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Backreaction of the Hawking radiation

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Abstract

Black holes create a vacuum matter charge to protect themselves from the quantum evaporation. A spherically symmetric black hole having initially no matter charges radiates away about 10% of the initial mass and comes to a state in which the vacuum-induced charge equals the remaining mass.

In Refs. [1,2] a new approach is developed to the problem of backreaction of the Hawking radiation. The present Letter is a report on the completion of this work. For earlier studies and the background material see the book [3].

The Hawking radiation is a semiclassical effect, and so is its backreaction [1]. If the collapsing matter has macroscopic parameters, there is a region of the expectation-value spacetime in which semiclassical theory is valid. This region is causally complete [1] and covers the entire evolution of the black hole from the macroscopic to the microscopic scale if the latter is reached. The ultraviolet ignorance of semiclassical theory is irrelevant to this region. In Refs. [1,2] and the present Letter, the collapsing matter is assumed spherically symmetric, uncharged, and having a compact spatial support. Then its only relevant parameter is its mass M which is also the ADM mass of the expectation-value spacetime. The principal condition of validity of the approach is $(\mu/M) \ll 1$ where μ is the Planckian mass (1 in the absolute units). An observable that, in the units of M, vanishes as $(\mu/M) \to 0$ is denoted as $\mathcal{O}$. An inequality of the form $X > |\mathcal{O}|$ assumes any $\mathcal{O}$ and signifies that X is a macroscopic quantity.

The key result [1] is that, in the semiclassical region of the expectation-value spacetime, the equations for the metric close purely kinematically leaving the arbitrariness only in the data functions. The data functions are two Bondi charges appearing as coefficients in the expansion of the metric at the future null infinity $\mathcal{I}^+$:

$$(\nabla r)^2\big|_{\mathcal{I}^+} = 1 - \frac{2\mathcal{M}(u)}{r} + \frac{Q^2(u)}{r^2} + O\left(\frac{1}{r^3}\right). \tag{1}$$

Here r is the luminosity parameter of the radial light, u is the retarded time labelling the radial future light cones, the coefficient $\mathcal{M}(u)$ is the gravitational charge, and $Q(u)$ is some matter charge.

Any spherically symmetric metric is completely specified by two local curvature invariants: $(\nabla r)^2$ and Δr where Δ is the D'Alembert operator in the Lorentzian subspace. In the semiclassical region, both invariants are expressed through the Bondi charges. On the other hand, the Bondi charges can be expressed through the metric in the semiclassical region by calculating the vacuum radiation against its background. As a result, the Bondi charges get expressed through themselves, i.e., one obtains closed equations for them and, thereby, for the metric in the semiclassical region. The first stage of this program: solving the kinematical equations for the metric in terms of the Bondi charges is accomplished in Ref. [1]. The second stage: calculation of the vacuum radiation against the thus obtained gravitational background is accomplished in Ref. [2]. The purpose of the present Letter is the solution of the final equations.

Two normalizations of the retarded time u figure in the problem: u^+ and u^-. The u^+ is counted out by an observer at infinity, and the u^- is counted out by an early falling observer [1]; du^+/du^- is the red-shift factor. The v below is the ad-

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vanced time labelling the radial past light cones and counted out by an observer at infinity.

The self-consistent equations for the Bondi charges obtained in Ref. [2] are of the form

$$-\frac{d\mathcal{M}}{du^+} = \frac{\mu^2}{48\pi}\kappa^2(1+\Gamma), \tag{2}$$

$$\frac{dQ^2}{du^+} = \frac{\mu^2}{24\pi}\kappa, \tag{3}$$

$$\kappa = \left(\mathcal{M}^2 - Q^2\right)^{1/2}\left[\mathcal{M} + \left(\mathcal{M}^2 - Q^2\right)^{1/2}\right]^{-2}, \tag{4}$$

where Γ is expressed through $\mathcal{M}$ and Q^2 [2]. Here and below, the numerical factors are given for the vacuum of spin-0 particles. Only the quantum s-mode contributes to the flux of Q^2, and the fact that this flux is nonvanishing is another key result [2]. The quantum modes with higher angular momenta contribute only to the flux of $\mathcal{M}$ through the term Γ but Γ is uniformly bounded and small: $\Gamma \leqslant (27/160)$. The spectral decomposition of the flux (2) is Planckian with the time-dependent temperature [2]

