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A global fit to the anomalous magnetic moment, $b \to X_s\gamma$ and Higgs limits in the constrained MSSM

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Abstract

New data on the anomalous magnetic moment of the muon together with the $b \to X_s\gamma$ decay rate are considered within the supergravity inspired constrained minimal supersymmetric model. We perform a global statistical χ^2 analysis of these data and show that the allowed region of parameter space is bounded from below by the Higgs limit, which depends on the trilinear coupling and from above by the anomalous magnetic moment a_μ. The newest $b \to X_s\gamma$ data deviate 1.7σ from recent SM calculations and prefer a similar parameter region as the 2.6σ deviation from a_μ.

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1. Introduction

Recently a new measurement of the anomalous magnetic moment of the muon became available, which suggests a possible 2.6 standard deviation from the Standard Model (SM) expectation [1]: $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (43 \pm 16) \times 10^{-10}$. The theoretical prediction depends on the uncertainties in the vacuum polarization and the light-by-light scattering, see, e.g., the discussion in [2]. However, even with a conservative estimate of the theoretical errors, one has a positive difference Δa_μ of the order of the weak contribution to the anomalous magnetic moment, which opens a window for “new physics”. The most popular explanation is given in the framework of SUSY theories [3–12], since the contribution of superpartners to the anomalous magnetic moment of the muon is of the order of the weak contribution and allows to explain the desired difference Δa_μ. It requires the Higgs mixing parameter to be positive [4] and the sparticles contributing to the chargino–sneutrino ($\tilde\chi^\pm$–$\tilde\nu_\mu$) and neutralino–smuon ($\tilde\chi^0$–$\tilde\mu$) loop diagrams to be relatively light [3].

The positive sign of μ_0 is also preferred by the branching ratio of the b-quark decaying radiatively into an s-quark — $b \to X_s\gamma$ — [13]. Last year the observed value of $b \to X_s\gamma$ was close to the SM expectation, so in this case the sparticles contributing to the chargino–squark ($\tilde\chi^\pm$–$\tilde q$) and charged Higgs–squark ($H^\pm$–$\tilde q$) loops have to be rather heavy in order *not* to contribute to $b \to X_s\gamma$.

However, it was recently suggested that in the theoretical calculation one should use the running c-quark mass in the ratio m_c/m_b, which reduces this ratio from 0.29 to 0.22 [14]. The SM value for $b \to X_s\gamma$ increases from $(3.35 \pm 0.30) \times 10^{-4}$ to $(3.73 \pm 0.30) \times 10^{-4}$ in this case. This value is 1.7σ above the most recent world average of $(2.96 \pm 0.46) \times 10^{-4}$, which is the average from CLEO ($(2.85 \pm 0.35_{\text{stat}} \pm 0.22_{\text{sys}}) \times 10^{-4}$) [15], ALEPH ($(3.11 \pm 0.80_{\text{stat}} \pm 0.72_{\text{sys}}) \times 10^{-4}$) [16] and BELLE

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$((3.36 \pm 0.53_{\text{stat}} \pm 0.42_{\text{sys}}(\pm^{0.50}_{0.54})_{\text{model}}) \times 10^{-4})$ [17]. For the error of the world average we added all errors in quadrature.

As will be shown, the small deviations from the SM for both a_μ and $b \to X_s\gamma$ require now very similar mass spectra for the sparticles.

In the Constrained Minimal Supersymmetric Model (CMSSM) with supergravity mediated breaking terms all sparticle masses are related by the usually assumed GUT scale boundary conditions of a common mass m_0 for the squarks and sleptons and a common mass $m_{1/2}$ for the gauginos. The region of overlap in the GUT scale parameter space, where both a_μ and $b \to X_s\gamma$ are within errors consistent with the data, is most easily determined by a global statistical analysis, in which the GUT scale parameters are constrained to the low energy data by a χ^2 minimization.

