Uncoupled thermoelastic analysis for a thick cylinder with radiation

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Abstract An attempt has been made to study the uncoupled thermoelastic response of thick cylinder of length 2h in which heat sources are generated according to the linear function of the temperature, with boundary conditions of the radiation type. This approach is based upon integral transform techniques, to find out the thermoelastic solution. The results are obtained in terms of Bessel functions in the form of infinite series. [doi:10.1063/2.1202105]

Keywords integral transform, Marchi-Fasulo transform, Marchi-Zgrablich transform, cylinder

The uncoupled thermoelasticity of structural problems is frequently referred to in literature, where the assumption is used in the advanced engineering design problems for the structures under thermal shock loads. Numerical methods, such as the finite element and boundary element techniques, have also been applied to solve this class of problems. A number of analytical solutions of one dimensional uncoupled thermoelasticity problems in rectangular and cylindrical geometries have been published. The uncoupled thermomechanical phenomenon is of great importance in situations where the structures are subjected to highly various thermal conditions such as a sudden change in temperature. A number of practical examples can be seen in nuclear reactors and in structures of spacecraft and propulsion systems. However, there are only a few studies concerned with the two dimensional steady-state thermal stress. Study of the one dimensional thermoelastic waves produced by an instantaneous plane source of heat in homogeneous isotropic infinite and semi-infinite bodies of the Green–Lindsay type is presented by Hetnarski and Ignaczak.[1] Also in generalized thermoelasticity analysis of the laser-induced waves propagating in an absorbing thermoelastic semi-space of the Green–Lindsay type is expressed by Hetnarski and Ignaczak.[2]

Bagri and Eslami[3] presented a solution for the generalized thermoelasticity of a disk. They employed the Laplace transform and the Galerkin finite element method to solve the governing equations. Eraslan and Orean[4] obtained the transient solution of the thermostatic-plastic deformation of internal heat-generating tubes by using the partial differential solver PDECOL for this purpose. The PDECOL is based on the method of lines and uses a finite element collocation procedure for the discretization of the spatial variable. Ting and Chen,[5] Li et al.,[6] and Eslami and Salehzadeh[7] applied the finite element method to solve thermoelastic problems by using governing equations formulated by Nowacki,[8] and Boley and Weiner.[9] Sharma and Sharma,[10] developed a mathematical model for predicting the response of a thick thermoelastic axisymmetric solid plate subjected to sudden lateral loading and thermal shock. Gosn and Sabbaghian,[11] obtained the one dimensional quasi-static coupled problems of thermoelasticity region.

This paper concerns with the uncoupled thermoelastic response of a thick cylinder with radiation type boundary conditions discussed by employing heat sources generated according to the linear function of temperature, further temperature distribution and stresses for thermoelastic problem presented by using integral transform technique defined in Refs. 13 and 14.

We consider a thick cylinder of length $2h$ and the inside and outside radius a and b, respectively. The material of the cylinder is isotropic and all the material properties are assumed to be uniform. For a radially symmetric loading condition, the thermoelasticity equations reduce to the following dimensionless equations (Gosn and Sabbaghian[11])

$$\frac{\partial}{\partial r'}\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(r', u'\right)\right] = \frac{\partial T'}{\partial r'}, \tag{1}$$

$$\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial}{\partial r'}\right) - \frac{\partial}{\partial t}\right]T' = \frac{C}{r}\frac{\partial}{\partial r'}\left(r'\dot{u}'\right), \tag{2}$$

where $u' = u'(r', t')$, $\quad T' = T'(r', t')$,

$$C = \frac{(3\lambda+2\mu)^2\ \alpha^2 T_0}{\rho\,(\lambda+2\mu)\ c} = \frac{\beta^2 T_0}{\rho^2\, c_1^2\, c} = \frac{\eta\beta_k}{\rho c_1^2}, \tag{3}$$

$$\beta = (3\lambda+2\mu)\,\alpha, \quad \eta = \frac{\beta T_0}{k},\ k = \frac{k}{\rho c}. \tag{4}$$

