Analysis of heat transfer due to stretching cylinder with partial slip and prescribed heat flux: A Chebyshev Spectral Newton Iterative Scheme

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Abstract This study is dedicated to analyze the combined effects of partial slip and prescribed surface heat flux when the fluid is moving due to stretching cylinder. A very moderate and powerful technique Chebyshev Spectral Newton Iterative Scheme is used to determine the solution of the present mathematical model. Involved physical parameters, namely the slip parameter, Casson fluid parameter, curvature parameter and Prandtl number are utilized to control the fluid moments and temperature distribution. The results show that the fluid velocity and the skin friction coefficient on the stretching cylinder are strongly influenced by the slip parameter. It is further analyzed that hydrodynamic boundary layer decreases and thermal boundary layer increases with the slip parameter. Influence of Casson fluid parameter on temperature profile provides the opposite behavior as compared to the slip parameter. The comparison of numerical values of skin friction coefficient and the local Nusselt number is made with the results available in the literature. The accuracy and convergence of Chebyshev Spectral Newton Iterative Scheme is compared with finite difference scheme (Keller box method) through tables. The CPU time is calculated for both schemes. It is observed that CSNIS is efficient, less time consuming, stable and rapid convergent.

1. Introduction

It is extremely difficult to find the exact or series solutions of every industrial and engineering problems. In this situation, scientists and engineers are taking much interest in finding the numerical solution of the problem, especially for nonlinear coupled equations. In the past, many researchers used different techniques to solve many laminar and turbulent flow problems

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Flow over a stretching cylinder has fascinated many researchers due to its industrial and engineering applications. Crane [10] was the first who investigated the flow over a stretching cylinder. Gupta and Gupta [11], Datta et al. [12], Chen and Char [13] extended the work of Crane [10] by including the heat and mass transfer analysis under different physical situations. Wang [14] was the pioneer, who investigated the flow over a stretching cylinder. Ishak et al. [15,16] have discussed uniform suction/blowing and MHD effects on flow and heat transfer due to stretching cylinder. Abbas et al. [17] dealt with the laminar MHD flow and heat transfer of an electrically conducting viscous fluid over a stretching cylinder in the presence of thermal radiation through a porous medium. Bachok and Ishak [18] investigated the steady laminar flow caused by a stretching cylinder immersed in an incompressible viscous fluid with prescribed surface heat flux.

In literature survey, it is discovered that the flow field obeys the no-slip condition. However, certain physical situations exist which do not cope with the said conditions that is why the replacement of no-slip boundary condition with slip boundary condition is highly essential. The role of the slip condition is vital in shear skin, hysteresis effects and spurts. Slips come into existence when the fluid is a rarefied gas, [19], or in the case when it is particulate such as blood, foam, emulsion or suspension [20]. Slip also arises on hydrophobic surfaces, especially in micro and nano-fluidics [21]. Recently, Mukhopadhyay [22–24] studied the effects of partial slip with MHD suspension [20]. Slip also arises on hydrophobic surfaces, especially in micro and nano-fluidics [21]. Recently, Mukhopadhyay [22–24] studied the effects of partial slip with MHD connection some useful research achievements are made for Casson fluid flow over a stretching surfaces [26–29].

In this article, we studied the heat and fluid flow of non-Newtonian Casson fluid due to stretching cylinder with partial slip and prescribed heat flux using the Chebyshev Spectral Newton Iterative Scheme (CSNIS). The effects of the parameters on velocity and temperature profiles are discussed with the assistance of graphs. The graphs of skin friction coefficient and the local Nusselt number are plotted against different values of parameters. Comparison of the numerical values of skin friction coefficient and the local Nusselt number is performed with the help of those results already found in the literature as well as the said comparison is done with finite difference method as shown in tables. It is found that the results are in excellent agreement.

2. Problem formulation

We considered the flow of non-Newtonian Casson fluid outside the stretching cylinder of radius r. The flow is assumed as steady and axi-symmetric subjected to laminar boundary
layer assumptions. The surface of the cylinder is heated due to prescribed heat flux. The physical model of the flow situation is shown in Fig. 1, where the x-axis is taken along the axis of the cylinder and the r-axis in the radial direction.