In this Letter we present such an analysis within the CMSSM assuming common scalar and gaugino masses and radiatively induced electroweak symmetry breaking. We use the full NLO renormalization group equations [18] to calculate the low energy values of the gauge and Yukawa couplings and the one-loop RGE equations for the sparticle masses with decoupling of the contribution to the running of the coupling constants at threshold. For the Higgs potential we use the full 1-loop contribution of all particles and sparticles. For details we refer to previous publications [19,20].

In principle, one can also require b–τ Yukawa coupling unification, which has a solution at low and high values of the ratio of vacuum expectation values of the neutral components of the two Higgs doublets, denoted $\tan\beta = \langle H_2^0\rangle/\langle H_1^0\rangle$ [19,20]. From Fig. 1 one observes that if the third generation Yukawa couplings at the GUT scale are constrained by the low energy top, bottom and tau masses, they become equal for $\mu < 0$ at $\tan\beta \approx 40$, while for $\mu > 0$ they never become equal, although the difference between the Yukawa couplings is less than a factor three. Since $\mu > 0$ is required by $\Delta a_\mu > 0$ (see below), we do not insist on Yukawa coupling unification and consider $\tan\beta$ to be a free parameter, except for the fact that the present Higgs limit of 113.5 GeV from LEP [21] requires $\tan\beta > 4.3$ in the CMSSM [13].

We found that the allowed area of overlap between $b \to X_s\gamma$ and a_μ can be increased considerably for positive values of the common trilinear coupling A_0 at the GUT scale. For $A_0 > 0$ the present Higgs limit becomes more stringent than for the no-scale models with $A_0 = 0$, as will be shown.

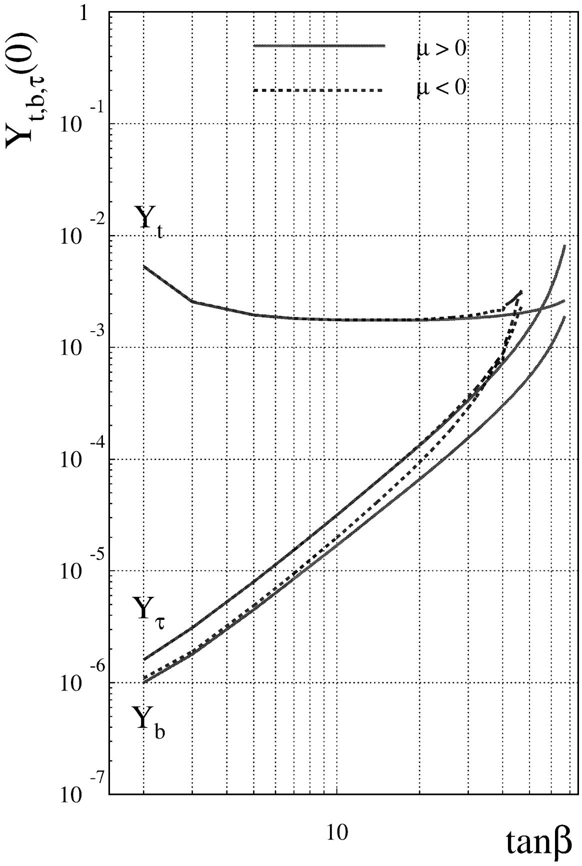

Fig. 1. The dependence of the third generation Yukawa couplings at the GUT scale as function of $\tan\beta$ for $\mu_0 > 0$ and $\mu_0 < 0$, obtained by fitting them to the low energy masses of the top, bottom and tau mass. The results are for a common mass $m_0 = m_{1/2} = 500$ GeV, but for different masses the curves look very similar, except that the 'triple' unification point for $\mu_0 < 0$ shifts between 42 and 48, if the common mass is shifted from 200 to 1000 GeV. Clearly, for $mu_0 > 0$ no b–τ Yukawa unification can be obtained within this CMSSM model.

