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# Born–Infeld quantum condensate as dark energy in the universe

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**Abstract**

Some cosmological implications of ultraviolet quantum effects leading to a condensation of Born–Infeld matter are considered. It is shown that under very general conditions the quantum condensate cannot act as phantom matter if its energy density is positive. On the other hand, it behaves as an effective cosmological constant in the limit where quantum induced contributions to the energy–momentum tensor dominate over the classical effects.


Evidence that the universe is undergoing a phase of accelerated expansion at the present epoch continues to grow. Not only is accelerated dynamics inferred in high redshift surveys of type Ia supernovae [1], it is now independently implied from observations of the anisotropy power spectrum of the Cosmic Microwave Background (CMB) [2–4]. The favoured explanation for this behavior is that the universe is presently dominated by some form of 'dark energy' contributing up to 70% of the critical energy density, with the remaining 30% comprised of clumpy baryonic and non-baryonic dark matter [3]. One of the central questions in cosmology today is the origin of this exotic matter. In the quintessence scenario, for example, the dark energy is a self-interacting scalar field that slowly evolves down a potential and thereby acts as a negative pressure source [5]. This paradigm has attracted attention because a wide class of models exhibit 'tracking' behavior at late times, where the dynamics of the field becomes independent of its initial conditions in the early universe. In principle, this may resolve the fine-tuning problem inherent in dark energy models based purely on a cosmological constant [6]. Nevertheless, there is at present no generally accepted origin for the quintessence field from a particle physics perspective.

Current observations constrain the effective equation of state of the dark energy to be within the region bounded by $-1.45 < w_{\rm DE} < -0.74$ at the 95% confidence level [3,7], where $w_{\rm DE} \equiv p_{\rm DE}/\rho_{\rm DE}$ and $p$ and $\rho$

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denote the pressure and energy density of the field, respectively. Phantom matter corresponds to the region of parameter space $w_{\mathrm{DE}} < -1$, where the scalar field has negative kinetic energy [8,9]. Although matter of this form is presently consistent with observations, the origin of such a scalar field with a non-conventional kinetic energy is not understood. On the other hand, it was recently noted that the phantom field need not necessarily be a scalar but may in principle have vector or tensor degrees of freedom [10]. Moreover, similarities between phantom matter and conformal field theory (CFT) were discussed in Ref. [11].

In view of the above developments, and given the absence of a favoured scalar field model, it is important to search for alternative candidates for the dark energy/phantom matter whose origin may be found within the context of string/M-theory. One of the more unusual types of matter that is predicted to arise in string/M-theory is the so-called Born–Infeld (BI) field. (For a review, see, e.g., Ref. [12].) The action for the BI field coupled to gravity is given by

$$S_{\mathrm{BI}} = -\lambda \int d^4x \left\{ \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})} - \sqrt{-\det g_{\mu\nu}} \right\}, \tag{1}$$

where $F_{\mu\nu}$ is the field strength for the gauge field, $g_{\mu\nu}$ is the spacetime metric and $\lambda$ is a coupling constant.

The question we address in this Letter is whether such a field can play the role of dark energy/phantom matter in the present-day universe. For simplicity, we assume throughout that the gauge group is abelian. It is known that standard abelian BI cosmology is necessarily anisotropic (or inhomogeneous) [13]. On the other hand, as shown in Ref. [14], non-abelian BI cosmology may be isotropic when the proper choice of gauge field configuration is made. Moreover, the equation of state of BI matter may become negative in some regions of parameter space [14]. However, it could be argued that such a complicated choice for the gauge field strength might be artificial. In the present Letter, therefore, we suggest that abelian BI theory may be employed as a toy model for isotropic BI cosmology where the field strength is time-dependent due to quantum effects (specifically effects similar to that of gluon condensation in QCD).

In a four-dimensional spacetime, it follows that

$$\det(g_{\mu\nu} + F_{\mu\nu}) = (-g)\left\{1 + \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16(-g)}\left(F_{\mu\nu}F^{*\mu\nu}\right)^2\right\}, \tag{2}$$

where $g \equiv \det g_{\mu\nu}$ and $F^{*\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. The energy–momentum tensor for the BI field is then derived by varying the action (1) with respect to the metric tensor:

$$T^{\mu\nu}_{\mathrm{BI}} \equiv \frac{1}{\sqrt{-g}}\frac{\delta S_{\mathrm{BI}}}{\delta g_{\mu\nu}} = -\frac{\lambda}{2}\left\{\frac{g^{\mu\nu}\left(1 + \frac{1}{2}F_{\rho\sigma}F^{\rho\sigma}\right) - F^{\mu}{}_{\rho}F^{\nu\rho}}{\sqrt{1 + \frac{1}{2}F_{\rho\sigma}F^{\rho\sigma} - \frac{1}{16(-g)}(F_{\rho\sigma}F^{*\rho\sigma})^2}} - g^{\mu\nu}\right\}. \tag{3}$$