It is assumed that cylinder is stretched in the axial direction with velocity \( U_w(x) \) and surface of the cylinder is subjected to variable heat flux \( q_w(x) \). The temperature at the cylinder surface \( T_u \) is constant and the ambient fluid temperature is \( T_\infty \), where \( T_u > T_\infty \). The rheological equation for an isotropic and incompressible flow of a Casson fluid is

\[
\tau_y = \begin{cases} 
2(\mu_b + p_i/\sqrt{2\pi})e_{ij}, & \pi > \pi_c, \\
2(\mu_b + p_i/\sqrt{2\pi})e_{ij}, & \pi < \pi_c.
\end{cases}
\]

Here \( \tau_y \) is the \((i,j)\)-th component of the stress tensor, \( \eta_{ij} = e_{ij}e_{ij} \) and \( \eta_g \) are the \((i,j)\)-th component of the deformation rate, \( \pi \) be the product of the component of deformation rate with itself, \( \pi_c \) is a critical value of this product based on the non-Newtonian model, \( \mu_b \) be the plastic dynamic viscosity of the considered fluid and \( p_i \) be the yield stress of the fluid. Therefore, if the applied shear stress is less than the yield stress, it behaves like a solid, whereas if the applied shear stress is greater than yield stress, it starts to move. By means of boundary layer approximation, the continuity, momentum and energy equations are

\[
\frac{\partial(uu)}{\partial x} + \frac{\partial(vr)}{\partial r} = 0, \quad (1)
\]

\[
\frac{\partial u}{\partial x} + vr = \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial v}{\partial r} \right), \quad (2)
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial v}{\partial r} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}, \quad (3)
\]

where \( u \) and \( v \) are the velocity components along \( x \) and \( r \) directions respectively, \( T \) is the temperature in the boundary layer, \( \nu \) is the kinematic viscosity, \( \beta = \mu_b \sqrt{2\pi}/p_i \) is Casson parameter, \( x \) is the thermal insuffusibility. The appropriate boundary conditions for the velocity components with velocity slip are given by

\[
u = 0 \text{ at } r = a, \quad u \to 0, \quad T \to T_\infty \text{ as } r \to \infty \quad (4)
\]

Here \( U_w = U_w(x/l) \) is the stretching velocity, \( B_i \) is velocity slip. The system of Eqs. (1)-(3) is transformed into the ordinary differential equations by using the following similarity transformation [18]

\[
\eta = \frac{r^2 - a^2}{2a} \sqrt{\frac{U_w}{v}}, \quad \psi = \sqrt{vU_w}af(\eta),
\]

\[
T_w = T_\infty + \frac{q_w}{k} \sqrt{\frac{v}{U_w}} \theta(\eta), \quad (5)
\]

where \( \eta \) is the similarity variable, \( \psi \) is the stream function defined as \( u = r^{-1} \partial \psi / \partial r \) and \( v = r^{-1} \partial \psi / \partial x \), which identically satisfies Eq. (1). The functions \( f(\eta) \) is the dimensionless stream function and \( \theta(\eta) \) is the dimensionless temperature function. From transformation (5), we obtain

\[
u = Uf'(\eta) \quad \text{and} \quad v = -\frac{a}{r} \sqrt{\nu U_0 / L} f(\eta). \quad (6)
\]

After substituting Eqs. (5) and (6) into Eqs. (2) and (3), we get the following governing equations in terms of ordinary differential equations

\[
(1 + 2\gamma \eta) \left( 1 + \frac{1}{\beta} \right) f'' + 2 \left( 1 + \frac{1}{\beta} \right) f' + f'^2 = 0, \quad (7)
\]

\[
(1 + 2\gamma \eta) \theta'' + 2\gamma \theta' + Pr(r\theta' - f') = 0, \quad (8)
\]

where primes denote differentiation with respect to \( \eta \), \( Pr \) be the Prandtl number and \( \gamma = \sqrt{\nu U_wa^2} \) is curvature parameter. The boundary conditions in Eq. (4) become

\[
f(0) = 0, \quad f'(0) = 1 + B \left( 1 + \frac{1}{\beta} \right) f'(0), \quad f'(\infty) = 0, \quad \theta'(0) = -1, \quad \theta(\infty) = 0. \quad (9)
\]

The physical quantities of interest are the skin friction coefficient \( C_f \) and the local Nusselt number \( Nu_s \), which are defined as

\[
C_f = \frac{\tau_w}{\nu u_w}, \quad Nu_s = \frac{xq_w}{\pi(T_w - T_\infty)}, \quad (10)
\]

in which, the wall skin friction and the wall heat flux are

\[
\tau_w = \mu_b \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial r} \right)_{r=a}, \quad q_w = -k \left( \frac{\partial T}{\partial r} \right)_{r=a}. \quad (11)
\]