2. a_μ and $b \to X_s\gamma$ in the CMSSM

The contributions to the anomalous magnetic moment of the muon from SUSY particles are similar to that of the weak interactions after replacing the vector bosons by charginos and neutralinos. The total contribution to a_μ can be approximated by [3]

$$\begin{aligned}\left|a_\mu^{\mathrm{SUSY}}\right| &\simeq \frac{\alpha(M_Z)}{8\pi \sin^2\theta_W}\,\frac{m_\mu^2}{m_{\mathrm{SUSY}}^2} \\ &\quad\times \tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{m_{\mathrm{SUSY}}}{m_\mu}\right) \\ &\simeq 140\times 10^{-11}\left(\frac{100\ \mathrm{GeV}}{m_{\mathrm{SUSY}}}\right)^2 \tan\beta, \qquad (1)\end{aligned}$$

where m_μ is the muon mass, m_{SUSY} is an average mass of supersymmetric particles in the loop (essentially the chargino mass). In our calculations we use the complete one-loop SUSY contributions from [4] with zero phase factors and the additional logarithmic suppression factor as in Eq. (1). The calculated value of a_μ is shown in Fig. 2 as function of $\tan\beta$. Clearly, it is approximately proportional to $\tan\beta$ and its sign depends on the sign of μ_0.[1] Only positive values of μ_0 are allowed for the positive deviation from the SM and in addition the sparticles have to be rather light. However, light sparticles contribute also substantially to the $b \to X_s\gamma$ decay rate. In the past this posed a conflict. However, if one uses in the $b \to X_s\gamma$ calculations the running mass for the charm quark, as suggested recently by Gambino and Misiak, the SM prediction is increased by 11%. In this case the newest world average on $b \to X_s\gamma$ is 1.7σ below the SM, as mentioned in the introduction. Such a deviation is most easily obtained for large $\tan\beta$ and not too heavy sparticles, as shown in Fig. 3. In the upper part the scale uncertainty of the low energy scale μ_b is displayed by the width of the theoretical curves, while in the lower part the dependence on the trilinear coupling A_0 is shown. The scale μ_b was varied between $0.5m_b$ and $2m_b$. For $\tan\beta \approx 40$ only positive values of the Higgs mixing parameter at the GUT scale μ_0 are allowed in agreement with the preferred sign of μ_0 by the anomalous magnetic moment. For intermediate sparticle masses and $\mu_0 > 0$ large values of A_0 and small values of the low energy scale ($\mu_b \approx 0.5m_b$) bring the calculated values of $b \to X_s\gamma$ closest to the data, as can be seen from the left-hand side of Fig. 3. Note that for heavy sparticles (right-hand side of Fig. 3) the effect of the trilinear coupling is small, because the stop mixing is small, if the left- and right-handed stops are much heavier than the top mass.

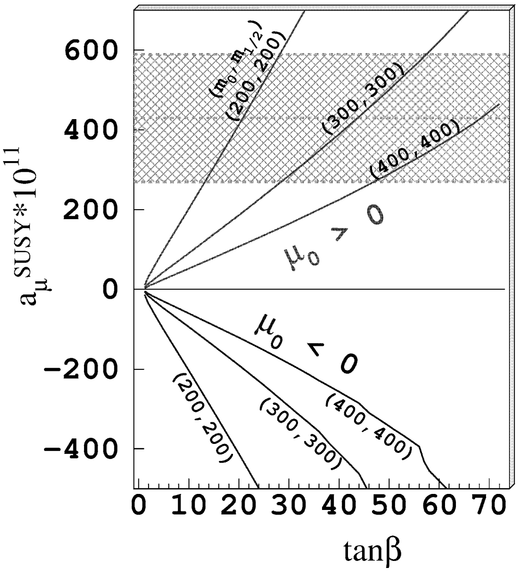

Fig. 2. The dependence of a_μ^{SUSY} versus $\tan\beta$ for various values of the SUSY breaking parameters m_0 and $m_{1/2}$. The horizontal band shows the discrepancy between the experimental data and the SM estimate. Good agreement with the data is only achieved at large $\tan\beta$ and for light sparticles. Clearly, the fit allows only the positive sign of μ.