Modifying system of Eqs. (1), (2) in two dimension form and employing heat sources Q which is generated according to the linear function of temperature, we get

$$k\left[\frac{\partial}{\partial r'}\left(\frac{1}{r'}\frac{\partial\theta'}{\partial r'}\right) + \frac{\partial^2\theta'}{\partial z'^2}\right] + Q\left(r', z', t', \theta'\right) = \frac{\partial\theta'}{\partial t'} \tag{5}$$

$$k\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial}{\partial r'}\right) - \frac{\partial}{\partial t}\right]\theta' + Q\left(r', z', t', \theta'\right) = \frac{C}{r}\frac{\partial}{\partial r'}\left(r', \theta', z'\right), \tag{6}$$

where

$$Q\left(r', z', t', \theta'\right) = \Phi\left(r', z', t'\right) + \psi(t')\theta'(r', z', t'), \tag{7}$$

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and

$$\theta'(r',z',t') = T'(r',z',t')\mathrm{e}^{-\int_0^t \psi(\varsigma)\mathrm{d}\varsigma},$$
$$\chi(r',z',t') = \Phi(r',z',t')\mathrm{e}^{-\int_0^t \psi(\varsigma)\mathrm{d}\varsigma}, \tag{8}$$

or for the sake of brevity, we consider

$$\chi(r',z',t') = \frac{\delta(r'-r_0)\delta(z'-z_0)}{2\pi r_0}\mathrm{e}^{-\omega t'}, \tag{9}$$

where $a \le r_0 \le b$, $-h \le z_0 \le h$, $\omega > 0$.

Substituting Eqs. (7), (8) and (9) into system of Eqs. (5) and (6), we obtain

$$k\left[\frac{\partial}{\partial r'}\left(\frac{1}{r'}\frac{\partial T'}{\partial r'}\right) + \frac{\partial^2 T'}{\partial z'^2}\right] + \chi(r',z',t') = \frac{\partial T'}{\partial t'}, \tag{10}$$

$$k\left[\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial}{\partial r'}\right) - \frac{\partial}{\partial t}\right]T' + \chi(r',z',t') = \frac{C}{r}\frac{\partial}{\partial r'}(r',z',T'). \tag{11}$$

Dimensionless quantities r', u', t', T', z' and σ'_{ij} are related to the dimensional quantities r, u, t, T, z and σ_{ij} as

$$r' = \frac{c_1}{k}r,\quad r' = \frac{c_2}{k}t,\quad u' = \frac{\rho c_3}{\beta k T_0}u,$$
$$T' = \frac{T-T_0}{T_0},\ z' = \frac{c_4}{k}z,\quad \sigma'_{ij} = \frac{\sigma_{ij}}{\beta T_0}. \tag{12}$$

Subject to the boundary conditions

$$L_t(T',\,1,\,0,\,0) = T_0 \quad \text{for}\ \ a \le r_0 \le b,\ -h \le z_0 \le h, \tag{13}$$

$$\left.\begin{aligned} L_r(T',\,1,\,k_1,\,a) = 0\\ L_r(T',\,1,\,k_2,\,b) = 0\end{aligned}\right\}\ \text{for all}\ -h \le z_0 \le h, \tag{14}$$

$$\left.\begin{aligned} L_z(T',\,1,\,k_3,\,h) = \exp(-\omega t')\delta(r'-r_0)\\ L_z(T',\,1,\,k_4,\,-h) = \exp(-\omega t')\delta(r'-r_0)\end{aligned}\right\}\ \text{for all}\ a \le r' \le b,\ t' > 0, \tag{15}$$

where $\delta(r'-r_0)$ is the Dirac Delta function, $a \le r' \le b$, $\omega > 0$ is a constant; $\exp(-\omega t')\delta(r'-r_0)$ is the additional sectional heat available on its surface at $z' = h$, T_0 is the reference temperature.

In order to solve the system of uncoupled Eqs. (10) and (11), applying the integral transform defined in Appendix referred with Eqs. (A) and (B) and taking their inversion, we obtain

$$T'(r',z',t') = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{1}{\lambda_m}\left[\wp_{n,m}\mathrm{e}^{-wt} + \left(\overline{\overline{T'}}_0 - \wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\right\}\cdot P_m(z')\,S_0(k_1,k_2,\mu_n r), \tag{16}$$

where $\wp_{n,m} = \frac{F(n,m)}{k(\lambda_{n,m}-w)}$ and

$$F(n,\,m) = \left\{\frac{P_m(h)k}{k_3} - \frac{P_m(-h)k}{k_4} + \frac{P_m(z_0)}{2\pi r_0}\right\}\cdot r_0 S_0(k_1,k_2,\mu_n r_0)$$

Taking into account the first equation of (8), we finally represent the temperature distribution by

$$\theta'(r',z',t') = \sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{1}{\lambda_m}\left[\wp_{n,m}\mathrm{e}^{-wt} + \left(\overline{\overline{T'}}_0 - \wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\cdot P_m(z')\,S_0(k_1,k_2,\mu_n r)\mathrm{e}^{-\int_0^t \psi(\varsigma)\mathrm{d}\varsigma}. \tag{17}$$

The function given in Eq. (17) represents the temperature at every instant and at all points of cylinder when there are conditions of radiation type.

