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Energy Procedia 17 (2012) 1124 – 1131

Energy

**Procedia**

2012 International Conference on Future Electrical Power and Energy Systems

## Dynamic Response of Power Transmission Towers under Wind Load

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### Abstract

This paper describes a method for evaluation on dynamic characteristics of power transmission towers coupled with power lines under wind load. The evaluation criteria of dynamic responses is developed and applied to the finite element analysis of a power transmission tower-line system under wind load. The numerical results indicate that the proposed energy evaluation criteria can be effectively utilized in the examination on structural dynamic performance.

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Keywords: power transmission tower-line system, dynamic response, energy response criteria, finite element.

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### 1. Introduction

Power transmission towers are connected with power lines, which show geometrically nonlinear behavior and have many complex vibration modes whose frequencies are normally very close. It has been shown that the dynamic response of a power transmission tower is strongly influenced by the behavior of the power lines [1-2]. Although Researchers and engineers have paid much attention to the performance of power transmission tower-line system under strong wind load, and many theoretical and experimental investigations have been carried out during the past three decades [3-6], the evaluation of energy properties of transmission tower-line system under strong wind excitations has not been effectively investigated up to know.

In this regard, the feasibility of using vibration energy response criteria to assess the dynamic characteristics of the power transmission tower-line system is actively carried out in this study. The analytical model of transmission tower-line system is developed first for both in-plane/out-of-plane vibration. Energy criteria are developed for performance assessment. A real transmission tower-line system is taken as an example to examine the feasibility and reliability of the proposed approach. The numerical results demonstrate that the energy criteria are effective on the performance assessment of large scale transmission tower subjected to strong wind load.

## 2. Model of Transmission Tower-Line System

The power line is modeled as several lumped masses connected through elastic elements. After establishing the kinetic energy and potential energy of a coupled power transmission tower-line system, the mass and stiffness matrices of a coupled system are determined as the coefficients to generalized velocity and generalized displacement respectively.

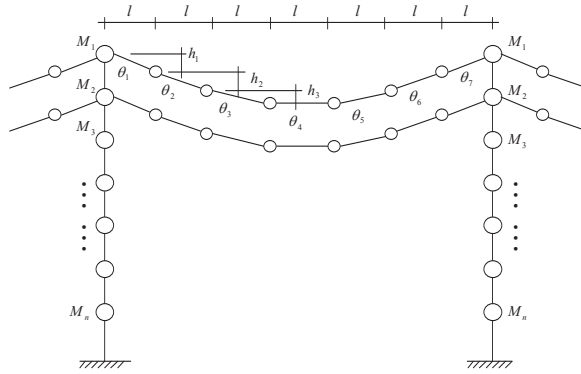


Figure 1. Analytical model for in-plane vibration of power transmission tower-line system

### 2.1. In-plane vibration

The analytical model for power transmission tower-line system is displayed in Fig. 1. The kinetic energy of the coupled system for in-plane vibration is

$$T = \sum_{i=1}^{n-1} \frac{1}{2} m_i (\dot{u}_i^2 + \dot{v}_i^2) + T_{tower} \tag{1}$$

$$= T_{line} (\dot{\xi}_2, \dot{\xi}_3, \dots, \dot{\xi}_{n-1}, \delta \dot{l}_1, \delta \dot{l}_2, \dots, \delta \dot{l}_n) + T_{tower}$$

Similarly, the potential energy of the coupled system is

$$U = \sum_{i=1}^{n-1} m_i g v_i + \sum_{j=1}^n \frac{EA}{2} \left( \frac{(l_{js} + \delta l_j)^2}{l_{j0}} - \frac{l_{js}^2}{l_{j0}} \right) + U_{tower} \tag{2}$$

where  $n$  is the number of discretized elements for transmission line.

By substituting (1) and (2) into the Lagrange equation, the mass matrix and stiffness matrix of the coupled tower-line system is determined by carrying out the differential calculations. A power transmission tower-line system is a complex system with many towers and transmission lines, which make it impossible to establish an analytical model involving all the towers and lines. The development of a simplified computational model with satisfactory accuracy is essential for dynamic analysis and performance assessment. For simplicity sake, the coupled system with two power transmission tower and three lines is adopted in this study. For in-plane vibration, it is observed that the mass matrices of transmission lines and tower are coupled with each other and there exist coupling parts in the global mass matrix of the tower-line system. The stiffness and mass matrices of the power transmission tower-line system are expressed as

$$\mathbf{K}^{in} = diag[\mathbf{K}_l^{in}, \mathbf{K}_t^{in}, \mathbf{K}_l^{in}] \tag{3}$$

$$\mathbf{M}^{in} = \begin{bmatrix} \mathbf{M}_l^{in} & \mathbf{M}_{leftcouple}^T & \\ \mathbf{M}_{leftcouple} & \mathbf{M}_t^{in} & \mathbf{M}_{rightcouple} \\ & \mathbf{M}_{rightcouple}^T & \mathbf{M}_l^{in} \end{bmatrix} \quad (4)$$

where  $\mathbf{M}_{leftcouple}$  and  $\mathbf{M}_{rightcouple}$  are the left and right coupled matrices, respectively. Similarly, the equivalent mass increment of towers can also be determined through differential analysis of kinetic energy of the tower-line system to simulate the coupled effects of tower due to transmission lines.