Fig. 4 shows the values of $b \to X_s\gamma$ and a_μ^{SUSY} as function of m_0 and $m_{1/2}$ for $\tan\beta = 35$. For $b \to X_s\gamma$ the ratio $m_c(\mu)/m_b^{\mathrm{pole}} = 0.22$ was used, while for the NLO QCD contributions the formulae from Ref. [23] were used. The calculated values have to be compared with the experimental values $BR(b \to X_s\gamma) = (2.96 \pm 0.46) \times 10^{-4}$ [15–17] and $\Delta a_\mu = (43 \pm 16) \times 10^{-10}$ [1], which shows once more that $b \to X_s\gamma$ and a_μ^{SUSY} prefer a relatively light supersymmetric spectrum.

To find out the allowed regions in the parameter space of the CMSSM, we fitted both the $b \to X_s\gamma$ and a_μ data simultaneously. The fit includes the following constraints: (i) the unification of the gauge couplings, (ii) radiative electroweak symmetry breaking, (iii) the masses of the third generation particles, (iv) $b \to X_s\gamma$ and Δa_μ, (v) experimental limits on the SUSY masses, (vi) the lightest superparticle (LSP) has to be neutral to be a viable candidate for dark matter. We do not impose b–τ unification, since it prefers $\mu_0 < 0$, as shown in Fig. 1, while Δa_μ requires $\mu_0 > 0$, as shown in Fig. 2. Yukawa unification

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[1] Our sign conventions are as in Ref. [22].

