Available online at www.sciencedirect.com
ScienceDirect
Procedia - Social and Behavioral Sciences 191 (2015) 1859 – 1864
Procedia
Social and Behavioral Sciences
ELSEVIER

WCES 2014

Problem Solving - Purpose And Means Of Learning Mathematics In School

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Abstract

Of all school subjects, mathematics introduces and develops the "problem-solving" concept, as fundamental component of school learning with a strong formative effect on students. In mathematics, solving problems represents the most effective concept to contextualization and re-contextualization of concepts, to operational and basic mathematical knowledge transfer to ensure a sustainable and meaningful learning. The resolvent conduct of the student also involves, in addition to the cognitive factors, factors aiming the affectivity and the experience of the student. In this context, the study conducted on a significant group of students at the end of the secondary school aimed at knowing *the importance given by students to solving problems, the preferred type of problems and the performance level achieved by the students in solving math problems*. Taking into account the complex intellectual activity, the nature of difficulties which the student faces in solving problems is varied, ranging from perceptual difficulties to those concerning his cognitive self-regulation.

Selection and peer-review under responsibility of the Organizing Committee of WCES 2014

Keywords: Problem, problem-situation, problem solving;

1. Problem. Problem solving. Selective conceptual field

From a general point of view, a problem is a question or a difficulty that needs to be solved. In a pedagogical situation, when giving a student a problem to solve it means that he is asked to react, in a satisfactory way, to solve it, by appealing to his knowledge (Raynal & Rieunier, 1997, p 295). According to the same authors, problem solving represents a higher intellectual activity, regarded by the majority of experts as the most complex level of cognitive activities that mobilizes at the same time, all of the individual's intellectual faculties: memory, perception, reasoning, conceptualization, language and they equally involve emotions, motivation, self-confidence and ability to control the

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Selection and peer-review under responsibility of the Organizing Committee of WCES 2014
doi:10.1016/j.sbspro.2015.04.332

situation. The complexity of solving a problem is also reflected in the structure of its conceptual field: problem, problematic situation, problematic space, resolvent conduct (Zlate, 2006). As regarding the problematic situation, Raynal & Rieunier (1997, p. 295) distinguishes between *the situation of execution* (a situation where the solving procedures are known by the individual and directly applicable) and *the problem-situation* (a position for which the individual does not have solving procedures because they either lack or they have resulted in a failure, ant they are dependent on the representation method of the problem). In mathematics, a problem-situation is a learning situation which the teacher imagines in order to create a space for reflection and analysis around a problem/ question to be solved. This situation should allow the student to improve his knowledge, through new representations, and therefore, to learn. In essence, every problem-situation should be a reason of contextualization and operationalizing of knowledge processed in mathematics lessons. For an individual, a given situation is defined as an execution situation or problem situation, not only in itself, but rather by "the relationship existing between the task and the abilities of the subject." (J.-F. Richard, cited Raynal & Rieunier 1997, p. 296). Thus, the problem that the teacher has to face is to assess the proposed difficulty and, in case of a problem-situation he has to introduce a level of guidance/ mediation sufficient to guide the student until the situation becomes a situation of execution for him. The notion of *space-problem* (Raynal & Rieunier, 1997, p. 295) corresponds to the research space specific to each problem: in order to build a good representation of the problem there must be identified a workable research space, where assumptions can be made, results can be interpreted, and solving phases can be shaped etc.