$$kT = \mu^2\frac{\kappa(u)}{2\pi}. \tag{5}$$

The data at $\mathcal{I}^+$, $\mathcal{M}$ and Q^2, are related to the data at the apparent horizon (AH) as [1]

$$r_{\mathrm{AH}} = \mathcal{M} + \left(\mathcal{M}^2 - Q^2\right)^{1/2}, \qquad \Delta r|_{\mathrm{AH}} = 2\kappa, \tag{6}$$

and the equation of the apparent horizon in the coordinates (u, v), $v = v_{\mathrm{AH}}(u)$, is given by the law [1]

$$\frac{d\ln\beta}{du^+} = \kappa\left(\frac{dv_{\mathrm{AH}}}{du^+} - 1\right), \qquad \beta = -\frac{dr_{\mathrm{AH}}}{du^+}. \tag{7}$$

The outgoing light rays $u = \mathrm{const}$ cross the AH twice, and there is a point, (u_0, v_0), at which the AH is tangent to an outgoing light ray [1]. Equivalently, the AH has two branches with the origin at (u_0, v_0). The equations above pertain to the second (later) branch, and their validity is limited to the range

$$u^+ > u_0^+ + O(M) \tag{8}$$

in which a significant radiation occurs. In this range, the redshift factor is given by the expression [1]

$$\frac{du^+}{du^-} = \frac{1}{2\beta_0}\exp\left(\int_{u_0^+}^{u^+}\kappa\, du^+\right), \qquad \beta_0 = \beta|_{u=u_0}. \tag{9}$$

Another and more important limitation on the validity of the equations above stems from the fact that they were derived under certain assumptions about the data functions [1]. These assumptions, deliberately valid at the beginning of the radiation process, could cease being valid at some late value of u, and it was envisaged that the solution for the metric obtained in Ref. [1] should then be cut off at this value of u. For later u, it is no longer valid. Now, that the data functions are obtained, it turns out that, of these assumptions, the crucial one is

$$\kappa > |\mathcal{O}|. \tag{10}$$

The equations above are easy to solve, and the result is this. $\mathcal{M}(u)$ decreases monotonically, and $Q^2(u)$ increases monotonically from the instant u_0 at which their values (up to $\mathcal{O}$) are

$$u = u_0: \quad \mathcal{M}_0 = M, \qquad Q_0^2 = 0 \tag{11}$$

to an instant u_1 at which $\mathcal{M}^2$ and Q^2 become equal:

$$u = u_1: \quad \mathcal{M}_1^2 = Q_1^2. \tag{12}$$

Approximately,

$$u_1^+ - u_0^+ = 96\pi\frac{M^3}{\mu^2}, \tag{13}$$

and

$$0.098 < \frac{M - \mathcal{M}_1}{M} < 0.112. \tag{14}$$

Here the lower bound accounts for the contribution of the s-mode alone. Thus only about 10% of M is radiated away by the instant u_1. The temperature of radiation first grows but only up to a maximum value which is slightly greater than $\mu^2/8\pi M$, and next decreases down to zero. At $u = u_1$, the red shift reaches its maximum:

$$\left.\frac{du^+}{du^-}\right|_1 = \exp\left(24\pi\frac{\mathcal{M}_1^2}{\mu^2}\right) \approx \exp\left(19.4\pi\frac{M^2}{\mu^2}\right). \tag{15}$$

Along the AH, r decreases monotonically from the value $2\mathcal{M}_0$ to the value $\mathcal{M}_1$, and $(r\Delta r)$ decreases monotonically from 1 to 0. Because of the limitations (8) and (10), Eq. (7) can be used only outside the $O(M)$ neighbourhoods of the end points u_0^+ and u_1^+. There, the equation of the apparent horizon is

$$\frac{dv_{\mathrm{AH}}}{du^+} = 1 + |\mathcal{O}|, \qquad u_1^+ - O(M) > u^+ > u_0^+ + O(M). \tag{16}$$

In the neighbourhoods of the end points, only unessential details of the behaviour of the AH are unknown. At $u = u_0$, the AH is null and tangent to the outgoing light ray $u = u_0$. At $u = u_1$, the AH is null and tangent to the incoming light ray $v = v_1$. Approximately,

$$v_1 - v_0 = 96\pi\frac{M^3}{\mu^2}. \tag{17}$$

The apparent horizon (the second branch) is shown in Fig. 1. At $u = u_1$, the assumption (10) breaks down, and the solution for later u is presently unknown.