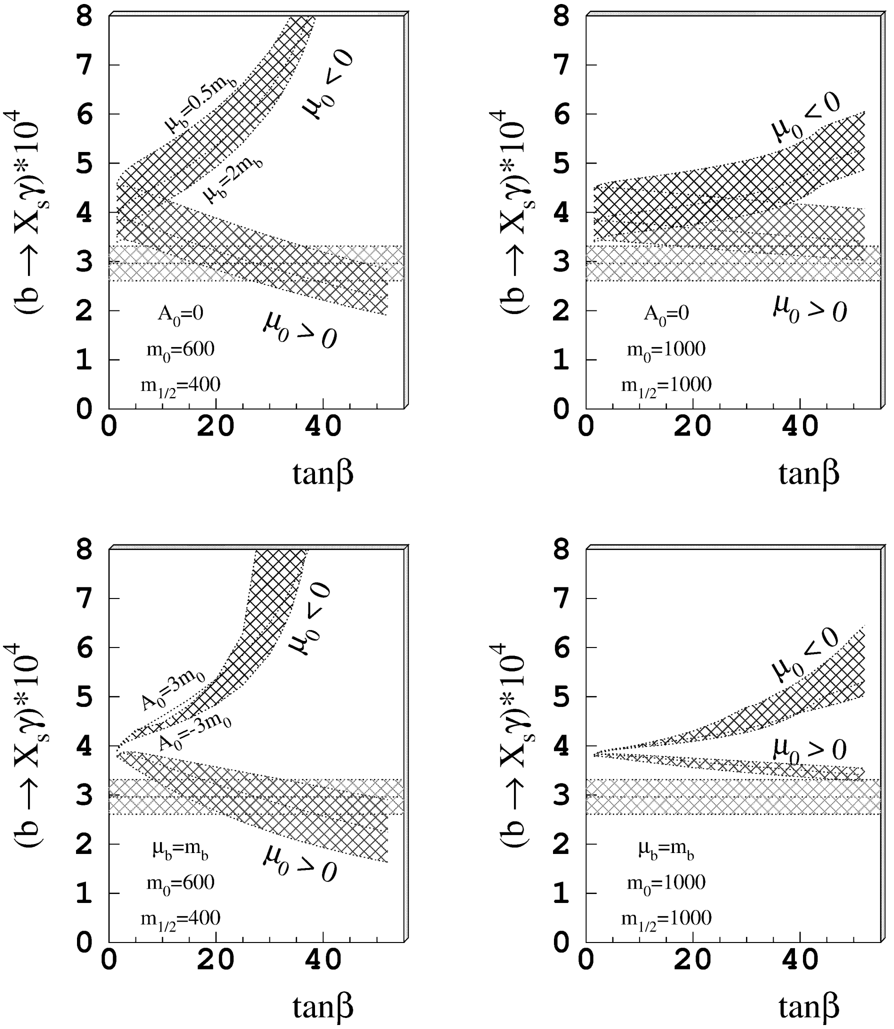

Fig. 3. The upper picture shows the dependence of the $b \to X_s\gamma$ rate on $\tan\beta$ for $A_0 = 0$ and $m_0 = 600$ (1000) GeV, $m_{1/2} = 400$ (1000) GeV at the left (right). For each value of $\tan\beta$ a fit was made to bring the predicted $b \to X_s\gamma$ rate (curved bands) as close as possible to the data (horizontal bands). The width of the predicted values shows the renormalization scale uncertainty from a scale variation between $0.5m_b$ and $2m_b$. The bottom picture shows the same dependence but for a fixed renormalization scale of $1m_b$. The width of the band is given by the variation of A_0 between $-3m_0$ and $3m_0$.

for $\mu_0 > 0$ can only be obtained by relaxed unification of the gauge couplings and nonuniversality of the soft terms in the Higgs sector [24].

The χ^2 contributions of $b \to X_s\gamma$ and the anomalous magnetic moment a_μ in the global fit are shown in Fig. 5 for $A_0 = 0$ and $\tan\beta = 35$. As expected, the χ^2 contribution from $b \to X_s\gamma$ is smallest for heavy sparticles, if $b \to X_s\gamma$ is calculated with $m_c/m_b = 0.29$, while the minimum χ^2 is obtained for intermediate sparticles, if $m_c/m_b = 0.22$ is used. With the newly calculated $b \to X_s\gamma$ values, one can see, that $b \to X_s\gamma$ and a_μ prefer a similar region of the m_0, $m_{1/2}$ plane. Fig. 6 shows the combined χ^2 contributions from $b \to X_s\gamma$ and a_μ^{SUSY} in the m_0, $m_{1/2}$ plane, both in 3D and 2D, for $A_0 = 0$ (top) and A_0 free (bottom). In the latter case the lower 2σ contour from $b \to X_s\gamma$ moves to the lower left corner, but for the preferred value $A_0 \approx 3m_0$, which is the

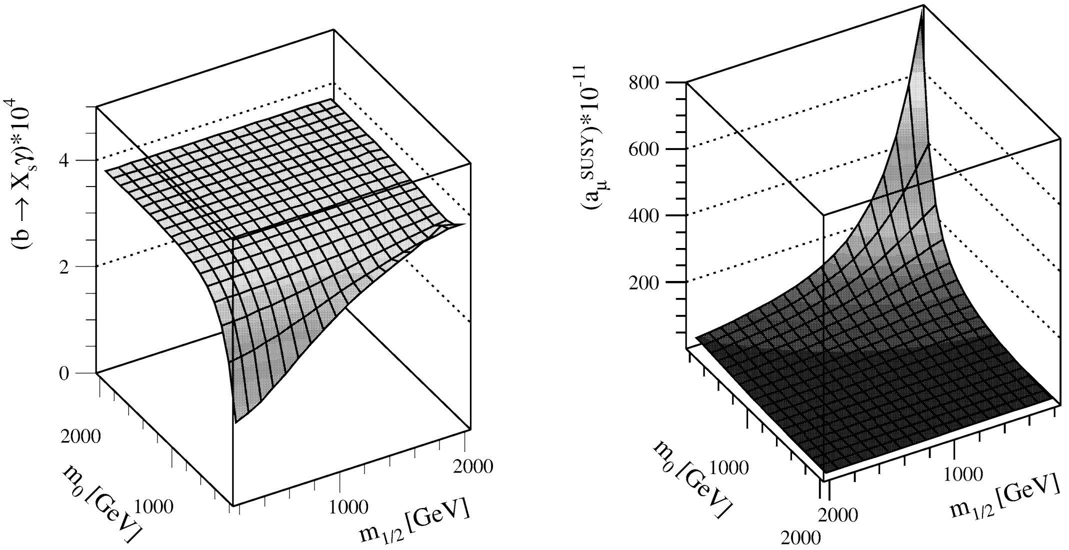

Fig. 4. The values of $b \to X_s\gamma$ and a_μ^{SUSY} in the m_0, $m_{1/2}$ plane for positive μ and $\tan\beta = 35$ to be compared with experimental data $b \to X_s\gamma = (2.96 \pm 0.46) \times 10^{-4}$ and $a_\mu^{\mathrm{SUSY}} = (43 \pm 16) \times 10^{-10}$. One can see that both $b \to X_s\gamma$ and a_μ^{SUSY} prefer relatively light sparticles.

