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On Golay sequences

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Abstract

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Golay sequences are two binary $(+1, -1)$ sequences with nonperiodic autocorrelation function zero. These sequences have a wide range of applications in constructing orthogonal designs and Hadamard matrices, in coding theory, in multislit spectrometry and in surface acoustic wave devices.

In this paper we develop an algorithm for constructing such sequences. We prove that Golay sequences of length $n = 2 \cdot 7^{2t}$ do not exist and we give new proofs of some known results. In particular we show there are no Golay sequences of length 98. We conjecture that there are no Golay sequences of length $2 \cdot q^{2t}$ where q is not the sum of two integer squares.

1. Introduction

Definition. Given the sequence $A = \{a_1, a_2, \ldots, a_n\}$ the nonperiodic autocorrelation function $N_A(s)$ is defined as

$$N_A(s) = \sum_{i=1}^{n-s} a_i a_{i+s} \quad s = 0, 1, \ldots, n-1. \tag{1}$$

If $A(z) = a_1 + a_2 z + \cdots + a_n z^{n-1}$ is the generating function, also called associated polynomial, of the sequence A, then

$$A(z)A(z^{-1}) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j z^{i-j} = N_A(0) + \sum_{s=1}^{n-1} N_A(s)(z^s + z^{-s}) \quad \forall z \neq 0. \tag{2}$$

If $A' = \{a_n, \ldots, a_1\}$ is the sequence A reversed, then

$$A'(z) = a_n + \cdots + a_1 z^{n-1} = z^{n-1} A(z^{-1}). \tag{3}$$

Now if $A = \{a_1, \ldots, a_n\}$, $B = \{b_1, \ldots, b_m\}$ are two sequences of length n and m, then their Kronecker product $A \times B$ is defined as

$$C = A \times B = \{a_1 B, a_2 B, \ldots, a_n B\} \tag{4}$$

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and is an $n \cdot m$ sequence with generating function

$$C(z) = a_1 B(z) + a_2 z^m B(z) + \cdots + a_n z^{(n-1)m} B(z)$$

or

$$C(z) = B(z)A(z^m). \tag{5}$$

Definition. If $A = \{a_1, \ldots, a_n\}$, $B = \{b_1, \ldots, b_n\}$ are two binary $\{+1, -1\}$ sequences of length n and

$$N_A(s) + N_B(s) = 0 \quad \text{for } s = 1, \ldots, n-1 \tag{6}$$

then A, B are called Golay sequences of length n (abbreviated GS(n)). See [4–5].

From this definition and relation (2) we conclude that two $\{+1, -1\}$ sequences of length n are GS(n) if and only if

$$A(z)A(z^{-1}) + B(z)B(z^{-1}) = 2n \quad \forall z \neq 0. \tag{7}$$

Lemma 1 (Golay [5], Turyn [11]). *If A, B are* GS(n) *and C, D are* GS(m), *then*

$$\begin{aligned} X &= A \times \left(\frac{C+D}{2}\right) + B \times \left(\frac{C-D}{2}\right), \\ Y &= A \times \left(\frac{C'-D'}{2}\right) - B \times \left(\frac{C'+D'}{2}\right) \end{aligned} \tag{8}$$

are GS($m \cdot n$).

Proof.

$$X(z) = \frac{C(z)+D(z)}{2} A(z^m) + \frac{C(z)-D(z)}{2} B(z^m),$$

$$Y(z) = z^{m-1}\left(\frac{C(z^{-1})-D(z^{-1})}{2}\right)A(z^m) - z^{m-1}\left(\frac{C(z^{-1})+D(z^{-1})}{2}\right)B(z^m).$$

Hence

$$X(z)X(z^{-1}) + Y(z)Y(z^{-1}) = \frac{(C(z)C(z^{-1}) + D(z)D(z^{-1}))(A(z)A(z^{-1}) + B(z)B(z^{-1}))}{2} = 2mn$$

and $X(z)$, $Y(z)$ are $\{+1, -1\}$ sequences. □

Golay [5], [6] found that GS(n) exist for $n = 2, 10, 26$ so GS(n) exist for $n = 2^a \cdot 10^b \cdot 26^c$ where a, b, c are nonnegative integers.