Upon using the similarity transformation (5), the skin friction coefficient and the local Nusselt number can be written as

\[
C_fRe_s^{1/2} = \left( 1 + \frac{1}{\beta} \right) f'(0), \quad Nu_s/Re_s^{1/2} = \frac{1}{\partial(0)}. \quad (12)
\]

### 3. Numerical method

In order to solve nonlinear Eqs. (7) and (8) subject to boundary conditions (9) for different values of involving parameters, an efficient numerical scheme namely: Chebyshev Spectral Newton Iterative Scheme (CSNIS) is used. Chebyshev Spectral Newton Iterative Scheme is mathematically simple and can be easily coded in Matlab algorithm. It is based on Newton iterative scheme having convergence of order 2. It is therefore rapidly convergent as shown in Tables 4 and 5. It is low-cost scheme with less CPU usage. The solution procedure is as follows:

In first step, we linearized Eq. (7) by using Newton iterative scheme. For \((i + 1)\)th iterates, we write

\[
f_{i+1} = f_i + \delta f_i, \quad (13)
\]
for all dependent variables. Using Eq. (13) in Eqs. (7) and (9) we obtained
\[c_1 \delta f_0'' + c_2 \delta f_1'' + c_3 \delta f_1' + c_4 \delta f_0 = R_i,\]  
subject to boundary conditions
\[\delta f_i(0) = -f_i(0), \quad \delta f_i'(0) = B(1 + 1/\beta) \delta f_i''(0) = 1 - f_i'(0) + B(1 + 1/\beta) \delta f_i''(0), \quad \delta f_i'(\infty) = -f_i'(\infty).\]  
The coefficients \(c_i\) for \(i = 1, 2, 3, 4\) and \(R_i\) are
\[c_1 = (1 + 2\eta) (1 + 1/\beta), \quad c_2 = 2\eta (1 + 1/\beta) + f_i, \quad c_3 = 0, \quad c_4 = f_i', \quad R_i = -(1 + 2\eta) (1 + 1/\beta) f_i'' - 2\eta (1 + 1/\beta) f_i' - f_i' + (f_i')^2.\]  
The obtained Eq. (14) with boundary conditions (15) is now linear and solved by using the Chebyshev Spectral Collocation method. For this purpose, the physical domain \([0, \infty]\) is truncated to \([0, L]\), where \(L\) is chosen sufficiently large. The reduced domain is transformed to \([-1, 1]\) by using transformation \(\xi = 2\eta/L - 1\). Nodes from \(-1\) to \(1\) are defined by \(\xi_j = \cos(\pi j/N), j = 0, 1, 2, \ldots, N\), and are known as Gauss–Lobatto collocation points. The Chebyshev Spectral Collocation method is based on differentiation matrix \(D\), which can be computed in different ways. Here, we used \(D\) as suggested by Trefethen. For \(i = 0\), Eqs. (13) and (14) become
\[f_i = f_0 + \delta f_0, \quad c_1 \delta f_0'' + c_2 \delta f_1'' + c_3 \delta f_1' + c_4 \delta f_0 = R_i,\]  
in which \(f_0 = 1 - e^{-x}\) is used as an initial guess and we found \(\delta f_0\) from first iteration. Similarly for \(i = 1, (13), (14)\) become
\[f_i = f_1 + \delta f_1, \quad c_1 \delta f_1'' + c_2 \delta f_1' + c_3 \delta f_1' + c_4 \delta f_0 = R_i,\]  
where \(f_0 = f_1 + \delta f_0\) is known function and we found \(\delta f_1\) from second iteration. This procedure is continued until the accuracy \((f_{i+1} - f_i) \approx 0\) is achieved. As Eq. (7) subject to the boundary condition (9) has been solved and solution of \(f_i\) is obtained, then Eq. (8) becomes linear and it is solved by using the Chebyshev Spectral Collocation method without applying the Newton Iterative scheme. This algorithm is developed in MATLAB R2010a.