### 2.2. Out-of-plane vibration

The mass and stiffness matrices of tower-line system for out-of-plane vibration can be expressed as

$$\mathbf{M}^{out} = \text{diag}[\mathbf{M}_l^{out}, \mathbf{M}_t^{out}, \mathbf{M}_l^{out}] \quad (5)$$

$$\mathbf{K}^{out} = \begin{bmatrix} \mathbf{K}_l^{out} & \mathbf{K}_{couple}^T & \\ \mathbf{K}_{couple} & \mathbf{K}_t^{out} & \mathbf{K}_{couple} \\ & \mathbf{K}_{couple}^T & \mathbf{K}_l^{out} \end{bmatrix} \quad (6)$$

where  $\mathbf{K}_{couple}$  is the coupled stiffness matrix

### 3. Equation of Motion of Coupled System

The equation of motion of the power transmission tower-line system for in-plane vibration under wind load can be expressed as [8]

$$\mathbf{M}^{in} \ddot{\mathbf{x}}(t) + \mathbf{C}^{in} \dot{\mathbf{x}}(t) + \mathbf{K}^{in} \mathbf{x}(t) = \mathbf{P}_w^{in}(t) \quad (7)$$

where  $\mathbf{M}^{in}$ ,  $\mathbf{C}^{in}$  and  $\mathbf{K}^{in}$  are mass, damping and stiffness matrices of the coupled system for in-plane vibration respectively,  $\mathbf{x}(t)$ ,  $\dot{\mathbf{x}}(t)$  and  $\ddot{\mathbf{x}}(t)$  are the displacement, velocity and acceleration responses for in-plane vibration, respectively,  $\mathbf{P}_w^{in}(t)$  is the wind load for in-plane vibration.

Similarly, the equation of motion of controlled tower-line system for out-of-plane vibration is

$$\mathbf{M}^{out} \ddot{\mathbf{x}}(t) + \mathbf{C}^{out} \dot{\mathbf{x}}(t) + \mathbf{K}^{out} \mathbf{x}(t) = \mathbf{P}_w^{out}(t) \quad (8)$$

The meanings of the symbols in Eq. (8) are similar to those in (7). The energy representation for the structural responses can be formed by integrating the individual force components in (7) and (8) over the corresponding relative displacement. The starting point for accumulating energy is from the beginning of vibration due to wind excitation. The absolute energy equation corresponding to (7) then becomes

$$E_K + E_D + E_S = E_I \quad (9)$$

where:

$$E_K = \frac{1}{2} \dot{\mathbf{x}}^T \mathbf{M} \dot{\mathbf{x}} \quad (10)$$

$$E_D = \int \dot{\mathbf{x}}^T \mathbf{C} \dot{\mathbf{x}} dt \quad (11)$$

$$E_S = \int \mathbf{x}^T \mathbf{K} d\mathbf{x} \quad (12)$$

$$E_I = - \int \ddot{\mathbf{x}}_g^T \mathbf{M} I d\mathbf{x} \quad (13)$$

The terms on the left hand side of (9) quantify the contribution of the various forms of energy that the structure employs to resist the earthquake.  $E_K$  is the structural kinetic energy,  $E_D$  is the energy dissipated by structural damping,  $E_S$  is the structural strain energy, and  $E_I$  is the total input energy from seismic excitation. The energy equation for the out-of-plane is the same to that of (9) which is not expressed here for simplicity.

#### 4.Simulation of Wind Loading

Following the statistical analytical of Davenport [9], the wind velocity power spectrum density in along wind direction can be expressed as

$$S_v(n) = 4k\bar{v}_{10}^{-2} \frac{x^2}{n(1+x^2)^{4/3}} \tag{14}$$

$$x = \frac{1200n}{\bar{v}_{10}} \tag{15}$$

$$n = \frac{\omega}{2\pi} \tag{16}$$

in which  $k$  is the coefficients to reflect the ground Roughness,  $\bar{v}_{10}$  is the average wind velocity at 10m height.