The displacement functions are represented by Goodier's thermoelastic displacement potential $\phi(r',z',t')$ and Michell's function M[12] given by the following relations

$$u'_r = \frac{\partial\phi}{\partial r'} - \frac{\partial^2 M}{\partial r'\,\partial z'}, \tag{18}$$

$$u'_z = \frac{\partial\phi}{\partial r'} + 2(1+\nu)\nabla^2 M - \frac{\partial^2 M}{\partial^2 z'}. \tag{19}$$

In which Goodier's thermoelastic potential must satisfy the equation

$$\nabla^2\phi = \left(\frac{1+\nu}{1-\nu}\right)\ \alpha_t T', \tag{20}$$

and the Michell's function M must satisfy the equation

$$\nabla^2(\nabla^2 M) = 0, \tag{21}$$

where

$$\nabla^2 = \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial}{\partial r'}\right) + \frac{\partial^2}{\partial z'^2}.$$

Substituting the value of $T'(r,z,t)$ from Eq. (16) into Eq. (20), one obtains the thermoelastic displacement function $\phi(r,z,t)$ as

$$\phi(r',z',t') = \frac{(1+\nu)}{(1-\nu)}\alpha'_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\lambda_{n,m})}\left[\wp_{n,m}\mathrm{e}^{-wt} + \left(\overline{\overline{T'}}_0 - \wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}P_m(z')\right]\right\}\cdot S_0(k_1,k_2,\mu_n r)\mathrm{e}^{\int_0^{t'}\psi(\varsigma)\mathrm{d}\varsigma}, \tag{22}$$

Similarly, the solution for Michell's function M is assumed so as to satisfy the governing condition of Eq. (21) as

$$M\left(r',z',t'\right)=\frac{(1+\nu)}{(1-\nu)}\alpha_t'\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m\left(\lambda_{n,m}\right)}\left[\wp_{n,m}\mathrm{e}^{-wt}+\left(\overline{\overline{T}}'_0-\wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\right\}\cdot$$
$$\left[B_{nm}\sinh(\mu_n z)+C_{nm}z\cosh(\mu_n z)\right]S_0(k_1,k_2,\mu_n r)\mathrm{e}^{\int_0^{t'}\psi(\varsigma)d\varsigma}, \tag{23}$$

Using Eqs. (22) and (23) in Eqs. (18) and (19), one obtains the radial and axial displacement as

$$u_r'=\frac{(1+\nu)}{(1-\nu)}\alpha_t'\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m\left(\lambda_{n,m}\right)}\left[\wp_{n,m}\mathrm{e}^{-wt}+\left(\overline{\overline{T}}'_0-\wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\right\}\cdot$$
$$\left[P_m(z)-(B_{nm}\mu_n+C_{nm})\cosh(\mu_n z)+C_{nm}z\sinh(\mu_n z)\right]S_0'(k_1,k_2,\mu_n r)\mathrm{e}^{\int_0^{t'}\psi(\varsigma)\mathrm{d}\varsigma}, \tag{24}$$

$$u_z'=\frac{(1+\nu)}{(1-\nu)}\alpha_t'\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m\left(\lambda_{n,m}\right)}\left[\wp_{n,m}\mathrm{e}^{-wt}+\left(\overline{\overline{T}}'_0-\wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\right\}\cdot$$
$$\left\{\left[\begin{array}{l}-a_m\left[Q_m\sin(a_m z)+W_m\cosh(a_n z)\right]\\-\mu_n^2\left(-1+2v\right)\left[B_{nm}z\sinh(\mu_n z)\right]\end{array}+C_{nm}z\cosh(\mu_n z)-2\left(-1+2v\right)C_{nm}\sinh(\mu_n z)\right]S_0(k_1,k_2,\mu_n r)+\right.$$
$$\left.\mu_n(2(1-\nu))\left[B_{nm}\sinh(\mu_n z)+C_{nm}z\cosh(\mu_n z)\right]\left[\mu_n S_0''(k_1,k_2,\mu_n r)\ \frac{S_0'(k_1,k_2,\mu_n r)}{r}\right]\right\}\mathrm{e}^{\int_0^{t'}\psi(\varsigma)\mathrm{d}\varsigma}. \tag{25}$$