Thus the Hawking process liberates more than half of the energy from the black hole. Only about 10% of it goes away in the form of thermal radiation. If one forgets about the radiation and just compares the initial state at $u = u_0$ and the final state at $u = u_1$, then in the initial state all of the available energy is in the black hole, and in the final state exactly one half. Most of the liberated energy remains in the compact domain outside the black hole in the form of the energy of a long-range field whose source is the charge Q. The black hole manufactures this charge from the vacuum to protect itself from the quantum evaporation. It is unexpected that the vacuum stress tensor develops a macroscopic value and even more surprising that this

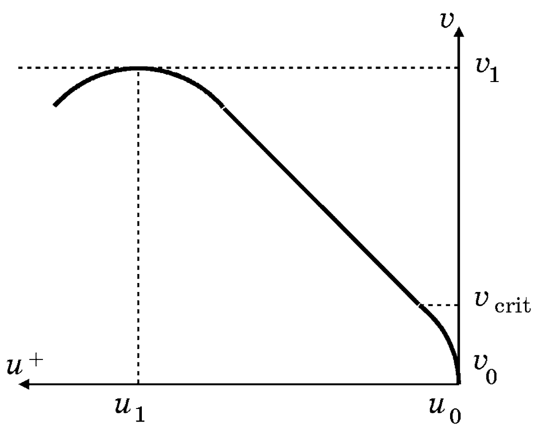

Fig. 1. The second branch of the apparent horizon in the coordinates u^+, v.

is the stress tensor of a long-range field. One may conclude that the massless quantum field assumed to be in the in-vacuum state [2] develops a nonvanishing expectation value. However, from the equations above, only the energy–momentum tensor of this field is available.

The state reached by the instant u_1 is not really final because the black hole does not stabilize in this state. This is seen from the fact that, at $u = u_1$, both u^+ and du^+/du^- have finite values. Only in the region of weak field [1] does the expectation-value metric appear as the Reissner–Nordstrom metric with slowly varying parameters. In the neighbourhood and interior of the AH, it is different [1]. The $\mathcal{M}(u)$ and $Q^2(u)$ come to the instant u_1 with vanishing derivatives and can be continued as constants but this is not true of $r_{\rm AH}$ and $\Delta r|_{\rm AH}$. The $r_{\rm AH}$ continues decreasing while $\Delta r|_{\rm AH}$ passes through zero and becomes negative. Therefore, the AH continues through the point (u_1, v_1) as shown in Fig. 1. At $u = u_1$, the present solution ceases being valid *but semiclassical theory does not*. It is a matter of generalizing the solution, to learn what next.

It will be emphasized that the term "black hole" is used here for the interior of the apparent horizon rather than of the event horizon. No event horizon has thus far been found in the solution. There is strictly speaking no black hole but at each instant of evaporation there is an "instantaneous black hole" [1]. At the beginning of the evaporation process, this is the "classical black hole" that corresponds to the sector $v_0 < v < v_{\rm crit}$ of the AH in Fig. 1. The value $v_{\rm crit}$ sets the limit to the validity of the correspondence principle [1]. In Ref. [1], it has been expressed through the constant β_0 that figures in Eq. (9) above. Now one is able to calculate it:

$$v_{\rm crit} - v_0 = 4M \ln \frac{M^2}{\mu^2} + O(M). \tag{18}$$

Remarkably, one is able to calculate also the value of u_0^+:

$$u_0^+ - u_{\rm early}^+ = 4M \ln \frac{M^2}{\mu^2} + O(M), \tag{19}$$

$$\left.\frac{du^+}{du^-}\right|_0 = \mathrm{const.}\frac{M^2}{\mu^2}. \tag{20}$$

Here $u_{\rm early}$ is any value of u at which the red shift is moderate:

$$\left.\frac{du^+}{du^-}\right|_{\rm early} < \frac{1}{|\mathcal{O}|}. \tag{21}$$

Eq. (19) gives the time instant at which the "classical black hole" forms, and Eq. (18) gives its life time. In the classical geometry, both are infinite. The expectation-value geometry contains two characteristic time scales: the one in Eq. (19) and the one in Eq. (13).

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