Golay sequences were conceived originally in connection with the optical problem of infrared multislit spectrometry [5] and later found applications in communications engineering [11] and in constructing orthogonal designs [4, 10, 11]. The basic properties were studied by Golay [5] who proved that GS(n)

exist for $n = 2^a \cdot 10^b \cdot 26^c$, Turyn [11] studied a number of interesting applications. Andres [1], Andres and Stanton [2] and James [8] have developed an algorithm and by exhaustive search have shown that for $n \leqslant 100$ the only undecided lengths are $\{72, 74, 82, 90, 98\}$. Griffin [7] has shown that GS(n) do not exist for $n = 2 \cdot 9^t$. The value $n = 18$ was previously excluded by a complete search by Golay [5], Kruskal [9] and Yang [12].

In this paper we develop an algorithm for constructing GS(n) which explores the concept of generating functions and is different from the Andres–Stanton–James algorithm. We also prove that no GS(n) exist for $n = 2 \cdot 7^{2t}$ and the computer output for $n = 98$ is given.

Finally we verified that the number N of non-isomorphic GS(n) given in [8] is as in the following table.

n	2	4	8	10	16	20	26	32
N	1	1	5	2	36	25	1	336

2. Properties

Lemma 2 (Golay [5]). *If* A, B *are* GS(n), *then*

$$a_i + b_i + a_{n-i+1} + b_{n-i+1} = \pm 2 \quad i = 1, \ldots, n. \tag{9}$$

Proof. Since a_i, b_i are equal to ± 1 we have

$$a_i b_i \equiv (a_i + b_i - 1) \bmod 4. \tag{10}$$

From $N_A(s) + N_B(s) = 0$, $s = 1, 2, \ldots, n-1$ we have

$$\sum_{i=1}^{n-s} (a_i a_{i+s} + b_i b_{i+s}) = 0$$

or

$$\sum_{i=1}^{n-s} (a_i + a_{i+s} + b_i + b_{i+s} - 2) \equiv 0 \bmod 4$$

or

$$\sum_{i=1}^{n-s} (a_i + b_i) + \sum_{i=s+1}^{n} (a_i + b_i) \equiv 2(n-s) \bmod 4, \quad s = 1, \ldots, n-1. \tag{11}$$

Setting $s - 1$ instead of s we have

$$\sum_{i=1}^{n-s+1} (a_i + b_i) + \sum_{i=s}^{n} (a_i + b_i) \equiv 2(n-s+1) \bmod 4, \quad s = 2, \ldots, n. \tag{12}$$

From (11) and (12) we obtain

$$a_{n-s+1} + b_{n-s+1} + a_s + b_s \equiv 2 \bmod 4, \quad s = 2, \ldots, n-1. \tag{13}$$

For $s=1$, (13) is also valid because

$$N_A(n-1)+N_B(n-1)=a_1a_n+b_1b_n=0,$$

i.e.

$$a_n+a_1+b_n+b_1\equiv 2 \bmod 4 \quad \square$$

Before describing the algorithm we need the following: Given a sequence $A=\{a_1,\ldots,a_n\}$ we define m subsequences, for some $m=2,3,\ldots,n$.

$$A_1=\{a_1,a_{1+m},\ldots,a_{1+s_1\cdot m}\},\quad \text{with } s_1=\left[\frac{n-1}{m}\right],$$

$$A_2=\{a_2,a_{2+m},\ldots,a_{2+s_2\cdot m}\},\quad \text{with } s_2=\left[\frac{n-2}{m}\right],$$

$$\vdots$$

$$A_m=\{a_m,a_{2m},\ldots,a_{m+s_m\cdot m}\},\quad \text{with } s_m=\left[\frac{n-m}{m}\right],$$

or

$$A_i=\{a_i,a_{i+m},\ldots,a_{i+s_i\cdot m}\},\quad \text{with } s_i=\left[\frac{n-i}{m}\right],\quad i=1,\ldots,m,$$

