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Random Weighting Estimation Method for Dynamic Navigation Positioning

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Abstract

This paper presents a new random weighting estimation method for dynamic navigation positioning. This method adopts the concept of random weighting estimation to estimate the covariance matrices of system state noises and observation noises for controlling the disturbances of singular observations and the kinematic model errors. It satisfies the practical requirements of the residual vector and innovation vector to sufficiently utilize observation information, thus weakening the disturbing effect of the kinematic model error and observation model error on the state parameter estimation. Theories and algorithms of random weighting estimation are established for estimating the covariance matrices of observation residual vectors and innovation vectors. This random weighting estimation method provides an effective solution for improving the positioning accuracy in dynamic navigation. Experimental results show that compared with the Kalman filtering, the extended Kalman filtering and the adaptive windowing filtering, the proposed method can adaptively determine the covariance matrices of observation error and state error, effectively resist the disturbances caused by system error and observation error, and significantly improve the positioning accuracy for dynamic navigation.

Keywords: estimation; navigation; error; random weighting estimation; dynamic navigation positioning; covariance matrix; kinematic model error; observation model error

1. Introduction

The Kalman filter is a commonly used computational method in aerospace navigation. It is required that both the state errors predicted from the kinematic model error and the observation model error be normally distributed with zero means. If the kinematic and observation models contain errors, the navigation estimates will be biased, and even divergent. However, it is unavoidable in practical engineering applications that the kinematic and observation models contain global or

local errors, due to the disturbances caused by singular observations and random factors in the dynamic environment. Therefore, it is necessary to estimate the deviations of the kinematic and observation models for improving the accuracy of dynamic navigation positioning.

In essence, the control of the influences caused by the kinematic model error and observation model error is to reduce the covariance matrix of observation vectors, thus sufficiently utilizing observation information to weaken the influence of the model errors on the state parameter vector^[1]. The windowing method is a commonly used method to adaptive estimation of the covariance matrix of observation noise^[2-5]. It uses m epochs of innovation vectors or residual vectors to estimate the current observation residual covariance matrix. In the case of innovation vectors, it is called the

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innovation-based adaptive estimation (IAE) filtering. Otherwise, it is called the residual-based adaptive estimation (RAE) filtering. However, this method requires residual vectors at each epoch be in the same type and the identical dimension and distribution, which is difficult to achieve in a highly dynamic environment^[2,4]. Jazwinski^[6] reported a method to compensate the model errors by using polynomial fitting. The use of a high-order polynomial may lead to the difficulty in solving the state parameters, while the use of a low-order polynomial may result in the low compensation accuracy. Mehra^[7] proposed an innovation-based adaptive estimation by windowing approximation. This method adaptively updates the covariance matrices of the observation equations and the state errors by using the observation information^[4]. However, it requires that the covariance matrices of observation vector and state errors be adaptively changed according to observation information, which is difficult to achieve in practical engineering. Yang, et al.^[1] reported an adaptive filter by combining the robust maximum-likelihood estimation with the state covariance matrix of the expansion model to improve the accuracy of dynamic navigation positioning. However, when the observational information is not sufficient at some epochs, it is difficult for this filtering method to estimate state parameters.

The robust adaptive filter is a method to control the influence of singular observations and the kinematic model errors by robustly estimating the covariance matrix of observation noise and adaptively adjusting the covariance matrix of the state noise through the adaptive factor^[8-10]. The robust adaptive filtering absorbs the merits of both robust estimation and adaptive filtering. It cannot only resist the disturbance of the observation, but also obtain reliable filtering results by using robust estimation principles for observation information. However, in the robust adaptive filtering, the iterative process for estimating the covariance matrix of the observation noise requires reliable state estimates. If the state estimates are disturbed by singular observation model error and the kinematic model error, the reliable equivalent covariance matrix cannot be obtained^[5,11]. The arithmetic mean estimation is a straightforward method for estimating the covariance matrix of the innovation vectors and the residual vectors^[12-13]. However, when calculating the covariance matrix of the observation noise vectors, there exists error in state prediction. If the state prediction error is large, the predicted residual is also large, thus decreasing the reliability in estimating the covariance matrices of the innovation vectors and the observation residual vectors.