The wind excitations of power transmission tower-line system are simulated by spectral representation approach. The stochastic wind fields  $f_j^0(t)$ ,  $j = 1, 2, \dots, n$ , can be simulated by the following series

$$f_j(t) = 2\sqrt{\Delta\omega} \sum_{m=1}^j \sum_{l=1}^N |H_{jm}(\omega_{ml})| \cos(\omega_{ml}t - \theta_{jm}(\omega_{ml}) + \phi_{ml}) \tag{17}$$

$$\Delta\omega = \frac{\omega_{up}}{N} \tag{18}$$

where  $N$  is sufficiently large number;  $\Delta\omega$  the frequency increment;  $\omega_{up}$  the upper cutoff frequency with the condition that when  $\omega > \omega_{up}$ , the value of  $S^0(\omega)$  is trivial;  $\phi_{ml}$  represents a sequence of independent random phase angles, uniformly distributed over the interval  $[0, 2\pi]$ ;  $\omega_{ml}$  is the double-indexing frequency.

$$\omega_{ml} = (l-1)\Delta\omega + \frac{m}{n}\Delta\omega, l = 1, 2, \dots, N \tag{19}$$

and  $H_{jm}(\omega)$  is a typical element of matrix  $\mathbf{H}(\omega)$ , which is defined with Cholesky decomposition of cross-spectral density matrix  $\mathbf{S}^0(\omega)$

$$\mathbf{S}^0(\omega) = \mathbf{H}(\omega) \mathbf{H}^{T*}(\omega) \tag{20}$$

$$\mathbf{H}(\omega) = \begin{bmatrix} H_{11}(\omega) & 0 & \dots & 0 \\ H_{21}(\omega) & H_{22}(\omega) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ H_{n1}(\omega) & H_{n2}(\omega) & \dots & H_{nn}(\omega) \end{bmatrix} \tag{21}$$

where  $\theta_{jm}(\omega)$  is complex angle of  $H_{jm}(\omega)$ . Since  $\mathbf{H}(\omega)$  is a function of  $\omega$ , it can be seen from the structure of (39) that the Cholesky decomposition has to be conducted separately for every frequency  $\omega_{ml}$ .

### 5. Case Study

#### 5.1. Description of a power transmission tower-line system

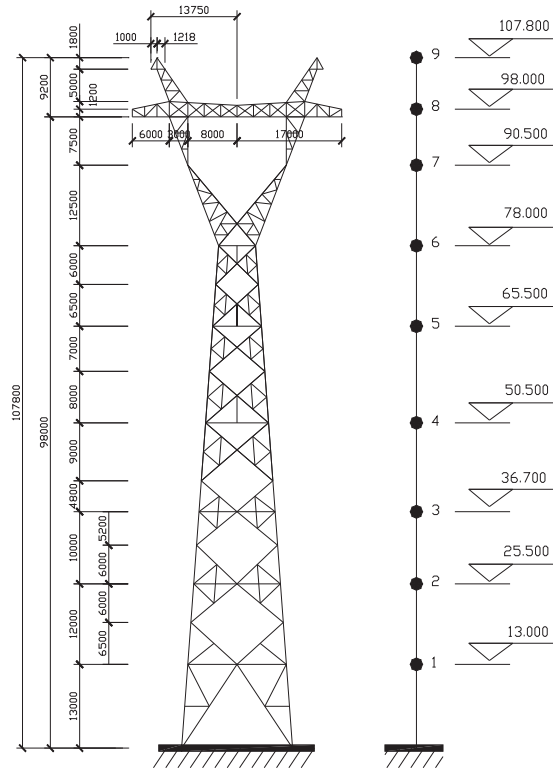


Figure 2. Elevation of a power transmission tower

To examine the performance of the proposed energy evaluation approach, a real power transmission tower-line system, as shown in Fig. 2, is selected as the analytical example. The tower has a height of 107.8m and the span of transmission line is 830m. The Rayleigh damping assumption is adopted to construct the structural damping matrix [7]. The coefficients of Rayleigh damping can be calculated as:

$$a_1 = 2 \left( \frac{\xi_1}{\omega_1} - \frac{\xi_2}{\omega_2} \right) / \left( \frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right) \tag{22}$$

$$a_2 = 2 (\xi_2 \omega_2 - \xi_1 \omega_1) / (\omega_2^2 - \omega_1^2)$$

where  $\xi_1$ ,  $\xi_2$  are damping ratio corresponding to  $\omega_1$ ,  $\omega_2$  which usually take value between 0.1~0.2. In our example,  $\omega_1$ ,  $\omega_2$  take the value of the first two eigenfrequencies. A two dimensional lumped mass model is constructed based on the original three dimensional finite element model, as displayed in Fig. 1.

#### 5.2. Dynamic properties

The eigenfrequencies of the selected power transmission tower in the coupled tower-line system are computed and listed in Tables 1 and 2 in comparison with those of the single tower. It is found that the

eigenfrequencies of the tower for in-plane vibration are smaller than those of single tower. This is because the influence of internal force of the power line on the dynamic properties of tower. As for the out-of-plane vibration, the effects of power line on the dynamic properties of the tower are relatively small.