### 4. Results and discussion

The nonlinear ordinary differential Eqs. (7) and (8), subject to the boundary conditions (9) have been solved numerically using Chebyshev Spectral Newton Iterative Scheme (CSNIS) and the results are compared with implicit finite difference scheme known as the Keller box method, which is described in the book by Cebeci and Bradshaw. In Table 1, the numerical values of the surface temperature \(\theta(\eta)\) are compared with previously published results available in the literature. It is observed that the results are in excellent agreement. In Tables 2 and 3, the comparison of the values of skin friction coefficient and local Nusselt number with the finite difference method is given. The main finding of the tables is that the CSNIS has advantage over finite difference scheme in terms of time consumption. CSNIS reduces the cost over the time, which is need of hour and we achieved excellent accuracy. In Tables 4 and 5, the values of skin friction coefficient \((Re_s^{1/2} C)\) and \(\theta(\eta)\) are presented to show the validity and convergence of the results obtained by CSNIS. In Table 4 it is observed that the values of skin friction coefficient \((Re_s^{1/2} C)\) converge rapidly after only 3 or 4
A Chebyshev Spectral Newton Iterative Scheme

Table 3  Variation of local Nusselt number $\text{Re}_{x}^{-1/2} \text{Nu}_{x}$ for different values of $\gamma$, $Pr$ and $\beta$ when $B = 0.5$.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\beta$</th>
<th>$Pr$</th>
<th>Finite difference</th>
<th>CPU time</th>
<th>Iterative scheme</th>
<th>CPU time</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.7</td>
<td>0.69609</td>
<td>12.793637</td>
<td>0.69609</td>
<td>0.0703620</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>7.0</td>
<td>2.51384</td>
<td>12.881962</td>
<td>2.51380</td>
<td>0.070446</td>
<td>$4.0 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.7</td>
<td>0.67216</td>
<td>5.916910</td>
<td>0.67216</td>
<td>0.072338</td>
<td>$10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>2.57226</td>
<td>6.022827</td>
<td>2.57222</td>
<td>0.070590</td>
<td>$4.0 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.7</td>
<td>0.66611</td>
<td>6.098248</td>
<td>0.66611</td>
<td>0.079756</td>
<td>$10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>2.57853</td>
<td>6.047961</td>
<td>2.57850</td>
<td>0.077398</td>
<td>$3.0 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4  Values of skin friction coefficient ($\text{Re}_{x}^{1/2}C_f$) at different iteration.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>$\gamma = 0$, $\beta = \infty$</th>
<th>$\gamma = 1$, $\beta = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Re}_{x}^{1/2}C_f$</td>
<td>$B = 0.1$</td>
<td>$B = 0.5$</td>
</tr>
<tr>
<td>1</td>
<td>$-0.86956217$</td>
<td>$-0.57142857$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.872081922$</td>
<td>$-0.591083974$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.872082478$</td>
<td>$-0.59195472$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.872082478$</td>
<td>$-0.59195485$</td>
</tr>
<tr>
<td>5</td>
<td>$-0.872082478$</td>
<td>$-0.59195485$</td>
</tr>
<tr>
<td>6</td>
<td>$-0.872082478$</td>
<td>$-0.59195485$</td>
</tr>
</tbody>
</table>

Table 5  Comparison of the results of Bachok and Ishak [18] with present CSNIS results.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>$\gamma = 0$</th>
<th>$\gamma = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta (0) \rightarrow$</td>
<td>$Pr = 0.72$</td>
<td>$Pr = 6.7$</td>
</tr>
<tr>
<td>1</td>
<td>$1.236657471$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>2</td>
<td>$1.236657471$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>3</td>
<td>$1.236657471$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>4</td>
<td>$1.236657471$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>5</td>
<td>$1.236657471$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>6</td>
<td>$1.236657471$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>7</td>
<td>$1.236657471$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>8</td>
<td>$1.236657471$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>Bachok and Ishak [15] (Analytical)→</td>
<td>$1.236657472$</td>
<td>$0.333303061$</td>
</tr>
<tr>
<td>Bachok and Ishak [15] (Numerical)→</td>
<td>$1.2367$</td>
<td>$0.3333$</td>
</tr>
</tbody>
</table>

iterations. Table 5 clearly indicates that after performing small number of iterations the present CSNIS results display an excellent agreement with the results of Bachok and Ishak [18]. It is also noted that in Table 5 for the case of stretching sheet ($\gamma = 0$) the results converge after first iteration. This authenticates the validity of the present scheme.