2. The formative nature of solving mathematical problems

Solving problems is the obvious way of manifestation and usefulness of mathematics, intellectually and beyond. This activity provides the student the opportunity to face a difficulty that he can overcome by exploiting the combination of knowledge he possesses (declarative, procedural and conditional) in an efficient manner in a well-defined context. It also favours the transfer of this knowledge between different fields of knowledge. "In addition, any problematic situation stimulates motivation towards an intellectual progress; it trains his creativity and applies mental behaviour which aims a better structure of his knowledge. Consequently, problems develop curiosity and the spirit of research demonstrates the usefulness of the taught subject by solving concrete problems." (Bair & others, 2000, pp. 20-22). In addition, any problem solving is "an initiation into a scientific research", by learning to analyze a certain situation, modelling, researching effective strategies, reasoning, using analogies, generalization, examining specific cases etc...The successful completion of the process of solving a math problem, whether it is a pure mathematical problem or a problem with a practical application, is subject to a positive way of thinking of the solver engaged in a successful project, supported by "the firm intention to reach the end." Bair & others (2000, pp. 20-22) highlights a number of conditions and qualities which he claims and are also developed by the mathematical problem solving: excellent mental condition which implies a disposition to work, to always achieve the best results, it gives strength to overcome the obstacles that inevitably come in the way of success, a thorough knowledge of the theory, a consistent, methodical and reflexive work, and also high moral qualities (courage, determination, ambition, patience, perseverance, initiative spirit and enthusiasm). There are also needed: motivation, pleasure, and self-confidence (essential for a positive attitude). Self-confidence is acquired by understanding matter through personal reflection on work, by analysing successes, even the partial ones, and also failures. The chances of a student to carry out the solving of a difficult problem are significantly reduced if he shows fear towards the statement and he is pessimistic as regarding his possibilities of success: such a negative mindset is often paralyzing and can lead to a real "mental block" right from the start. To avoid these situations, the teacher must provide the conditions to achieve "successful experiences" in solving problems, to give him a positive self-image in relation to this activity.
In conclusion, when solving a problem, three types of factors constantly interact and they constitute the "ideal space" specific to each individual (Bair & others, 2000, p.14): *emotional factors* (stress, interest, motivation, anxiety, perseverance, endurance to abandonment); *experience factors* (age, maturity, mathematical experience, degree of familiarity with the methods, the context, content, ..); *cognitive factors* (reading skills, memory, logic, analysis ability, ...) As regarding **the conditions of solving problems**, problems can be solved *individually* or *in groups*, with repercussions on the resolvent process. In literature (cited Zlate, 2006) there is the idea that solving problems in group is more productive than solving them individually due to the interactions between the group members, the emulation of compensatory and complementary phenomena functioning within the group, provided

that the group is well organized (as size, distribution of statutes and roles, types of interactions, relationships, as value guidance of the component members, etc..)

3. Some aspects aimed at solving math problems

The study was made on a sample of 350 students, with ages between 14 and 16 and it was focused on the variables 1 and 2:

3.1 Variable 1: the intentions of middle school students with regard to learning of mathematics

The set of items below refer to the intentions of students with regard to learning of mathematics in middle school.

By learning mathematics I intend:

I 1: *to acquire theoretical mathematics knowledge;*
I 2: *to improve my calculation skills;*
I 3: *to form the capacity to reason and solve a problem.*

The first statistical data analysis can be performed on the basis of the starting parameters shown in the table below.

Table 1. Statistic coefficients for "the intentions of middle school students with regard to learning of mathematics"

By learning mathematics I intend:		to acquire theoretical mathematics knowledge	to improve my calculation skills	to form the capacity to reason and solve a problem
N	Valid	350	349	349
	Missing	0	1	1
Mean		4,0143	4,5186	4,8911
Median		4,0000	5,0000	5,0000
Mode		4,00	6,00	6,00
Std. Deviation		1,46098	1,41129	1,34534
Variance		2,134	1,992	1,810
Minimum/ Maximum		1,00*/6,00**	1/6	1/6

Interpretation: * *1- this never happens* ** *6- this always happens*

The signification level for the difference between statistic ambient corresponding to these items is given by **TNW**, the results of which are shown in the Table below:

Table 2. Wilcoxon Signed Ranks Test for "the intentions of middle school students with regard to learning of mathematics"

By learning mathematics I intend:	to form the capacity to reason and solve a problem – to improve my calculation skills	to improve my calculation skills – to acquire theoretical mathematics knowledge
Z	-4,071(a)	-5,795(a)
Asymp. Sig. (2-tailed)	**,000**	**,000**

a Based on negative ranks.