TABLE I. EIGENFREQUENCIES OF POWER TRANSMISSION TOWER FOR IN-PLANE VIBRATION (HZ)

<i>Freq.</i>	<i>Single tower</i>	<i>Tower in tower-line system</i>
$f_1$	1.348	1.044 (29.1%)
$f_2$	3.641	2.850 (27.8%)
$f_3$	4.993	4.712 (6.0%)

TABLE II. EIGENFREQUENCIES OF POWER TRANSMISSION TOWER FOR OUT-OF-PLANE VIBRATION (HZ)

<i>Freq.</i>	<i>Single tower</i>	<i>Tower in tower-line system</i>
$f_1$	1.049	1.049 (0.0%)
$f_2$	4.393	4.401 (0.2%)
$f_3$	7.219	7.216 (0.04%)

### 5.3. Dynamic energy

Fig. 3 indicates the energy responses of the selected power transmission tower under wind load. Figs. 4 and Fig. 5 display the time histories of the strain energy and kinetic energy, respectively. It is easy to understand that the energy input from wind load is equal to the sum of kinetic energy, strain energy and the energy dissipated by structural damping. Most of the inputting energy from wind excitations of the tower is dissipated by the structural damping. In reality, some parts of the inputting energy of the tower are transferred between the kinetic energy and strain energy. The status with peak kinetic energy is actually the status with minimum strain energy and vice versa. The comparison between kinetic energy and strain energy of the tower are carried out and plotted in Fig.6. It is clear that the peak values of kinetic energy and strain energy occurs alternatively due to the internal energy transfer of the tower during vibration.

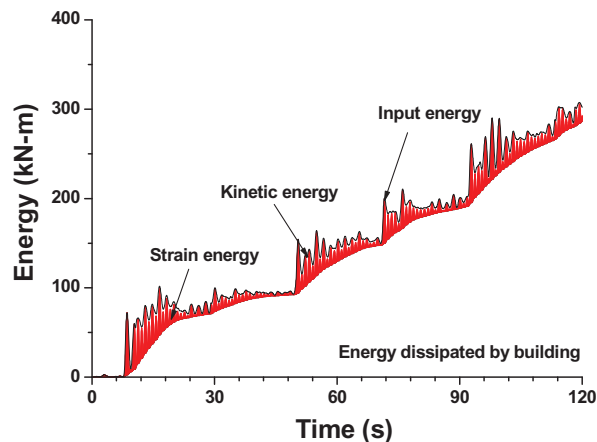


Figure 3. Energy responses of the power transmission tower

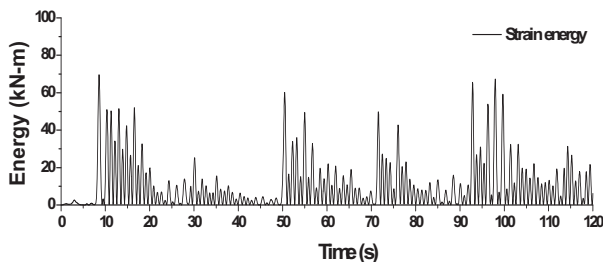


Figure 4. Time history of strain energy of the power transmission tower

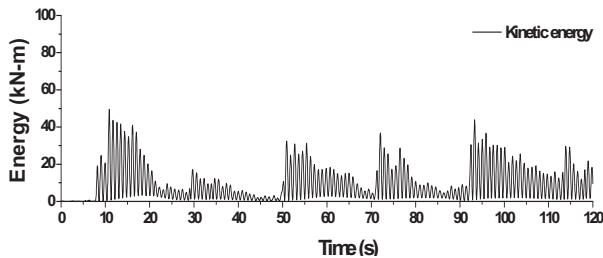


Figure 5. Time history of kinetic energy of the power transmission tower

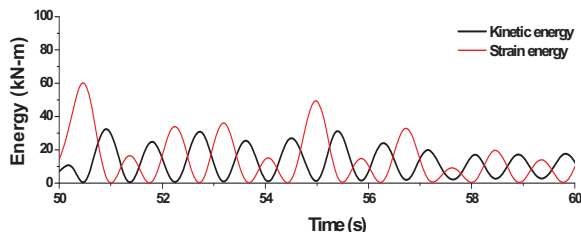


Figure 6. Comparison between kinetic energy and strain energy of the power transmission tower

**6.Conclusions**

The feasibility of using energy criteria to assess the dynamic characteristics of a large scale power transmission tower-line system under strong wind load is actively investigated in this study. A coupled system with two power transmission tower and three lines is taken as an example to examine the feasibility and reliability of the proposed approach. The developed energy criteria are effective in the performance assessment of structural dynamic responses.

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