The stress functions on the traction free function are given by

$$\sigma_{z'z'}=\ \sigma_{r'z'}=0,\quad \text{at}\quad z'=\pm h. \tag{26}$$

The components of the stresses are represented by the use of the potential ϕ and Michell's function M as

$$\sigma_{r'r'}=2G\left[\left(\frac{\partial^2\phi}{\partial r'^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z'}\left(\nu\,\nabla^2 M-\frac{\partial^2 M}{\partial r'^2}\right)\right], \tag{27}$$

$$\sigma_{\theta'\theta'}=2G\left[\left(\frac{1}{r'}\frac{\partial^2\phi}{\partial r'}-\nabla^2\phi\right)+\frac{\partial}{\partial z'}\left(\nu\,\nabla^2 M-\frac{\partial M}{\partial r'}\right)\right], \tag{28}$$

$$\sigma_{z'z'}=2G\left\{\left(\frac{\partial^2\phi}{\partial z'^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z'}\left[(2-\nu\,)\,\nabla^2 M-\frac{\partial^2 M}{\partial r'^2}\right]\right\}, \tag{29}$$

and

$$\sigma_{r'z'}=2G\left\{\frac{\partial^2\phi}{\partial z'\partial r'}+\frac{\partial}{\partial z'}\left[(1-\nu\,)\,\nabla^2 M-\frac{\partial^2 M}{\partial z'^2}\right]\right\}. \tag{30}$$

Using Eqs. (22) and (23) in Eqs. (27) to (30), the stress functions are obtained as

$$\sigma_{r'r'}=2G\frac{(1+\nu)}{(1-\nu)}\alpha_t'\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m\left(\lambda_{n,m}\right)}\left[\wp_{n,m}\mathrm{e}^{-wt'}+\left(\overline{\overline{T}}'_0-\wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\right\}\cdot$$
$$\left\{-P_m(z')\left[\frac{S_0'(k_1,k_2,\mu_n r')}{r'}-a_m^2 S_0(k_1,k_2,\mu_n r')\right]+\mu_n^2(\nu-1)\Big\{\mu_n\big\{B_{nm}\cosh(\mu_n z')+C_{nm}[z\sinh(\mu_n z')+\right.$$
$$\cosh(\mu_n z')]\big\}\Big\}S_0''(k_1,k_2,\mu_n r')+\mu_n\nu\Big\{B_{nm}\cosh(\mu_n z')\mu_n+$$
$$C_{nm}\left[z\cosh(\mu_n z)+\cosh(\mu_n z)\right]\Big\}\left[\frac{S_0'(k_1,k_2,\mu_n r')}{r'}+\mu_n S_0(k_1,k_2,\mu_n r')\right]\cdot$$
$$\left.2vC_{nm}\mu_n^2 z\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)\right\}\mathrm{e}^{\int_0^{t'}\psi(\varsigma)\mathrm{d}\varsigma}, \tag{31}$$

$$\sigma_{\theta'\theta'}=2G\frac{(1+\nu)}{(1-\nu)}\alpha_t'\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m\left(\lambda_{n,m}\right)}\left[\wp_{n,m}\mathrm{e}^{-wt'}+\left(\overline{\overline{T}}'_0-\wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\right\}\cdot$$
$$\left\{-P_m(z')\left[\mu_n^2 S''{}_0(k_1,k_2,\mu_n r')-a_m^2 S_0(k_1,k_2,\mu_n r')\right]+\right.$$

$$\frac{\mu_n(\nu-1)}{r}\left[(\mu_n B_{nm}+C_{nm})\cosh(\mu_n z')+\mu_n C_{nm} z'\sinh(\mu_n z')\right]S_0'(k_1,k_2,\mu_n r')+$$
$$\mu_n^2\nu\left[(\mu_n B_{nm}+C_{nm})\cosh(\mu_n z')+\mu_n C_{nm} z'\sinh(\mu_n z')\right]+\left[S''_0(k_1,k_2,\mu_n r')+\mu_n S_0(k_1,k_2,\mu_n r')\right]\cdot$$
$$\left.2vC_{nm}\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r')\right\}\mathrm{e}^{\int_0^{t'}\psi(\varsigma)\mathrm{d}\varsigma}, \tag{32}$$