It results that the differences between the averages are significant, the null hypothesis H_0 can be rejected in both cases, at a significance threshold of $p < .01$. Thus, we can say that, by learning mathematics, mid school students aim, first of all, *to form the capacity to reason and solve a problem,* then *to improve their mathematic calculation skills* and, in the end, *to acquire theoretical mathematics knowledge*.

3.2 Variable 2: preferences of students regarding the method of solving math problems

Generally, there are two major types of problem-solving procedures: algorithmic procedures (consisting of an ordered sequence of operations which, if executed correctly, allow us to definitely get the result; they apply *procedures mentally available*) and heuristic procedures (provide, on the contrary, prescriptions and possible strategies for solving problems, but without guaranteeing the success, they require *developing original strategies*) (Astolfi et alii, 1997, pp.139-140).

The next set of items concerns *students' preferences* regarding *methods of solving* mathematics problems.

I prefer math problems:

I 4: *whose solving is based on reliable methods for solving (algorithms taught by the teacher);*

I 5: *for which I must find myself the method of solving.*

Table 3. Statistic coefficients for "types of problems prefered by the students"

I prefer math problems:		whose solving is based on reliable methods for solving (algorithms taught by the teacher)	for which I must find myself the method of solving
N	Valid	350	350
	Missing	0	0
Mean		4,7686	3,4657
Median		5,0000	3,0000
Mode		6,00	4,00
Std. Deviation		1,42054	1,51505
Variance		2,018	2,295
Minimum/ Maximum		1,00*/6,00**	1,00

Interpretation: * *1- this never happens* ** *6- this always happens*

From the starting parameters table above, it can result that students prefer math problems whose solving is based on *algorithmic methods.* The difference between statistical averages belonging to the two items is significant, as the **TNW** presented in Table....also shows.

Table 4. Wilcoxon Signed Ranks Test for "types of problems prefered by the students"

I prefer math problems:	for which I must find myself the method of solving – whose solving is based on reliable methods for solving (algorithms taught by the teacher)
Z	-10,325(a)
Asymp. Sig. (2-tailed)	**,000**

a Based on positive ranks.

3.3 Variable 3: the level of students' results in using mathematics to solve real-life problems

The application of mathematics for solving problems situated in the real world, called mathematical modeling can be conceived as a complex process containing several stages: understanding the present situation, the construction of a mathematical model which describes the essence of the significant elements and relationships involved in the situation, the application of the mathematical model to solve problems, the contextual interpretation of the results provided by the model applied (Crahay & others, 2005). In the experiences conducted within more extensive studies (Crahay & others, 2005, p. 159), the majority of students showed an absence or a weak trend to take into account real-world knowledge in solving problems.

To the same result has also led the research carried out on a sample of 300 papers written by students aged 14-16 which followed the results achieved by the students in solving an arithmetic problem that was presented in a concrete situation from real life .

Father and son are 60 years old together. Their ages report values is 2,75.

a) Find out how old the son is.

b) How many years ago the father's age was three times greater than the son's?

The first step in solving this subject consists of writing the appropriate mathematical model. This can be an equation or a system of two equations, if the solving is made with algebraic methods or the figurative representation, in case of solving with arithmetical methods.