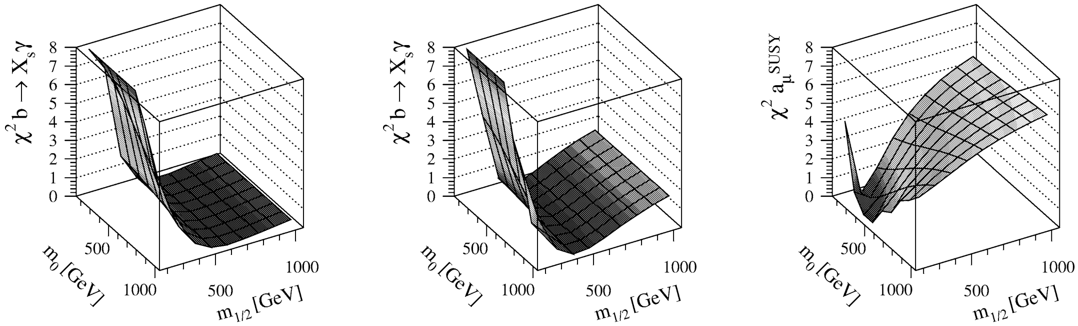

Fig. 5. The individual contributions to χ^2 from $b \to X_s\gamma$ and a_μ in the m_0, $m_{1/2}$ plane for $\tan\beta = 35$, $\mu > 0$ and $A_0 = 0$. On the left handside we show the old contribution from $b \to X_s\gamma$, as calculated with $m_c/m_b = 0.29$, which has the lowest χ^2 for heavy supersymmetric particles. In the middle the contribution from $b \to X_s\gamma$ for $m_c/m_b = 0.22$ is shown, which now has a minimum for intermediate masses. The χ^2 contribution from a_μ is shown on the right handside, which clearly prefers light sparticles.

maximum allowed value in the fit in order to avoid negative stop- or Higgs masses and colour breaking minima, the Higgs bound moves up considerably. The total allowed region is similar in both cases, as shown by the light shaded areas in the contour plots. The 2σ contours from the individual contributions are in good agreement with previous calculations [6,9], but in these paper a simple scan over the parameter space was performed without calculating the combined probability. In addition, $A_0 = 0$ was assumed.

We repeated the fits for $\tan\beta = 20$ and 50, as shown in Fig. 7. For smaller values of $\tan\beta$ the allowed region decreases, since a_μ becomes too small. At larger $\tan\beta$ values the region allowed by a_μ and $b \to X_s\gamma$ increases towards heavier sparticles, as