$$\sigma_{z'z'}=2G\frac{(1+\nu)}{(1-\nu)}\alpha_t'\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\lambda_{n,m})}\left[\wp_{n,m}\mathrm{e}^{-wt'}+\left(\overline{\overline{T}}'_0-\wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\right\}\cdot$$
$$\left\{-\mu_n P_m(z')\left(\frac{S_0'(k_1,k_2,\mu_n r')}{r'}+\mu_n S_0''(k_1,k_2,\mu_n r')\right)+\mu_n^2\left[B_{nm}\cosh(\mu_n z')+C_{nm}z\sinh(\mu_n z')\right](2-\nu)+\right.$$
$$\left.\left(\mu_n S_0''(k_1,k_2,\mu_n r')+r^{-1}S_0'(k_1,k_2,\mu_n r')\right)\right\}+$$
$$(2-\nu)\left\{\left[\frac{S_0'(k_1,k_2,\mu_n r')}{r'}+\mu_n S_0''(k_1,k_2,\mu_n r')\right](1-\nu)\,\mu_n^2 S_0(k_1,k_2,\mu_n r')\right\}\mathrm{e}^{\int_0^{t'}\psi(\varsigma)\mathrm{d}\varsigma}, \tag{33}$$

$$\sigma_{r'z'}=2G\frac{(1+\nu)}{(1-\nu)}\alpha_t'\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\lambda_{n,m})}\left[\wp_{n,m}\mathrm{e}^{-wt'}+\left(\overline{\overline{T}}'_0-\wp_{n,m}\right)\mathrm{e}^{-(kt\lambda_{n,m})t'}\right]\right\}\cdot$$
$$\left\{\left\{-\mu_n a_m\left[Q_m\sin(a_m z')+W_m\sin(a_m z')\right]\right\}S_0'(k_1,k_2,\mu_n r')+\left[B_{nm}\sinh(\mu_n z')+C_{nm}z\cosh(\mu_n z')\ \mu_n^2\cdot\right.\right.$$
$$\left.\left[v\mu_n+\frac{(1-\nu)}{r'}\right]-2v\mu_n^2 C_{nm}\sinh(\mu_n z)\right\}S_0'(k_1,k_2,\mu_n r')+$$
$$\left.(1-\nu)\left[B_{nm}\sinh(\mu_n z')+C_{nm}z\cosh(\mu_n z')\right]\left[\mu_n^3 S_0'''(k_1,k_2,\mu_n r')-\frac{S_0'(k_1,k_2,\mu_n r')}{r'}\right]\right\}\mathrm{e}^{\int_0^{t'}\psi(\varsigma)\mathrm{d}\varsigma}. \tag{34}$$

To find B_{nm} and C_{nm}, applying boundary condition (26) to Eqs. (33) and (34), we obtain

$$B_{nm}=\frac{P_m(h)\,\overline{A}\left[h\cosh(\mu_n h)\,\overline{B}-\overline{C}\right]-a_m\bar{l}\mu_n S''_0(k_1,k_2,\mu_n r')\overline{m}\left[(2-\nu)\,\overline{A}+(1-\nu)\,\mu_n S_0(k_1,k_2,\mu_n r')\right]}{(2-\nu)\,\overline{A}+(1-\nu)\,\mu_n S_0(k_1,k_2,\mu_n r')}\cdot$$
$$\left\{(h\mu_n)\cosh(\mu_n h)\left[h\cosh(\mu_n h)\,\overline{B}-\overline{C}\right]\right\}-\overline{m}\sinh(\mu_n h)\,\overline{B}, \tag{35}$$

and

$$C_{nm}=\frac{P_m(h)\,\overline{A}\left[\sinh(\mu_n h)\,\overline{B}\right]-a_m\bar{l}\mu_n S''_0(k_1,k_2,\mu_n r')\left[(2-\nu)\,\overline{A}+(1-\nu)\,\mu_n S_0(k_1,k_2,\mu_n r')\right]}{(2-\nu)\,\overline{A}+(1-\nu)\,\mu_n S_0(k_1,k_2,\mu_n r')}\cdot$$
$$\left\{(h\mu_n)\cosh(\mu_n h)\left[h\cosh(\mu_n h)\,\overline{B}-\overline{C}\right]\right\}-\overline{m}\sinh(\mu_n h)\,\overline{B}, \tag{36}$$