Figs. 2–7 are plotted for various physical parameters namely: curvature parameter ($\gamma$), Casson fluid parameter ($\beta$), slip parameter ($B$) and Prandtl number $Pr$ against both velocity profile $f(\eta)$ and temperature profile $\theta(\eta)$. In Figs. 2 and 3, the domain truncation parameter ‘L’ and number of collocation points ‘N’ are set as 15 and 82, respectively, whereas in remaining figures $L$ and $N$ are set as 25 and 120, respectively. In Fig. 2, influence of velocity profile for various values of curvature parameter ($\gamma$) is developed. It depicts that velocity profile increases with increase of curvature parameter ($\gamma$) and growth in boundary layer thickness is noticed. Fig. 3 demonstrates the variation in the temperature profile $\theta(\eta)$ for various values of curvature parameter ($\gamma$). As surface area of the cylinder will squeeze with increase in curvature parameter ($\gamma$), consequently, less surface area provides low heat transfer rate in other words temperature profile decreases with increase of curvature parameter ($\gamma$). In Fig. 4 effects of Casson fluid parameter ($\beta$) on velocity profile are presented. It is noticed that the increase in the non-Newtonian parameter ($\beta$) provides more resistance in fluid motion and resultantly velocity of the fluid gets slow down with an increase in Casson fluid parameter ($\beta$). Influence of temperature profile with Casson fluid parameter ($\beta$) is plotted in Fig. 5. It is important to mention that highly viscous fluid provides more heat transfer rate as
compared to the Newtonian fluid. These noticeable effects can be observed in Fig. 5 that with an increase of Casson fluid parameter ($b$) temperature profile increases. In Fig. 6, the mainstream velocity, has been plotted against $\eta$ for various values of slip parameter $B$. It is noted that velocity profiles decrease near the wall with increase of $B$. It is due to the reason that when slip parameter increases in magnitude, the fluid near the wall no longer moves with the stretching velocity of surface. Increase in the value of $B$ the surface of the cylinder becomes smoother so that the pulling of the stretching surface rarely transmitted to the fluid. It is obvious that $B$ has a substantial effects on the solutions. In Fig. 7, temperature profiles
are plotted against $\eta$ for various values of slip parameter $B$. It is depicted that temperature of the fluid enhances with increase in slip parameter $B$. Fig. 8 presents the variation in temperature due to increase in Prandtl number ($Pr$). It is seen that the increment in $Pr$ reduces the thermal boundary layer thickness. Prandtl number denotes the ratio of kinematic viscosity to thermal diffusivity. As the viscosity of the fluid increases, the heat transfer rate enhances due to which the temperature of fluid decreases. Fig. 9 is plotted for skin friction coefficient against slip parameter ($B$) for the different values of $\beta$. The absolute value of skin friction gives higher friction with the wall for $\beta = 0.5$ and $1$ as compared to Newtonian fluid ($\beta \to \infty$). It is also observed that with increase in the value of $B$, the drag in the fluid with the wall increases for both Newtonian and non-Newtonian fluids. Fig. 10 is plotted for skin friction coefficient against curvature parameter ($c$). The value of skin friction coefficient is decreasing with the increase of curvature parameter $c$. These results also validate the findings in Fig. 2. Fig. 11 is drawn for local Nusselt number against slip parameter $B$ for the different values of $\beta$. For small value of $B$, increase of $\beta$ results in reduction of heat transfer rate and for large value of $B$, heat transfer rate enhances. Fig. 12 is plotted for local Nusselt number against Casson fluid parameter $\beta$ for the different values of curvature parameter $c$. With increase in $\gamma$ the surface area of cylinder reduces due to which heat transfer rate increases and same effects are observed for increasing values of $\beta$. 

Figure 8 Temperature profile for various values of $Pr$ with $B = 0.5$, $\beta = 1$ and $\gamma = 0.5$.

Figure 9 Variation of skin friction coefficient against slip parameter $B$ for the different values of $\beta$.

Figure 10 Skin friction coefficient against Casson fluid parameter $\beta$ for the different values of $\gamma$.

Figure 11 Variation of local Nusselt number against slip parameter $B$ for the different values of $\beta$.

Figure 12 Variation of local Nusselt number against Casson fluid parameter $\beta$ for the different values of curvature parameter $\gamma$. 
5. Concluding remarks

The numerical investigation of heat and fluid flow of non-Newtonian Casson fluid due to stretching cylinder with partial slip and prescribed heat flux is performed. For computation purpose, the Chebyshev Spectral Newton Iteration Scheme (CSNIS) is utilized. It is observed that the CSNIS is efficient, less time consuming, stable and rapid convergent and has excellent agreement with analytical solution (see Table 1) and finite difference method (see Table 2). The present investigations helps to conclude that

- Velocity is decreasing function of $\beta$ and temperature profile is increasing with increase in $\beta$.
- Momentum and thermal boundary layer thickness increases with increase of curvature of cylinder.
- Absolute skin friction gives higher friction for small $\beta$ as compared to Newtonian fluid ($\beta \to \infty$) and absolute value of skin friction coefficient increases with the increase of curvature of cylinder.
- For small values of $B$, reduction in heat transfer rate has been observed with increase of $\beta$.

References