The random weighting method is an emerging computing method in statistics^[14-16]. It has many benefits, such as the unbiased estimation, the simplicity in computation, the suitability for large samples, and no need to know the accurate probability distribution of objective characteristic parameters. The random weighting method can also be used to calculate statis-

tics with a probability density function, since the resultant statistical distribution provides a probability density function. Therefore, the random weighting method has been used to solve different problems^[14-19]. Just recently, Gao, et al.^[19] adopted the concept of random weighting estimation to fusion of multi-dimensional position data. However, the random weighting estimation is only used to estimate position data from single sensors. To the best of our knowledge, there has been very limited research to use random weighting method for dynamic navigation positioning.

In this paper, a new method is presented for dynamic navigation positioning. This method estimates the covariance matrices of observation noise and state noise to control the disturbances of singular observations model error and the kinematic model error on the state parameter estimation. It satisfies the practical requirements of the residual vector and innovation vector to sufficiently utilize observation information, thus weakening the disturbing effect of the kinematic model error and observation model error on the state parameter estimation. Experiments and comparison analysis with the existing methods are conducted to comprehensively evaluate the performance of the proposed method.

2. Principle of Random Weighting Estimation

Assume that X_1, X_2, \dots, X_n are independent and identically distributed random variables with common distribution function $F(x)$, and the corresponding empirical distribution function is

$$F_n(x) = \frac{1}{n} \sum_{i=1}^n I_{(X_i \leq x)}$$

The random weighting estimation^[14-18] of $F(x)$ can be defined as

$$H_n(x) = \sum_{i=1}^n V_i I_{(X_i \leq x)} \quad (1)$$

where $I_{(X_i \leq x)}$ is the characteristic function, i.e.,

$$I_{(X_i \leq x)} = \begin{cases} 1 & X_i \leq x \\ 0 & X_i > x \end{cases}$$

and $[V_1 \ V_2 \ \dots \ V_n]$ is a random vector subject to Dirichlet distribution $D(1, 1, \dots, 1)$, that is, $\sum_{i=1}^n V_i = 1$

and the joint density function of $[V_1 \ V_2 \ \dots \ V_n]$ is

$$f(V_1, V_2, \dots, V_n) = \Gamma(n)$$

where $[V_1 \ V_2 \ \dots \ V_n] \in \mathbf{D}_n$, and

$$\mathbf{D}_{n-1} = \left\{ [V_1 \ V_2 \ \dots \ V_{n-1}] : \begin{aligned} &V_k \geq 0 \ (k=1, 2, \dots, n-1), \\ &\sum_{k=1}^{n-1} V_k \leq 1 \end{aligned} \right\}$$

3. Covariance Matrices of Observation Residual Vectors and Innovation Vectors

Assume that the kinematic model is defined as^[1]

$$X_k = \Phi_{k,k-1} X_{k-1} + W_{k-1} \quad (2)$$

where X_k and X_{k-1} are the m -dimensional state parameter vectors at epoch t_k and t_{k-1} , respectively, $\Phi_{k,k-1}$ is an $m \times m$ state transition matrix, and W_k the model error vector whose mathematical expectation is zero and the covariance matrix is

$$\Sigma_{W_i} = \begin{cases} \Sigma_{W_k} & k = i \\ 0 & k \neq i \end{cases} \quad (3)$$

where W_k is a Gaussian white noise vector.

Assume that the random observation vectors are Z_1, Z_2, \dots, Z_{k-1} , and the optimally estimated value \hat{X}_{k-1} of the state parameter at time t_{k-1} is obtained from the previous $k-1$ steps of filtering. Also assume that observation vector Z_k can be obtained at time t_k . Then, the relationship between Z_k and system state parameter vector X_k at time t_k , that is, the observation model at time t_k , can be defined as^[1]

$$Z_k = A_k X_k + e_k \quad (4)$$

where A_k is an $n \times m$ design matrix (observation matrix), and e_k the observation noise vector whose mathematical expectation value is zero and covariance matrix is

$$\Sigma_{e_i} = \begin{cases} \Sigma_{e_k} & k = i \\ 0 & k \neq i \end{cases} \quad (5)$$

Apparently, e_k is a Gaussian white noise vector. When $i=k$, the covariance matrices of W_k and e_k are Σ_{W_k} and Σ_{e_k} , respectively. Here, W_k, W_i, e_k and e_i are independent of each other.