with generating function

$$A_i(z)=\sum_{j=0}^{s_i}a_{i+j\cdot m}z^j,\quad i=1,\ldots,m.$$

Then

$$A(z)=A_1(z^m)+zA_2(z^m)+\cdots+z^{m-1}A_m(z^m)$$

or

$$A(z)=\sum_{i=1}^{m}z^{i-1}A_i(z^m). \tag{14}$$

Theorem 1. *If $A=\{a_1,\ldots,a_n\}$, $B=\{b_1,\ldots,b_n\}$ are $\{+1,-1\}$ sequences of length n, then they are* GS(n) *if and only if for some $m=1,\ldots,n$*

$$\sum_{i=1}^{m}(A_i(z^m)A_i(z^{-m})+B_i(z^m)B_i(z^{-m}))=2n,$$

$$\sum_{i=1}^{m-s}(A_i(z^m)A_{i+s}(z^{-m})+B_i(z^m)B_{i+s}(z^{-m})+z^m\sum_{i=1}^{s}(A_{i+m-s}(z^m)A_i(z^{-m})$$

$$+B_{i+m-s}(z^m)B_i(z^{-m}))=0,\quad s=1,\ldots,\left[\frac{m}{2}\right] \tag{15}$$

for every $z\neq 0$.

Proof. Writing $A(z)$, $B(z)$ as in (14) and equating all coefficients of z^t in (7) where $t \equiv s \bmod m$ we find the above relations (15) for $s = 1, \ldots, m-1$. By taking complex conjugates we see that it is enough to take $s = 1, \ldots, [m/2]$. □

For $m = 2$ we have

$$A(z) = A_1(z^2) + zA_2(z^2), \qquad B(z) = B_1(z^2) + zB_2(z^2). \tag{16}$$

From (15) and (16) we conclude (Golay [5]) that there are 6 isomorphic transformations for the GS(n) A, B, i.e.

(i) interchange them,
(ii) reverse the first sequence,
(iii) reverse the second sequence,
(iv) negate the first sequence,
(v) negate the second sequence,
(vi) negate alternate elements in both sequences. (17)

Corollary 1. *If A, B are* GS(n) *and*

(i) $$k_{im} = \sum_{j \equiv i \bmod m} a_j, \qquad r_{im} = \sum_{j \equiv i \bmod m} b_j, \tag{18}$$

(ii) $$K_m = \{k_{1m}, \ldots, k_{mm}\}, \qquad R_m = \{r_{1m}, \ldots, r_{mm}\}, \tag{19}$$

(iii) $$N_K(s) = \sum_{i=1}^{m-s} k_{im}k_{i+s,m}, \qquad N_R(s) = \sum_{i=1}^{m-s} r_{im}r_{i+s,m}, \tag{20}$$

then

$$N_K(0) + N_R(0) = k_{1m}^2 + \cdots + k_{mm}^2 + r_{1m}^2 + \cdots + r_{mm}^2 = 2n, \tag{21}$$

$$N_K(s) + N_R(s) + N_K(m-s) + N_R(m-s) = 0, \quad s = 1, \ldots, \left[\frac{m}{2}\right]. \tag{22}$$

Proof. If we set $z^m = 1$, then $A_i(1) = k_{im}$, $B_i(1) = r_{im}$ and (15) becomes

$$\sum_{i=1}^{m} (k_{im}^2 + r_{im}^2) = 2n,$$

$$\sum_{i=1}^{m-s} (k_{im}k_{i+s,m} + r_{im}r_{i+s,m}) + \sum_{i=1}^{s} (k_{im}k_{i+m-s,m} + r_{im}r_{i+m-s,m}) = 0,$$

i.e.

$$N_K(0) + N_R(0) = 2n,$$

$$N_K(s) + N_R(s) + N_K(m-s) + N_R(m-s) = 0, \qquad s = 1, \ldots, \left[\frac{m}{2}\right]. \qquad \square$$

