lowest possible level (Models II and III vs. Model I). AvgSBW and MaxMinSBW can be further reduced by not taking MaxMinSPAIW into account (Model III vs. Model IV). This, however, results in unbalanced nurse–patient assignments in terms of distributing SPAIW among nurses, which is a population-based acuity metric. Incorporating both individualized (in this paper, based on a survey of nurses) and population-based (in this paper, based on patient acuity metrics from a commercial PCS) workload models is important for nurse–patient assignment and therefore future work will focus on modification of Model III by relaxing the constraint on population-based MaxMinSPAIW by using a multiple of its optimal level.

We envision that a clinical application for the suggested models has the following components:

- *Conduct periodic surveys for determining nurses' perceptions of the impact of various patient acuity indicators on their workload.* These surveys should be conducted periodically (*e.g.*, monthly) over a certain time period (*e.g.*, six months) to establish an accurate model of perceived workload for a each individual nurse.
- *Run an appropriate nurse–patient assignment model from Table 4 for the current patient census at the beginning of each shift.* This will ensure that nurses start their shift with an assignment that is both balanced and optimized for each individual nurse.
- *Dynamically assign incoming patients.* Based on the characteristics of an incoming patient (*i.e.*, his/her acuity indicators), who arrives intra-shift, and current assignment of patients to nurses, calculate how various performance measures described in Section 2.4 change if this patient is assigned to different nurses. Assign the patient to a nurse, which results in the most favorable change in the performance measures.

Future work will also focus on more comprehensive and realistic workload models. In particular, we will develop statistical models to predict types and frequencies of nursing activities from patient acuity indicator data and assess predictive power of patient acuity indicators in explaining the amount of nursing activity. These predictive models will then be used to improve nurse workload models and nurse–patient assignment process. In addition, we will consider other human factors methods to improve the nurse workload models. For example, link analysis can provide important information about the motion of nurses between different locations in a nursing unit, since the demands that comprise nurse workload do not only include caring for patients, but are also based on non-patient care tasks and aspects of the physical and psychosocial environment. Methods such as cognitive pathway analysis can also be used to determine the impact of cognitive shifts and incorporate them into workload models.

Conflict of Interest

The authors do not have any conflict of interests to declare.

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Appendix A. Nursing survey for identifying the associations between various patient acuity indicators and perceived workload

The purpose of this survey is to gather information to understand nurses' perceptions on the impact of various patient acuity indicators identified in the QuadraMed AcuityPlus patient classification system to classify patients on nurses' workload. Participating in this survey is completely voluntary. All of your responses are completely anonymous and will be viewed only by the research team. The entire survey should require approximately 10 min to complete. For the sake of the study, please give honest responses. Thank you in advance for participating.

1. What is your Age ? ____________

2. How many years of experience do you have working as a nurse?

☐ 1-5 ☐ 6-10 ☐ 11-15 ☐ 15-20 ☐ 21-25 ☐ Other (please specify): ____________

3. What is your gender? ☐ Female ☐ Male ☐ Transgender

4. What is your highest nursing degree?

☐ Certified Nursing Assistant (CNA) ☐ Licensed Practical Nurse (LPN)
☐ Registered Nurse (RN) ☐ Advanced Practical Registered Nurse (APRN)
☐ Nurse Practitioner ☐ Doctor of Nursing Practice (DNP)

5. Which department do you work for? ☐ Surgery ☐ Oncology

6. Please rate the following patient acuity indicators from QuadMed AcuityPlus patient classification system in terms of their impact on your workload?

	No impact	Slight impact	Some impact	Moderate impact	High impact	Extreme impact
Indicator 1	○	○	○	○	○	○
Indicator 2	○	○	○	○	○	○
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Indicator 26	○	○	○	○	○	○

7. Are there any other factors, which are not included in the patient acuity indicators above that affect your workload? if there are, please list them in order.

Factor 1: ____________
Factor 2: ____________
Factor 3: ____________
Factor 4: ____________
Factor 5: ____________

Thank you very much for completing this survey!.

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