The state predicted vector can be written as

$$\bar{X}_k = \Phi_{k,k-1} \hat{X}_{k-1} + W_k \quad (6)$$

Its solution may be written as

$$\hat{X}_k = \bar{X}_k + K_k (Z_k - A_k \bar{X}_k) \quad (7)$$

where

$$K_k = \Sigma_{\bar{X}_k} A_k^T (A_k \Sigma_{\bar{X}_k} A_k^T + \Sigma_{e_k})^{-1} \quad (8)$$

$$\Sigma_{\bar{X}_k} = \Phi_{k,k-1} \Sigma_{\hat{X}_{k-1}} \Phi_{k,k-1}^T + \Sigma_{W_k} \quad (9)$$

The residual vector and the innovation vector may be written as

$$V_k = A_k \hat{X}_k - Z_k \quad (10)$$

and

$$\bar{V}_k = A_k \bar{X}_k - Z_k \quad (11)$$

In order to control the influence of residual vector V_k and innovation vector \bar{V}_k on the accuracy of navigation system, under the conditions of satisfying the Dirichlet distribution, the random weighting factor is

chosen according to the values of V_k and \bar{V}_k . The larger/smaller the value of V_k , the smaller/larger the weighting factor. The same process can also be applied to \bar{V}_k . The covariance matrices of V_k and \bar{V}_k are

$$\Sigma_{V_k} = \Sigma_k - A_k \Sigma_{\hat{X}_k} A_k^T \quad (12)$$

and

$$\Sigma_{\bar{V}_k} = \Sigma_k + A_k \Sigma_{\bar{X}_k} A_k^T \quad (13)$$

According to Eq.(10), residual vector V_k can be obtained by state parameter \hat{X}_k at time t_k , which contains the information of observation vector Z_k at time t_k . Innovation vector \bar{V}_k can be calculated from predicted state vector \bar{X}_k at time t_k . Since V_k is related to the state modified by Z_k and \bar{V}_k is related to the state unmodified by Z_k , it is readily known that innovation vector \bar{V}_k is better than V_k in reflecting the disturbance of the dynamic system^[1].

4. Random Weighting Estimation of Observation Noise Covariance Matrix by Windowing

This paper adopts the concept of random weighting estimation to the windowing filtering process, and further establishes the random weighting estimation filtering method. Similar to the windowing filtering process, the random weighting estimation filtering method estimates the current observation residual covariance matrix based on the m epochs of either innovation vectors or residual vectors. The former is called the innovation-based random weighting estimation (IRWE) filtering, and the latter is called the residual-based random weighting estimation (RRWE) filtering.

4.1. IRWE filtering by windowing

Assume that the observation error approximately obeys the normal distribution and the calculating window is m . Then, according to Ref.[5], we can define

$$\hat{\Sigma}_{\bar{V}_k} = \frac{1}{m} \sum_{j=0}^{m-1} \bar{V}_{k-j} \bar{V}_{k-j}^T \quad (14)$$

where $\hat{\Sigma}_{\bar{V}_k}$ is the IAE estimation value of $\Sigma_{\bar{V}_k}$.

Correspondingly, the IRWE of $\Sigma_{\bar{V}_k}$ can be written as

$$\hat{\Sigma}_{\bar{V}_k}^* = \sum_{j=0}^{m-1} w_j \bar{V}_{k-j} \bar{V}_{k-j}^T \quad (15)$$

where w_j is random weighting factor.