Note that

$$k_{jm} = k_{j,2m} + k_{j+m,2m}, \qquad r_{jm} = r_{j,2m} + r_{j+m,2m}, \qquad j = 1, \ldots, m. \tag{23}$$

Another restriction for k_{im}, r_{im} is found from relation (9), i.e.

$$a_i + b_i + a_{n-i+1} + b_{n-i+1} \equiv 2 \bmod 4, \quad i = 1, \ldots, n$$

summing over all $i \equiv j \bmod m$ we have

$$k_{jm} + r_{jm} + k_{n-j+1,m} + r_{n-j+1,m} \equiv 2\left(\left[\frac{n-j}{m}\right] + 1\right) \bmod 4, \quad j = 1, \ldots, \left[\frac{m+1}{2}\right] \tag{24}$$

because there are $[(n-j)/m] + 1$ terms a_i, b_i with $i \equiv j \bmod m$. Here $n - j + 1$ is taken $\bmod m$. Whenever m is odd and $2j - 1 \equiv n \bmod m$, then (24) is equivalent to

$$k_{jm} + r_{jm} \equiv \left(\left[\frac{n-j}{m}\right] + 1\right) \bmod 4 \tag{25}$$

because then $j \equiv (n - j + 1) \bmod m$.

Theorem 2. *There are no* GS(n) *for* $n = 2 \cdot 7^{2t}$.

Proof. For convenience take $A_1(z) = z^4 \cdot A(z)$, $B_1(z) = z^4 \cdot B(z)$ where $A(z) = \sum_{i=1}^{n} a_i z^{i-1}$, $B(z) = \sum_{i=1}^{n} b_i z^{i-1}$ are the generating functions of GS(n).

From (7) we have the necessary and sufficient conditions

$$A_1(z)A_1(z^{-1}) + B_1(z)B_1(z^{-1}) = 2^2 \cdot 7^{2t} \quad \forall z \neq 0. \tag{26}$$

Let $k_j = \sum_{i \equiv j \bmod 14} a_i$, $r_j = \sum_{i \equiv j \bmod 14} b_i$, $j = 1, 2, \ldots, 14$. Note that k_j, r_j are odd as the sum of 7^{2t-1} terms and from (24)

$$k_i + k_{15-i} + r_i + r_{15-i} \equiv 2 \bmod 4, \quad i = 1, \ldots, 7. \tag{27}$$

From the equivalence relations (17) we can always take

$$\sum_{i=1}^{7} k_{2i-1} \geq \sum_{i=1}^{7} k_{2i} \geq 0, \qquad \sum_{i=1}^{7} r_{2i-1} \geq \sum_{i=1}^{7} r_{2i}. \tag{28}$$

If we set $z = 1$ or $z = -1$ in (26) we have

$$\left(\sum_{i=1}^{7} (k_{2i-1} + k_{2i})\right)^2 + \left(\sum_{i=1}^{7} (r_{2i-1} + r_{2i})\right)^2 = 2^2 \cdot 7^{2t},$$

$$\left(\sum_{i=1}^{7} (k_{2i-1} - k_{2i})\right)^2 + \left(\sum_{i=1}^{7} (r_{2i-1} + r_{2i})\right)^2 = 2^2 \cdot 7^{2t}.$$

If the sum of two squares is a multiple of 7 then we have $x^2 + y^2 \equiv 0 \bmod 7$ or $x^2 \equiv -y^2 \bmod 7$. Suppose $7 \nmid x$ then the Legendre symbol $(\frac{-1}{7}) = 1$, i.e. -1 is quadratic residue of 7 This is not true, so the only solution to $x^2 + y^2 \equiv \bmod 7$ is if $7 \mid x$ and $7 \mid y$.

