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Finite Element Modeling and Analysis of Functionally Graded (FG) Composite Shell Structures

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Abstract

This article deals with the finite element modeling and analysis of functionally graded (FG) shell structures under different loading such as thermal and mechanical. Free vibration analysis of functionally graded (FG) spherical shell structure has also been presented. In order to study the influences of important parameters on the responses of FG shell structures, different types of shells have been considered. The responses obtained for FG shells are compared with the homogeneous shells of pure ceramic (Al_2O_3) and pure metal (steel) shells and it has been observed that the responses of the FGM shells are in between the responses of the homogeneous shells. Based on the analysis, some important results are presented and discussed for thick as well as thin shells.

Keywords: FG shell structures; FE analysis; thermal and mechanical loading ; free vibration analysis

1. Introduction

Laminated composites have various advantages over conventional materials such as high specific stiffness and lightweight. Their major drawback is the weakness of interfaces between adjacent layers, known as delamination phenomena. When laminated composites are subjected to extreme temperatures, the effects of interlaminar stresses become even more severe that lead to failure of composite structure due to delamination. These problems can be reduced if the sudden change of material properties of the laminated composite structure is prevented. Functionally graded material (FGM) is a kind of material in which the individual material composition varies continuously along certain directions

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in a controllable way. There are many areas of application for FGM. The concept is to make a composite material by varying the microstructure from one material to another material with a specific gradient. Many works have been reported in the broad area of FGM structures, some of the important works in the direction of finite element analysis of FG structures have been presented in the following paragraphs.

Lakshminarayana et al. [1] performed a significant research on the analysis of FGM composite structures. Gunes et al. [2] investigated the thermal response of FGM plates. Kashtalyan [3] addressed a three-dimensional elasticity solution for the bending of functionally graded rectangular plates. Zhong and Shang [4] studied the three-dimensional exact analysis of a simply supported functionally graded piezoelectric plate. Charles et al. [5] developed a new finite element to evaluate the global response of a structure composed of laminated beams which contain de-laminations. Bambole et al. [6] developed a FE formulation based on the minimum energy principle for the analysis of thick/thin laminated composite beams. Ray and Sachade [7] derived an exact solution for functionally graded plates integrated with layers of piezoelectric reinforced composite. Croce and Venini [8] developed a finite element for the analysis of shear-deformable FGM Midlin plates. Zenkour [9, 10] investigated the functionally graded sandwich of ceramic-metal panels. Kargarnovin et al. [11] introduced the limit analysis of FGM circular plates subjected to arbitrary rotational symmetric loads. Bouchafa and Benzair [12] presented an analytical modeling of thermal residual stresses in exponential functionally graded material system. Anastasia and Muliana [13] introduced a micromechanical model for predicting the effective thermo-visco-elastic behaviors of a functionally graded material (FGM). Senthil et al. [14] developed an exact solution for the thermo-elastic deformations of functionally graded thick rectangular plates. Shen [15] introduced the post buckling of simply supported and clamped cylindrical panels subject to axial compression and temperature. Bilgili [16] used the shear strains in a non-homogeneous rubber like slab subjected to a thermal gradient in the thickness direction. Liew et al [17] analyzed the problem of thermal stresses in FGM hollow cylindrical shells. Yang et al. [18] reported the bending response of thick FGM plates subjected to lateral pressure and uniform temperature using higher-order shear deformation theory combined with a first-order perturbation technique. Vidal and Polit [19] developed a new three-node multi-layered beam finite element for analysis of thermo-mechanical behaviour of sandwich and laminated beams. Senthil et al. [20] addressed three-dimensional analysis of transient thermal stresses in functionally graded plates and developed an analytical solution for three-dimensional of thermo-mechanical deformations of a simply supported functionally graded (FG) rectangular plate subjected to time-dependent thermal loads on its top and/or bottom surfaces. Reddy and. Cheng [21] introduced three-dimensional solutions of smart functionally graded plates. From the exhaustive literature review, it has been observed that, while a number of works are available in the form of beam and plate finite elements analysis of FG structures, not many works are available in the form of analysis of FG shell structures. In the present work, thus, an attempt has been made for the analysis of FG shell structures in order to study the responses of various FG shell structures for both deep as well as shallow shells.