Substituting Eq.(15) into Eq.(12), the random weighting estimation for the covariance matrix of the observation vector Σ_k at t_k may be written as

$$\hat{\Sigma}_k^* = \hat{\Sigma}_{\bar{V}_k}^* - A_k \Sigma_{\bar{X}_k} A_k^T \quad (16)$$

4.2. RRWE by windowing

Similar to Eq.(14), the covariance matrix Σ_{V_k} of observation residual vector V_k can be defined as

$$\hat{\Sigma}_{V_k} = \frac{1}{m} \sum_{j=0}^m V_{k-j} V_{k-j}^T \quad (17)$$

Correspondingly, the IRWE of Σ_{V_k} can be written as

$$\hat{\Sigma}_{V_k}^* = \sum_{j=0}^m w_j V_{k-j} V_{k-j}^T \quad (18)$$

From Eq.(13), the random weighting estimation for the covariance matrix of observation vector Σ_k at time t_k is obtained as

$$\hat{\Sigma}_k^* = \hat{\Sigma}_{V_k}^* + A_k \Sigma_{\hat{X}_k} A_k^T \quad (19)$$

Note that, in Eq.(19), residual vector V_k and $\Sigma_{\hat{X}_k}$ at time t_k are used to calculate the random weighting estimation $\hat{\Sigma}_k^*$ adaptively. However, $\hat{\Sigma}_k^*$ must be obtained before solving $\Sigma_{\hat{X}_k}$ and V_k . Thus, the covariance matrix of observation vector Σ_k can be obtained by using the information at m epochs before time t_{k-1} , i.e.,

$$\hat{\Sigma}_{V_{k-1}}^* = \sum_{j=1}^m w_j V_{k-j} V_{k-j}^T \quad (20)$$

Correspondingly, Eq.(19) can be written as

$$\hat{\Sigma}_{k-1}^* = \hat{\Sigma}_{V_{k-1}}^* + A_{k-1} \Sigma_{\hat{X}_{k-1}} A_{k-1}^T \quad (21)$$

5. Experimental Results and Analysis

A prototype system has been developed for dynamic navigation positioning by using the proposed random weighting estimation method. Experiments have been conducted to comprehensively evaluate and analyze the performance of the proposed method. The comparison with the existing methods is also discussed in this section.

Trials are conducted to observe a glacier at the Alps^[2]. A Trimble 4000 SSI GPS receiver is installed at the base station, and a NovAtel Millenium GPS receiver is mounted to the aircraft. The aircraft takes off after the initialization of 8 min, and the flight time is about 100 min. The state equation is described as a constant velocity model. The variances of position, velocity and C/A code are 0.2 m^2 , $9.0 \times 10^{-5} \text{ m}^2/\text{s}^2$ and 1 m^2 , respectively. The spectral density of velocity is $0.2 \text{ m}^2/\text{s}^2$. The models reported in Ref.[2] are adopted as the variance and the covariance matrix of the state vector. When the aircraft is encountering atmospheric interference or conducting high-flexibility maneuvers, oscillation occurs, resulting in noise to the kinematic state of the aircraft.

For the comparison purpose, experiments are con-

ducted to estimate the dynamic positioning error of the aircraft under the same conditions by the proposed random weighting estimation filtering method as well as the existing methods such as the classical Kalman filtering, the extended Kalman filtering and the adaptive windowing filtering^[5], respectively. Fig.1 shows the filtering result obtained by the classical Kalman filtering. It can be seen that there are obvious oscillations in the filtering curve, and the positioning error is within $\pm 8 \text{ m}$. This shows that the classical Kalman filtering is significantly influenced by the disturbances of the kinematic model error and observation model error.

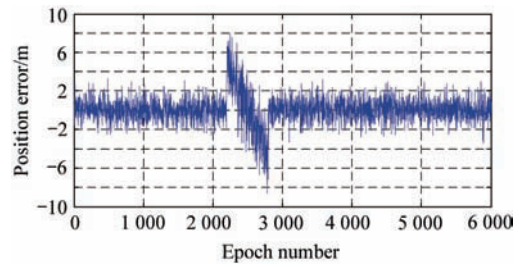


Fig.1 Classical Kalman filtering.

Fig.2 shows the filtering result generated by the extended Kalman filtering. The positioning error is within $\pm 5 \text{ m}$, which is much smaller than that by the classical Kalman filtering. This demonstrates that the extended Kalman filtering has a better performance than the classical Kalman filtering.