Also $\sum_{i=1}^{7} k_{2i-1}$, $\sum_{i=1}^{7} k_{2i}$ are odd. Hence we have the solution

$$\sum_{i=1}^{7} k_{2i-1} + \sum_{i=1}^{7} k_{2i} = 2 \cdot 7^t, \qquad \sum_{i=1}^{7} r_{2i-1} + \sum_{i=1}^{7} r_{2i} = 0,$$
$$\sum_{i=1}^{7} k_{2i-1} - \sum_{i=1}^{7} k_{2i} = 0, \qquad \sum_{i=1}^{7} r_{2i-1} - \sum_{i=1}^{7} r_{2i} = 2 \cdot 7^t,$$

i.e.

$$\sum_{i=1}^{7} k_{2i-1} = 7^t, \qquad \sum_{i=1}^{7} k_{2i} = 7^t, \qquad \sum_{i=1}^{7} r_{2i-1} = 7^t, \qquad \sum_{i=1}^{7} r_{2i} = -7^t. \tag{29}$$

Observe that

$$\begin{aligned}
&\Big(3(k_4 + k_{11}) - \frac{1}{2}\sum_{\substack{i=1\\ i\neq 4}}^{7} (k_i + k_{i+7})\Big)^2 + \Big(3(r_4 + r_{11}) - \frac{1}{2}\sum_{\substack{i=1\\ i\neq 4}}^{7} (r_i + r_{i+7})\Big)^2 \\
&\quad + \frac{7}{4}\Big(\sum_{\substack{1\le i<j\le 7\\ i,j\neq 4}} (((k_i + k_{i+7}) - (k_j + k_{j+7}))^2 \\
&\quad + ((r_i + r_{i+7}) - (r_j + r_{j+7}))^2)\Big) \\
&= 9\sum_{i=1}^{14} (k_i^2 + r_i^2) - 3\sum_{s=1}^{6} (N_K(s) + N_R(s) + N_K(14-s) \\
&\quad + N_R(14-s)) + 18(N_K(7) + N_R(7)) = 9 \cdot 2^2 \cdot 7^{2t}.
\end{aligned} \tag{30}$$

Because from (21), (22) we have

$$\begin{aligned}
&N_K(s) + N_R(s) + N_K(14-s) + N_R(14-s) = 0, \quad s = 1, \ldots, 6, \\
&N_K(7) + N_R(7) = 0, \qquad \sum_{i=1}^{14} (k_i^2 + r_i^2) = 2^2 \cdot 7^{2t}.
\end{aligned} \tag{31}$$

Hence every square in (30) is a multiple of 7^{2t}. Similarly observe that

$$\begin{aligned}
&\Big(3(k_4 - k_{11}) + \frac{1}{2}\sum_{\substack{i=1\\ i\neq 4}}^{7} (k_i - k_{i+7})(-1)^{i-1}\Big) + \Big(3(r_4 - r_{11}) + \frac{1}{2}\sum_{\substack{i=1\\ i\neq 4}}^{7} (r_i - r_{i+7})(-1)^{i-1}\Big)^2 \\
&\quad + \frac{7}{4}\Big(\sum_{\substack{1\le i<j\le 7\\ i,j\neq 4}} (((k_i - k_{i+7})(-1)^{i-1} - (k_j - k_{j+7})(-1)^{j-1})^2 \\
&\quad + ((r_i - r_{i+7})(-1)^{i-1} - (r_j - r_{j+7})(-1)^{j-1})^2)\Big) \\
&= 9\sum_{i=1}^{14} (k_i^2 + r_i^2) - 3\sum_{s=1}^{6} (N_K(s) + N_k(14-s))(-1)^s - 18(N_K(7) + N_R(7)) \\
&= 9 \cdot 2^2 \cdot 7^{2t}.
\end{aligned} \tag{32}$$

So every square in (32) is a multiple of 7^{2t}.

The solutions of (29), (30), (32) do not satisfy (27) so no solution exists. Therefore no GS(n) for $n = 2 \cdot 7^{2t}$ exist. □

3. The algorithm

Since it is difficult to find directly the values of $a_1, \ldots, a_n, b_1, b_2, \ldots, b_n$ we find the values of $k_{1m}, \ldots, k_{mm}, r_{1m}, \ldots, r_{mm}$ as defined in (18). Our algorithm relies on Corollary 1.