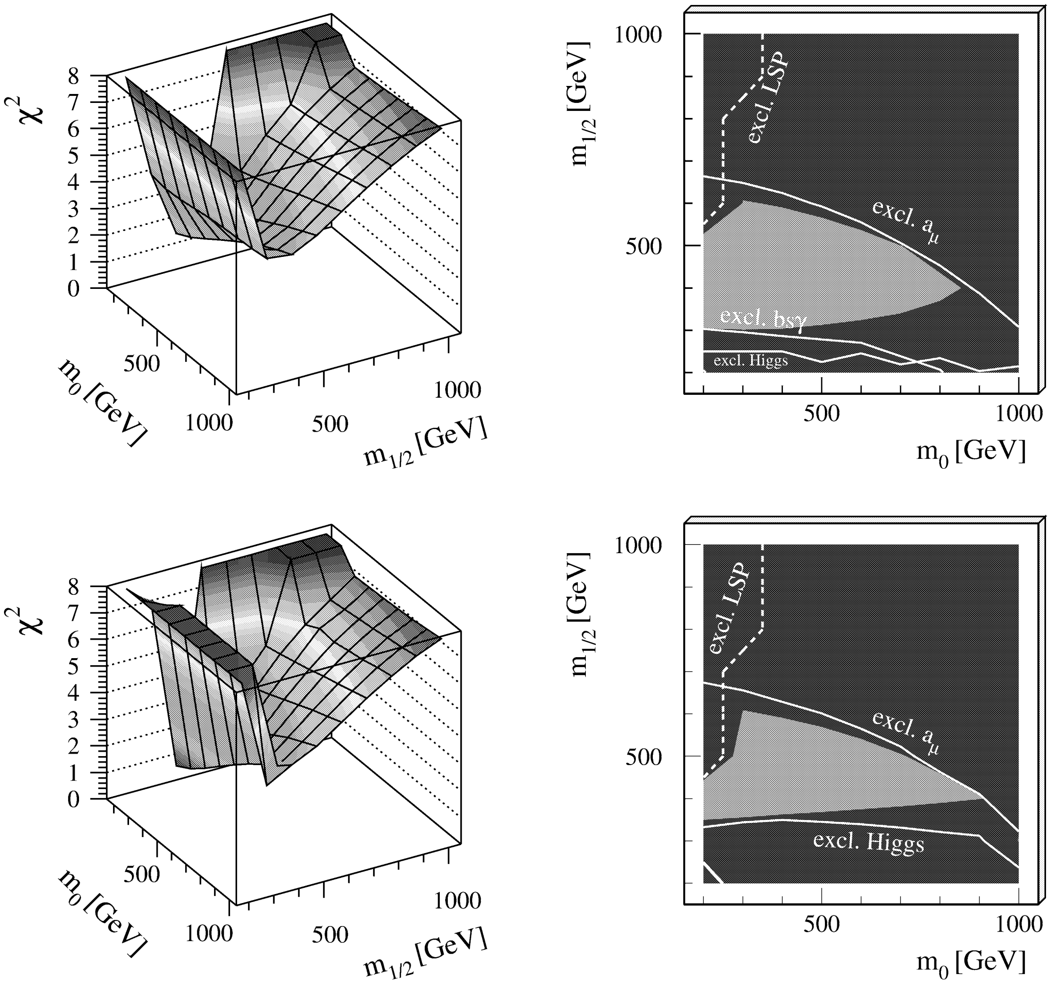

Fig. 6. The upper part shows the χ^2 contribution (left) and its projection (right) in the m_0, $m_{1/2}$ plane for $A_0 = 0$ and $\tan\beta = 35$. The light shaded area is the region, where the combined χ^2 is below 4. The regions outside this shaded region are excluded at 95% C.L. The white lines correspond to the "two-sigma" contours, i.e., $\chi^2 = 4$ for that particular contribution. The lower row shows the same for the fit, where A_0 was left free, in which case $A_0 \approx 3m_0$ (its maximum allowed value in our fit) is preferred in the region where the stop mixing is important, i.e., regions where the left- and right-handed stops are not very heavy compared with the top mass. One observes that with A_0 as a free parameter the Higgs limit becomes the most important lower bound on the SUSY sparticles, while for the no-scale models with $A_0 = 0$ (top) the $b \to X_s\gamma$ rate determines mainly the lower bound.

expected from Eq. (1), but it is cut by the region where the charged stau lepton becomes the Lightest Supersymmetric Particle (LSP), which is assumed to be stable and should be neutral. A charged stable LSP would have been observed by its electromagnetic interactions after being produced in the beginning of the universe. Furthermore, it would not be a candidate for dark matter. The increase of the LSP-excluded area is due to the larger mixing term between the left- and right-handed staus at larger $\tan\beta$.