2. Modeling of FGM properties

Different types of gradations laws are available in the literature. In the present work power law gradation   has been considered in order to calculate the material properties of FG structures. FGM consisting of two constituent materials has been considered. The top surface is assumed to be rich in material-2 (ceramic) and bottom surface is assumed to be rich in material-1 (metal).The region between the two surfaces consists of a combination of the two materials with continuously varying mixing ratios of two materials. Volume fraction following power law distribution can be written as [21].

$$V_c(z) = \left(\frac{(z+h)}{2h}\right)^m$$

$$V_c(z) = 1 - V_{mat}(z) \qquad\qquad (1)$$

$$P(z) = P_{mat} + (P_c - P_{mat})V_c(z)$$

Where z is the distance from mid–surface and m is the power law index, a positive real number. P_{mat} is the material property of metal (steel) surface and P_c is material property of ceramic (Al_2O_3) surface. $V_c(z)$ And $V_{mat}(z)$ are the volume fractions of the ceramic and metal surface. In the present analysis, the material properties are varying though the thickness from bottom to top. The material properties such as young's modulus, thermal expansion, conductivity and density are varying in thickness direction only and poisons ratios is constant throughout the thickness as shown in Fig 2. The material properties used for modeling and analysis of FG structures is presented in the Table 1.

Table.1. Thermo-elastic properties for metallic (Steel) and ceramic (Al2O3) phases

Material	Thermal expansion coefficient $\alpha(^oc^{-1})$	Poisons' ratio μ	Young's modulus E (GPa)	Density(kg/m³)	Conductivity(k) (W/m⁰K)
AL₂O₃	6.9×10^{-6}	0.25	390	3.89×10^3	25
Steel	14×10^{-6}	0.25	210	7.85×10^3	40

3. Modeling of FG Structures using ANSYS

The material properties of the FGM change throughout the thickness; the numerical model has been divided into various layers in order to make the changes in properties. Each layer has the finite portion of the thickness and treated like isotropic material. Material properties have been calculated at the mid-plane of each of this layer by using the power law gradation. The thickness of FG beam has been discretised into several layers as shown in Fig 1. Similarly, shell structures have also been discretised through the thickness into several layers in order to model FG shell structures. The finite element (FE) modelling has been carried out using ANSYS. An eight nodded layered shell element (SHELL99) has been used for modelling of FG structures. The element has six degrees of freedom at each node. Translations are in the nodal *x*, *y*, and z directions and rotations about the nodal *x, y,* and *z*-axis. In this paper finite element modelling of functionally graded beam as well as different shell structures have been done. A typical FG shell is shown in Fig 2.
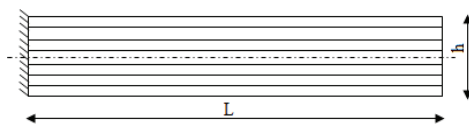

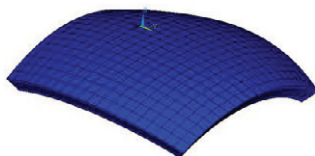Fig.1. Thickness discretization of beam

Fig.2. FG shell structure

4. Results and Discussions

Based on the FE analysis as discussed in the section 2, two types of FG structures (viz. beam and shell) have been analyzed using ANSYS. Results obtained of different analysis (such as static, free vibration and thermal) have been presented in the following subsections.

4.1. Static Analysis of FG Beam

For validaion of present analysis, a FG cantilever beam has been considered. The three layered system i.e steel-FGM-ceramic has been analyzed. The FG cantilever beam of 0.5 m length subjected to transverse distributed load (100 kN) on upper surface has been taken. Width is taken as 0.001m. The top most material is ceramic which has a thickness of 0.005m and bottom layer is metal of thickness 0.005m. In between these layers there is a FGM layer of 0.01m and the FGM layer is divided into five layers i.e. total number of layers of the model is seven. Eight nodded SHELL99 element is used for FE modeling of the FG beam and the material properties are calculated by using the power law. Material properties used for this analysis are listed in the Table 1. Four numbers of elements are taken in width and twenty numbers of elements are taken in length. The beam is subjected to a uniform distributed loading (q =100 kN/m). Fig 3 shows the results i.e. depth wise stress distribution in the beam using the ANSYS.

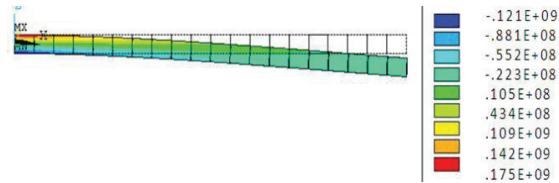|	
■	-.121E+09
■	-.881E+08
■	-.552E+08
■	-.223E+08
■	.105E+08
■	.434E+08
■	.109E+09
■	.142E+09
■	.175E+09

Fig.3. Depth wise stress variation in the beam

Depth wise stress distributions obtained using ANSYS are also tabulated in Table 2. It has been observed from the Table 2 that the results obtained from present analysis are within acceptable limits and are in excellent agreement with the already published results [22].