where

$$\overline{A}=\frac{S_0'(k_1,k_2,\mu_n r')}{r'}+S_0''(k_1,k_2,\mu_n r'),$$
$$\overline{B}=\mu_n^2\left[v\mu_n+(1-\nu)\right]\frac{S_0'(k_1,k_2,\mu_n r')}{r'}(1-\nu)\left(\mu_n^3 S_0'''(k_1,k_2,\mu_n r')-\frac{S_0'(k_1,k_2,\mu_n r')}{r'^2}\right),$$
$$\overline{C}=2v\mu_n^2\sinh(\mu_n h)S_0'(k_1,k_2,\mu_n r'),\quad \bar{l}=W_m\cos(a_m h)+Q_m\sin(a_m h),$$
$$\overline{m}=\cosh(\mu_n h)+(\mu_n h)\sinh(\mu_n h).$$

To interpret the numerical computations, we consider material properties of aluminum metal, which can be commonly used in both wrought and cast forms. The low density of aluminum results in its extensive use in aerospace industry and other transportation fields. Its resistance to corrosion leads to its use in food and chemical handling (cookware, pressure vessels, etc.) and architectural uses.

Figures 1–3 give the plot of temperature distribution along the radial direction of uncoupled thermoelasticity of thick cylinder. It is observed that due to the thickness of the cylinder, a steep decrease in temperature can be found at the beginning of the transient period. As expected, temperature increases more and

Table 1. Material properties and parameters used in this study. Property values are nominal.

Modulus of elasticity, $E/(\mathrm{N \cdot cm^{-2}})$	6.9×10^6
Shear modulus, $G/(\mathrm{N \cdot cm^{-2}})$	2.7×10^6
Poisson ratio, υ	0.281
Thermal expansion coefficient, $\alpha_t/(1 \cdot \mathrm{C^{-1}})$	2.5×10^{-6}
Thermal diffusivity, $\kappa/(\mathrm{cm^2 \cdot s^{-1}})$	0.86
Thermal conductivity, $\lambda/[\mathrm{W \cdot (mC)^{-1}}]$	200.96
Outer radius, b/cm	0.5
Thickness, h/cm	1

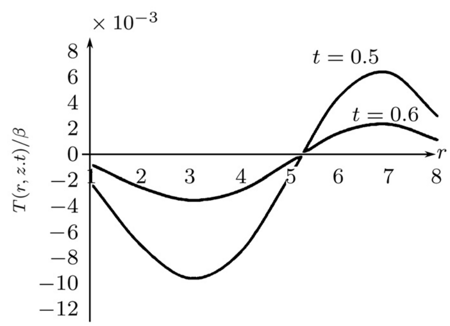

Fig. 1. $T(r, z, t)$ versus r for different values of t.

more gradually along the central portion of thickness direction. Finally temperature distribution further increases and attains zero value at the extreme end. The intensity of the waves of $T(r, z, t)/\beta$ shows a decrease from the inner to the outer side along the radius, and the waves decrease with time t.

The intensity of the wave pattern of thermoelastic displacement function increases along the radius from the centre towards the outer radius of the cylinder; but decreases with time.

The radial displacement increases along the radius, but with the increase of time the radial displacement decreases and comes to zero.

In this paper, the temperature distribution, displacement and thermal stresses of uncoupled thermoelastic response of cylinder due to axisymmetrical heating have been determined. The Marchi-Fasulo transform and Marchi-Zgrablich transform have been used to obtain the numerical results. The obtained results can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

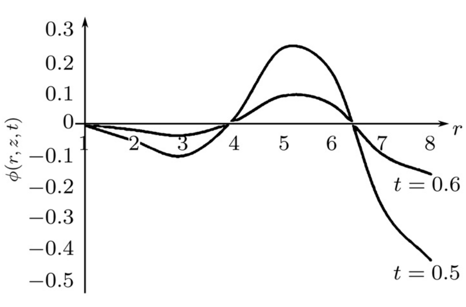

Fig. 2. $\phi(r, z, t)$ versus r for different values of t.