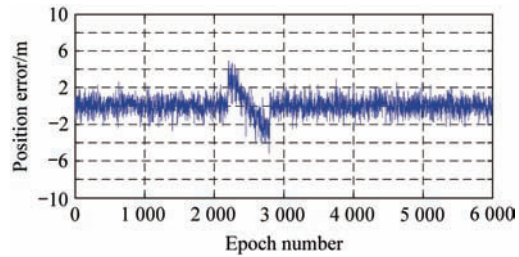


Fig.2 Extended Kalman filtering.

As shows in Fig.3, the filtering curve generated by the adaptive windowing filtering involves small oscillations. The positioning error is within $\pm 2.5 \text{ m}$, which is much smaller than those by the classical Kalman filtering and extended Kalman filtering. This demonstrates that the adaptive windowing filtering has the capability to restrain the disturbances of the kinematic

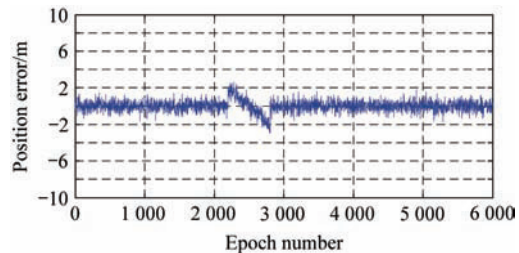


Fig.3 Adaptive windowing filtering.

model error and observation model error and the filtering performance is much better in comparison with the classical Kalman filtering and extended Kalman filtering.

Fig.4 shows the filtering result obtained by the proposed random weighting estimation method. It can be seen that there is no obvious oscillation in the filtering curve, and the curve is almost in the stable state during the flight time. The positioning error is within ± 1 m, which is much smaller than that obtained by the adaptive windowing filtering. This demonstrates that the proposed method outperforms the adaptive windowing filtering, and it can effectively resist the disturbances caused by the kinematic model error and observation model error.

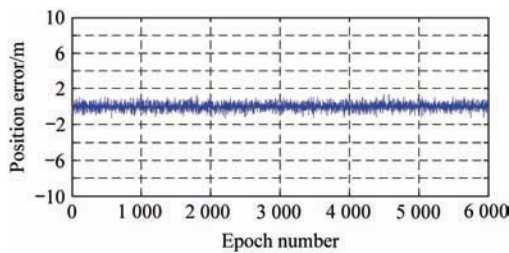


Fig.4 Random weighting adaptive filtering.

Comparison between Fig.4 and Figs.1-2 reveals that the proposed method has much higher positioning accuracy than the classical Kalman and extended Kalman filtering.

Table 1 shows the detailed error analysis by comparing the proposed random weighting estimation method with the classical Kalman filtering, extended Kalman filtering and adaptive windowing filtering methods.

Table 1 Error analysis of filtering output

Filtering method	Mean error /m	Standard deviation /m	Range of the positioning error/m
Kalman filtering	0.855	1.161	± 8.0
Extended Kalman filtering	0.742	0.879	± 5.0
Adaptive windowing filtering	0.623	0.714	± 2.5
Random weighting adaptive filtering	0.501	0.622	± 1.0

From the above experiments, it can be seen that the classical Kalman filtering and extended Kalman filtering cannot resist the disturbances of the kinematic model error and observation model error. The adaptive windowing filtering estimates the covariance matrix of observation noise by using the IAE and RAE methods. Since it provides the ability for resisting the disturbances of the kinematic model error and observation model error and the observation noise, the filtering performance is better than the classical Kalman and extended Kalman filtering. However, the adaptive windowing filtering does not only require the information on the dynamic body at each epoch be in the same type and identical dimension and distribution, but also re-

quires that the observation noise at the current epoch be close to the average observation noise at each epoch in the window. In contrast, the random weighting adaptive filtering proposed in this paper is much more effective than the adaptive windowing filtering in terms of resisting the disturbances of the kinematic model error and observation model error. It also has much higher positioning accuracy for dynamic navigation than the classical Kalman filtering and the extended Kalman filtering. Further more, it is simple in computation.