To avoid calculating isomorphic GS(n), we can always take, applying the six properties given in (17),

$$k_{11} \geqslant r_{11} \geqslant 0, \qquad k_{12} \geqslant k_{22} \geqslant 0, \qquad r_{12} \geqslant r_{22}. \tag{33}$$

Step 1: Find all k_{11}, r_{11}: k_{11}, r_{11} even, $k_{11} \geqslant r_{11} \geqslant 0$, $k_{11}^2 + r_{11}^2 = 2n$.

Step 2: For every pair k_{11}, r_{11} and a given $m \in \{2, 3, \ldots, n\}$ find $k_{1m}, \ldots, k_{mm}, r_{1m}, \ldots, r_{mm}$ satisfying

(i) $k_{11} = k_{1m} + \cdots + k_{mm}$, $r_{11} = r_{1m} + \cdots + r_{mm}$,

(ii) k_{jm}, r_{jm} both odd or both even if $[(n-j)/m]+1$ is odd or even, $|k_{jm}| \leqslant [(n-j)/m]+1$, $|r_{jm}| \leqslant [(n-j)/m]+1$, $j = 1, \ldots, m$,

(iii) $k_{jm} + k_{n-j+1,m} + r_{jm} + r_{n-j+1,m} \equiv 2([(n-j)/m]+1) \bmod 4$, $j = 1, \ldots, m$,

(iv) $k_{1m}^2 + \cdots + k_{mm}^2 + r_{1m}^2 + \cdots + r_{mm}^2 = 2n$,

(v) $N_K(s) + N_K(m-s) + N_R(s) + N_R(m-s) = 0$, $s = 1, \ldots, [m/2]$, where $N_K(s) = \sum_{j=1}^{m-s} k_{jm} \cdot k_{j+s,m}$, $N_R(s) = \sum_{j=1}^{m-s} r_{jm} \cdot r_{j+s,m}$.

Step 3. For every $k_{1m}, \ldots, k_{mm}$, $r_{1m}, \ldots, r_{mm}$ found in step 2, find $k_{1,2m}, \ldots, k_{2m,2m}, r_{1,2m}, \ldots, r_{2m,2m}$ satisfying

(i) $k_{jm} = k_{j,2m} + k_{j+m,2m}$, $r_{jm} = r_{j,2m} + r_{j+m,2m}$, $j = 1, \ldots, m$.

(ii) Go to step 2(ii), 2(iii), 2(iv), 2(v), setting $2m$ instead of m.

Step 4. Stop, when $m \geqslant n$ and examine if $N_K(s) + N_R(s) = 0$, $s = 1, \ldots, m-1$ because for $m \geqslant n$, $k_{jm} = 0, 1, -1$, $r_{jm} = 0, 1, -1$.

Example. The case $n = 98$.

There is one solution for $m = 1$: $k_{11} = 14$, $r_{11} = 0$.

There are 4 solutions for $m = 7$:

k_{17}	k_{27}	k_{37}	k_{47}	k_{57}	k_{67}	k_{77}	r_{17}	r_{27}	r_{37}	r_{47}	r_{57}	r_{67}	r_{77}
4	4	4	−10	4	4	4	0	0	0	0	0	0	0
2	2	2	2	2	2	2	2	2	2	−12	2	2	2
2	2	2	2	2	2	2	−2	−2	−2	12	−2	−2	−2
0	0	0	14	0	0	0	0	0	0	0	0	0	0

There are no solutions for $m = 14$, so there are no GS(98).

Conjecture. There are no GS(n) for $n = 2 \cdot q^{2t}$ for every $q \equiv 3 \bmod 4$, q prime.

Comment. In light of the number of theoretic properties being used it is likely that a conjecture can be made whenever q is not the sum of two squares. After discussion and comment by J. Seberry and A.L. Whiteman we conjecture the following.

Conjecture. There are no GS(n) for $n = 2 \cdot q^{2t}$ for every q not the sum of two squares.

To prove the non-existence or to find GS(n) for $n = 72$, 74, 82, 90 we must choose the appropriate value of m. However for the values of m we have tried $m = 2, 3, \ldots, 11$ the number of solutions increased beyond the available CPU time.

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