Table 5. Students' results in solving the math problem

Name of the step	Success	Failure	Omission
Solve completely subject a)	32,24%	33,88%	33,88%
Solve completely subject b)	18,18%	23,96%	57,86%

Note:
** By **Success** we designated the situation in which that procedure was successfully completed by the student: the judgment used was well motivated scientifically and the elaboration was correct and complete.*
*** By **Failure** we designated the situation in which specific procedures of a step were initiated, but either the judgment was wrong, not motivated or not finalized, or the elaboration was incorrect and/or incomplete, or the result achieved was incorrect.*
**** By **Omission** we designated the situation in which that procedure wasn't approached at all by the student.*

Result interpretation:

For subpoint a), we notice that the percentage of students who completely solved the subject (32,24%), who solved it incorrectly (33,88%) and of those who didn't approached it at all (33,88%) are approximately equal, a not encouraging situation, especially if we consider that solving this problem does not require developed knowledge of mathematics. The solving of subpoint b) depends on the results achieved at the first subpoint. So, there can be noticed a very important reduction of the percentage of those who started solving this subject. Only 18,18% of the students finished solving the problem, indicating the correct answer, while 57,86% avoided this subject. The percentage of 23,96% indicates the students which either gave another answer than the correct one or gave an incomplete solution. We have to make a remark on the way in which the students included in the study represented for themselves that problem (which is a practical application of mathematical knowledge): if we take into account the great number of answer impossible from the practical point of view formulated by the students to this problem, we tend to believe that they actually hadn't have a representation of the problem and hadn't made any connection with the real situation. For example, we give some of these results: 20,75 years old; 31,375 years old; -240/7 years old; 92 years ago; in 132 years' time; the sun is 55 year old and the father is 49 years old etc. Here we see the phenomenon of „putting within parentheses" (or „omission of the signification") commonly found in solving math problems. This is associated to the explicit belief that there is a „gap" between the artificial world of arithmetic problems presented in school and the real world outside the school. This tendency is explained in „the game rules" (Crahay&alii, 2005, p. 163), those hidden rules and postulates which seem to be taken into account, implicitly and tacitly, by students so that the "didactic contract" specific for problem solving functions efficiently.

These situations are determined mainly by two aspects of the practice and culture of teaching of math problems solving in school:

1. The nature of the problems presented: only few problems invite or encourage students to activate daily knowledge and experiences
2. The nature/type of the conversations and activities related to these problems within traditional math classes

Problems are exercises of relationship between real life situations and the mathematical models studied in school and they are occasions to reflect on complex relations between reality and mathematics. Students' capacity to issue judgments on the problems' reality reflects a positive attitude towards a realistic mathematical modeling.

Besides this aspect of putting within parentheses the signification, the failure of students who perfectly possess the procedures required to solve a problem may be caused by *inadequate decoding* of the problem or by the *cognitive overload* of the student. The more complex is the structure of the problem, the greater are the chances to occur the cognitive overload situation. Baddeley and Miller (apud Crahay& alii, 2005, p. 179) showed that human mind is limited in its ability to handle information. According to cognitive psychology the probability of procedures mobilization is even weaker as the number of procedures to activate is higher. Once the knowledge necessary for problem solving is recovered, the subject engaged in an solving endeavour must have a *planning cognitive function* to order his procedures, the mobilization and their correct execution and the evaluation of the extent to which the sequence of the procedures lead to the solution of the problem (Schoenfeld, apud Crahay& alii, 2005). It's about the **monitoring function** that contributes in a significant way to the cognitive self-regulation, which is absolutely necessary to complete any task.

4. Conclusions

Students realize the importance of solving math problems for learning mathematics, but they prefer problems that are solved through algorithmic methods. The difficulty of this problem lies identifying the type of problem and the solving effort is reduced to the correct application of the algorithm leading surely to the expected result. But the true intellectual challenge is given by problems that can be solved through heuristic methods, where the effort is focused on searching the solution. Solving such problems require in a greater degree student emotions and experience, but the formative effect is more obvious as it better prepares them to solve real problems that they will face in life. The study conducted on students with a significant experience in solving math problems have shown that their results are quite low, even if the problem to be solved doesn't present a high degree of difficulty for that level. So, teaching students to solve math problems is still a huge challenge for the math teacher. Especially as, beyond the actual content of the problems, as a priority and traditional area of his preoccupations, there is the student with his peculiarities, which the teacher must take into account a greater extent.

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