Experiments are also conducted to evaluate the proposed random weighting estimation filtering method in terms of the positioning accuracy for a strap-down inertial navigation system/synthetic aperture radar (SINS/SAR) integrated navigation system.

Suppose the aircraft initial position is 34.2° in latitude, 108° in longitude; its height is 10 000 m and initial velocity 800 m/s. The gyro constant drift is 0.01 ($^\circ$)/h, and the white noise 0.001 ($^\circ$)/ \sqrt{h} . The accelerometer's zero bias is $10^{-4}g$, and the random drift $10^{-5}g \cdot \sqrt{s}$. The SAR computing time for image matching is 5 s, the discretization period of the system state 1 s, and the update time of the filtering observation 5 s. The accuracy of the altimeter is 10 m, the SINS's initial position error is 10 m, initial velocity error 2 m/s, initial alignment error zero, and simulation time 1 500 s.

Experiments are conducted to estimate the position errors of the aircraft under the same conditions by the windowing filtering and the proposed random weighting estimation filtering method, respectively. Fig.5 shows the position error obtained by the windowing filtering. After 150 s, the longitude error is within ± 6 m, the latitude error within ± 5 m, and the height error within ± 4 m. Instead, Fig.6 shows the position error obtained by the random weighting adaptive filtering. After 150 s, the longitude error is within ± 1 m, the latitude error ± 1 m, and the height error ± 1 m. The experiments and comparison analysis have demonstrated that the proposed method is much more effective for improving the performance of SINS/SAR integrated navigation system than the windowing filtering.

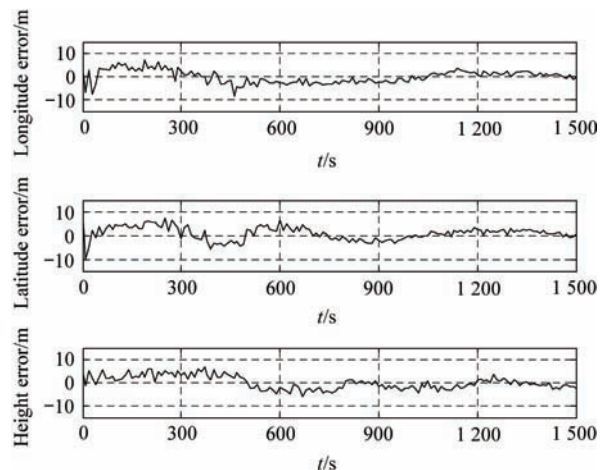


Fig.5 Position error obtained by windowing filtering.

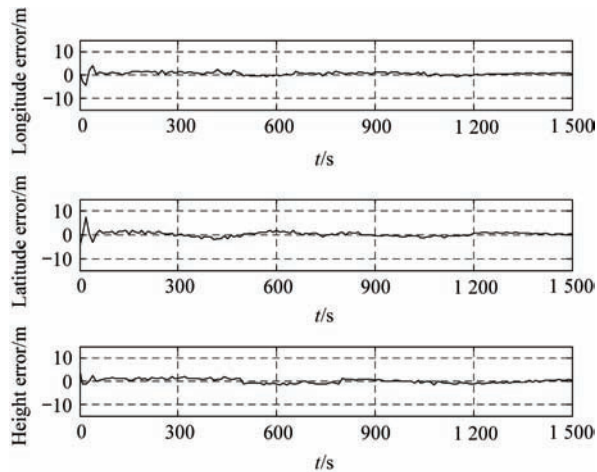


Fig.6 Position error obtained by random weighting adaptive filtering.

6. Conclusions

This paper presents a new random weighting estimation method for dynamic navigation positioning. This random weighting estimation method provides an effective solution for improving the positioning accuracy in dynamic navigation. Experimental results and comparison analysis demonstrate that the proposed method can not only adaptively determine the covariance matrices of observation noise and state noise, but also effectively resist the disturbances of singular observations and kinematic model noises. The proposed method is also simple in computation.

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