Table.2. Depth wise stress distribution for the FGM beam

Thickness (m)	Stress distribution(Pa) ($\times 10^8$) ANSYS	Stress distribution(Pa) ($\times 10^8$) Published [22]
0.01	1.75	1.64
0.008	1.47	1.3
0.006	1.09	0.9
0.004	0.76	0.65
0.002	0.4	0.35
-0.002	-0.105	-0.2
-0.004	-0.22	-0.3
-0.006	-0.582	-0.65
-0.008	-0.89	-0.85
-0.01	-1.21	-1.2

4.2. FE analysis of FG shell structure under different loading

A generalized shell structure is depicted in Fig 4. After validation, a simply supported spherical shell (R_1 = R_2) on square based ($a = b$) has been analysed under mechanical as well as thermal loading. Finite element (FE) analyses have been done using eight noded shell element (SHELL99). A 10 × 10 FE mesh been used with total degrees of freedom = 2046, number of elements = 100 and number of nodes = 341. Thickness of the shell (h) has been considered as 0.02m. The mechanical and thermal material properties used in the present study have been listed in the Table 1. The simply supported spherical shell has been subjected uniformly distributed load (q = 100 kN) as show in Fig.5.

Central displacements of the shell structures under uniformly distributed load for various (R/a) ratios have been listed in Table 3. From this table it has been observed that the center displacements of FG shells are in between the center displacements of the homogeneous shells of ceramic and metal. Figs 6 to 8 show the displacement variation through the surface for ceramic, FG and metallic shells under mechanical loading. The shell model has also been analysed under thermal loading. In thermal analysis the shell top surface is subjected to 130^0C and the shell bottom surface is subjected to 30^0C. The displacements of the shell structures due to the applied temperature loading are tabulated in Table 4. Figs 9 to 11 show the displacement variation through the surface for ceramic, FG and metallic shells under thermal loading. Free vibration analysis of the spherical FG shell has also been carried out for thick (a/h = 10) as well as thin shell *(a/h =100)*. The natural frequencies of thin and thick shells have been tabulated in Table.5. Figs 12 and 13 show the first and second modes of vibration for thick shell and Figs.14 and 15 show the first and second mode of vibration for thin shell.
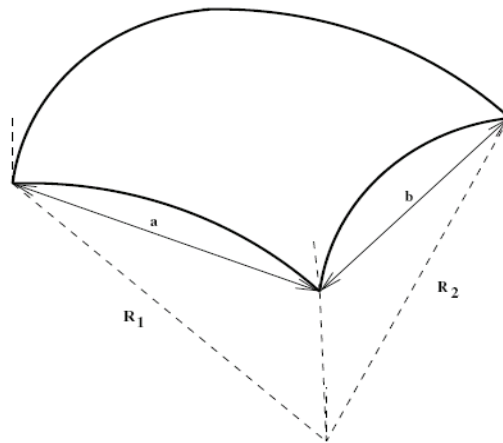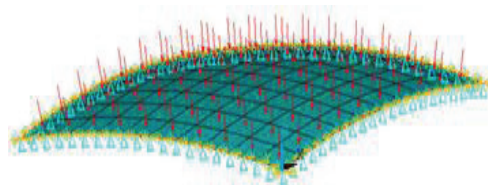

Fig.4. Generalized shell structure

Fig.5. Simply supported shell with uniformly distributed load at top surface.
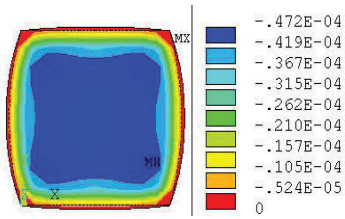
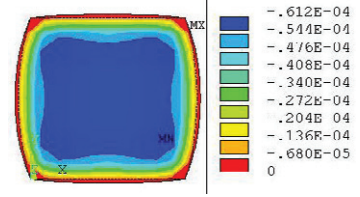Fig.6. Displacement variation for pure ceramic shell

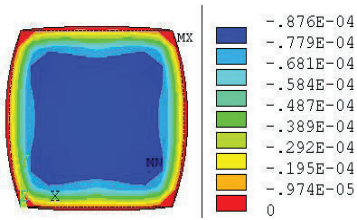Fig.7. Displacement variation for FG shell