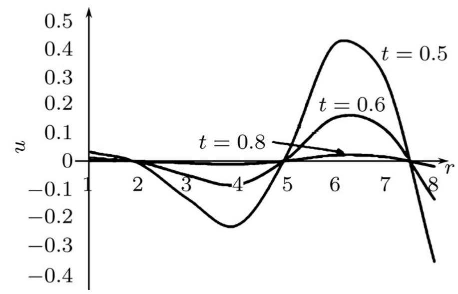

Fig. 3. U versus r for different values of t.

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1. R. B. Hetnarski, and J. Ignaczak, J. Therm. Stresses **16**, 473 (1993).
2. R. B. Hetnarski, and J. Ignaczak, J. Therm. Stresses **17**, 377 (1994).
3. A. Bagri, and M. R. Eslami, J. Therm. Stresses **27**, 691 (2004).
4. A. N. Eraslan, and Y. Orean, J. Therm. Stresses **25**, 559 (2002).
5. E. C. Ting, and H. C. Chen, J. Compos. Struct. **15**, 279 (1982).
6. Y. Y. Li, H. Ghoneim, and Y. Chen, J. Thermal Stresses **6**, 353 (1982).
7. M. R. Eslami, and A. Salehzadeh, Application of Galerkin method to coupled thermoelasticity problem. In: Proc 5th Int Modal Anal Conf, April 6–9, 1987.
8. W. Nowacki, *Thermoelasticity* (Pregman Press, Oxford, 1962).
9. B. A. Boley, and J. H. Weiner, *Theory of Thermal Stresses* (Wiley, New York, 1960).
10. R. L. Sharma, and J. N. Sharma. Response of thermoelastic solid thick plate subjected to lateral loading. In: Proc 3rd Int Cong Thermal Stresses, June 13–17, 1999.
11. A. H. Gosn, and M. Sabbaghian, J. Therm. Stresses **5**, 299 (1982).
12. N. Noda, and R. B. Hetnarski, and Y. Tanigawa, *Thermal Stresses*, 2nd Ed., (Taylor and Francis, New York, 2003)
13. E. Marchi, and G. Zgrablich. Bull. Calcutta Maths. **22**, 73 (1946).
14. E. Marchi, and A. Fasulo. Sci. Torino CI. Sci. Fis. Mat. Natur. **101**, 373 (1966/1967).

APPENDIX

We first introduce the integral transform[13] of order n over the variable r'. Let n be the parameter of the transform. Then the integral transform and its inversion are given by

$$\overline{f}(n) = \int_a^b r' f(r') S_p(k_1, k_2, \mu_n r') dr',$$
$$f(r') = \sum_{n=1}^{\infty} \left(\overline{f}_p(n)/C_n\right) S_p(k_1, k_2, \mu_n r'). \tag{A}$$

The operational property is

$$\int_a^b x \left(\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} - \frac{p^2}{x^2} f\right) S_p(k_1,\, k_2,\, \mu_n\, x) \mathrm{d}x = \frac{b}{k_2} S_p(k_1,\, k_2,\, \mu_n\, b) \left[f + k_2 \frac{\partial f}{\partial x}\right]_{x=b} - \frac{a}{k_1} S_p(k_1,\, k_2,\, \mu_n\, a) \left[f + k_1 \frac{\partial f}{\partial x}\right]_{x=a} - \mu_n^2 \overline{f}(n).$$

We introduce another integral transform[14] corresponding to the boundary condition of type (15)

$$\overline{f}(m,t) = \int_{-h}^{h} f(z', t') P_m(z') \mathrm{d}z',$$
$$f(z', t') = \sum_{m=1}^{\infty} \frac{\overline{f}(m, t')}{\lambda_m} P_m(z'),$$
$$P_m(z') = Q_m \cos(a_m z') - w_m \sin(a_m z'), \tag{B}$$

where

$$Q_m = a_m\,(k_3 + k_4) \cos(a_m h),$$
$$W_m = 2\cos(a_m h) + (k_3 - k_4)\, a_m \sin(a_m h),$$
$$\lambda_m = \int_{-h}^{h} P_m^2(z) \mathrm{d}z = h\left[Q_m^2 + W_m^2\right] + \frac{\sin(2a_m h)}{2a_m}\left[Q_m^2 - W_m^2\right].$$

The eigen values a_m are the positive roots of the characteristic equation

$$[k_3 a \cos(ah) + \sin(ah)]\, [\cos(ah) + k_4 a \sin(ah)] = [k_4 a \cos(ah) - \sin(ah)]\, [\cos(ah) - k_3 a \sin(ah)]\,.$$