We conclude that the a_μ measurement strongly restricts the allowed region of the parameter space in the CMSSM, since it excludes the $\mu_0 <$ solution, which was the preferred one from b–τ Yukawa unification. In addition, it prefers large $\tan\beta$ with relatively light

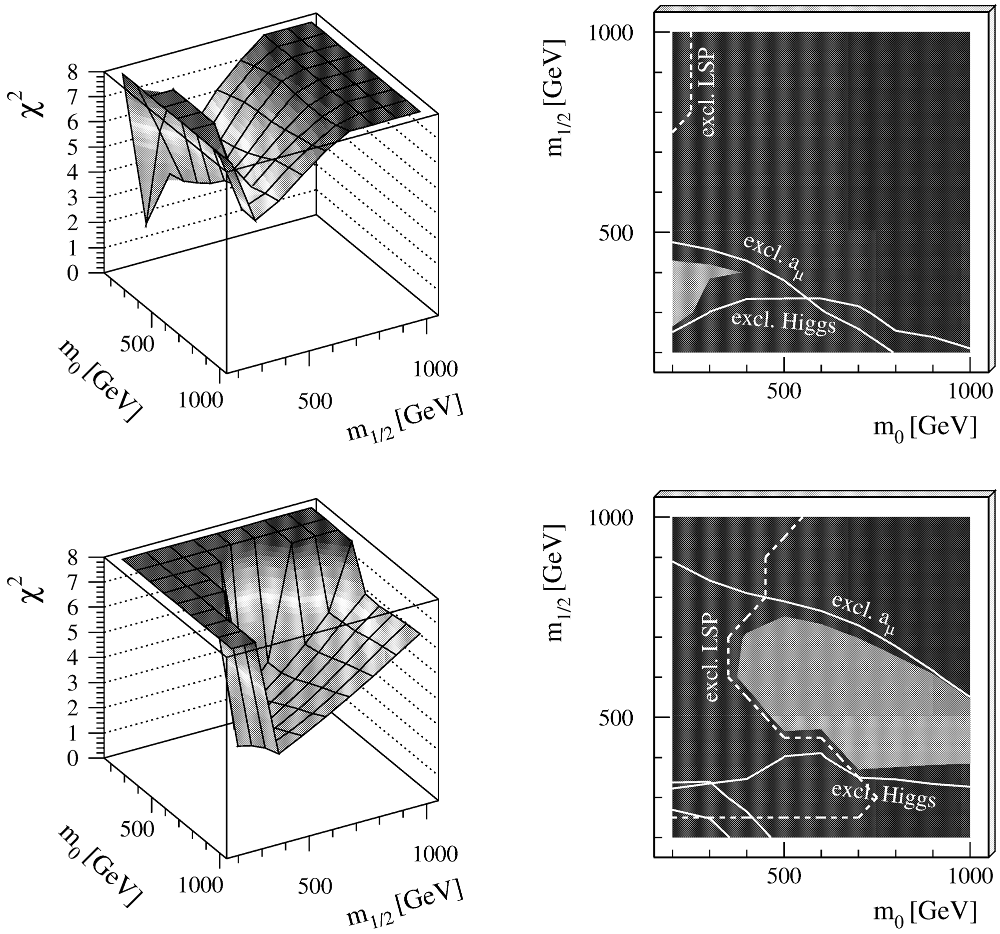

Fig. 7. The total χ^2 and the allowed regions in the parameter space for $\mu > 0$ and $\tan\beta = 20$ (top) and 50 (bottom), with A_0 free, as in Fig. 6 (bottom).

sparticles, if the present deviation from the SM of 2.6σ persists.

At large $\tan\beta$ a global fit including both $b \to X_s\gamma$ and a_μ as well as the present Higgs limit of 113.5 GeV leaves a quite large region in the CMSSM parameter space. Here we left the trilinear coupling to be a free parameter, which affects both the Higgs limit constraint and the $b \to X_s\gamma$ constraint, but in opposite ways, so that the preferred region is similar for the no-scale models with $A_0 = 0$ and models which leave A_0 free.

The 95% lower limit on $m_{1/2}$ is 300 GeV (see Figs. 6, 7), which implies that the lightest chargino (neutralino) is above 240(120) GeV. The 95% upper limit on $m_{1/2}$ is determined by the lower limit on a_μ^{SUSY} and therefor depends on $\tan\beta$ (see Fig. 2). For $\tan\beta = 35(50)$ one finds $m_{1/2} \leqslant 610(720)$ GeV, which implies that the lightest chargino is below 500(590) GeV and the lightest neutralino is below 260(310) GeV.

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