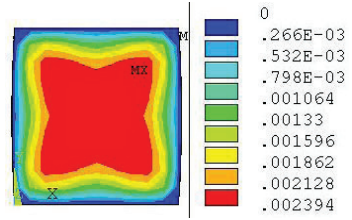Fig.8. Displacements variation for pure metallic shells

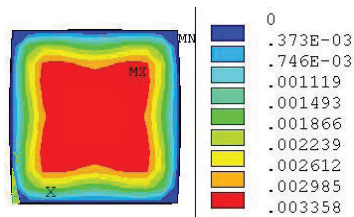Fig. 9. Displacement variation for pure ceramic shell

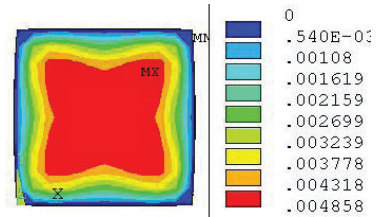Fig.10. Displacement variations for FG shell

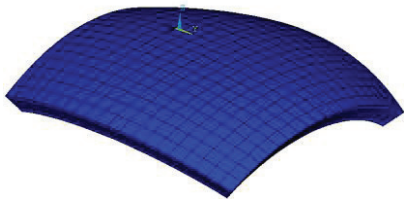Fig. 11. Displacement variation for pure metallic shell
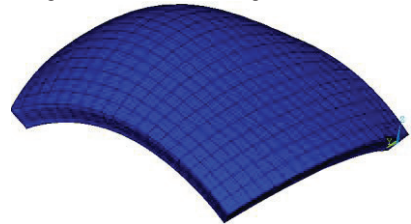

Fig.12. First mode of vibration for *a*/*h* =10

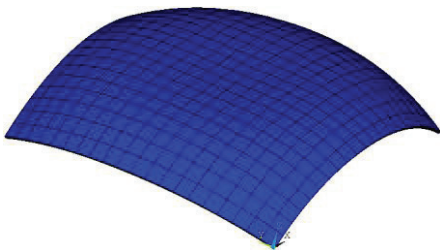

Fig.13. Second mode of vibration for *a*/*h* = 10
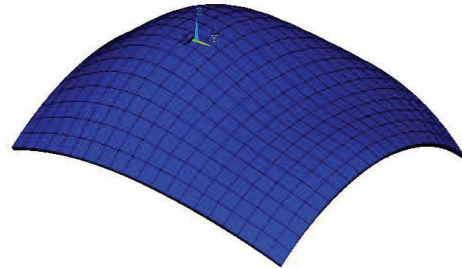

Fig.14. First mode of vibration for *a*/*h* =100

Fig.15. Second mode of vibration for *a*/*h* = 100

Table.3. Center displacement of shell with various *R/a* ratios for thin shell (*a/h*=100)

R/a	Center displacement of the shell(mm)		
	Pure ceramic	FG	Pure metal
1	-0.047	-0.061	-0.0876
2	-0.216	-0.28	-0.56
5	-1.43	-1.8	-2.655
10	-3.19	-5.04	-7.039
20	-5.969	-8.06	-11.05
50	-7.068	-9.618	-13.126
100	-7.262	-9.894	-13.486

Table.4. Center displacements of shell with various *R/a* ratios for thin shell (*a/h* =100)

R/a	Center displacement of the shell(mm)		
	Pure ceramic	FG	Pure metal
2	-2.394	-3.358	-4.858
5	-6.186	-8.778	-12.552
10	-8.209	-11.839	-16.656

Table.5. Comparison of natural frequencies for thick and thin spherical shells

a/h	Natural Frequencies (Hz)				
	1	2	3	4	5
10	6340.2	7980.9	8254.1	10297.0	10501.0
100	550.41	563.13	563.92	575.34	583.97

5. Conclusions

An eight nodded layered shell element (SHELL99) has been used for the modeling and analysis of functionally graded (FG) composite shell structures for deep as well as shallow shells. Important conclusions obtained from the present study have been presented here. Static analysis of the FG beam structure subjected to uniformly distributed load has been done and validated with the published results. Responses of FG spherical shell structure have been analyzed under mechanical and thermal loading and compared with the responses of the homogeneous shell structures, and it has been noted that the response of FG shells are between the responses of the homogeneous shells of the ceramic and metal. Free vibration analysis of thick and thin FG shell structures has been done and first and second mode shapes have been presented. It has also been observed that at the deep region the responses are changing rapidly and converging towards the plate region.

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