We now assume that the spacetime metric corresponds to the spatially flat, isotropic, Friedman–Robertson–Walker (FRW) universe:

$$ds^2 = -dt^2 + a^2(t)\,dx^2, \tag{4}$$

where $a(t)$ represents the scale factor of the universe. If the electric or magnetic component of the field strength, $F^{\mu\nu}$, becomes non-trivial, the isometry of the metric (4) is broken. However, it is known that in the case of QCD, the vacuum expectation value of the square of the field strength becomes non-trivial due to the condensation of the gluon. Phenomenologically, the gluon condensation has been observed by using the operator product expansion [15]. In QCD the condensation can be derived by using the trace anomaly induced effective action [16] (for a general discussion, see [17]). If we take into account such effects, we may impose as our main assumption that

$$\left\langle F_{\mu\nu}F^{\mu\nu}\right\rangle_V = \alpha(t), \qquad \left\langle F_{\mu\nu}F^{*\mu\nu}\right\rangle_V = \beta(t)\sqrt{-g}, \tag{5}$$

where $\langle\,\rangle_V$ denotes vacuum expectation values and the functions $\alpha(t)$ and $\beta(t)$ may in general depend on time but are constant on the spatial hypersurfaces. Due to the isometry of the spatial hypersurfaces, we may further assume that

$$\left\langle F^{0}{}_{\rho}F^{0\rho}\right\rangle_V = \frac{\alpha_t}{4}g^{00}, \qquad \left\langle F^{i}{}_{\rho}F^{j\rho}\right\rangle_V = \frac{\alpha_s}{4}g^{ij}, \qquad \alpha_t + 3\alpha_s = 4\alpha, \tag{6}$$

where $\alpha_s = \alpha_s(t)$, $\alpha_t = \alpha_t(t)$.

We emphasize that the origin of Eq. (5) is purely quantum in nature and arises from the condensate that appears due to vacuum fluctuations. In general, the vacuum expectation values (5) are not independent. However, the relationship between them can only be determined by a direct calculation to some finite order in loop corrections (normally chosen to be the one-loop level). Moreover, such a calculation depends on the choice of the background, the origin of the BI field itself, as well as the compactification scheme and particular string theory under consideration. In Ref. [18], the one-loop effective potential (and static potential) for a toroidal D-brane described by the BI-action in constant electric and magnetic fields was evaluated. In the case of the one-loop potential, it was found that the presence of a magnetic background may stabilize the D-brane, whereas, in contrast, a constant electrical field leads to destabilization. The main conclusion to be drawn from such a study is that the consideration of quantum effects in BI theory is extremely involved even for the case of a stationary background. Since the explicit calculation of the time-dependence of (5) is beyond the scope of the present work, we adopt a more phenomenological approach. Our aim is to discuss the possible role of the quantum condensate in cosmological settings and, in particular, to determine the conditions that $\alpha$ and $\beta$ would need to satisfy in order for the BI field to act as a viable candidate for dark energy.

The classical thermodynamics of the radiation must also be accounted for. The classical contribution to the energy density and pressure of the BI field is determined by averaging over the spatial volume, as in Ref. [20]. By identifying the electric and magnetic components such that $E_i \equiv F_{i0}$ and $B_i \equiv \frac{1}{2}\epsilon_{ijk}F^{jk}$, respectively, we may specify $\left\langle\sum_i E_i^2\right\rangle = \left\langle\sum_i B_i^2\right\rangle \equiv \epsilon(t)$ by invoking the equipartition principle [20]. Since $\langle F^0{}_\rho F^{0\rho}\rangle = \left\langle\sum_i E_i^2\right\rangle$ and $\langle F_{i\rho}F_j{}^\rho\rangle = -\langle E_iE_j\rangle + 2\langle B_iB_j\rangle$, it is also consistent to further assume that $\langle E_iE_j\rangle = \langle B_iB_j\rangle = \epsilon g_{ij}/3$, and in this case, it follows that

$$\alpha_t = \alpha - 4\epsilon, \qquad \alpha_s = \alpha + \frac{4}{3}\epsilon. \tag{7}$$

The parameters $\alpha_t$ and $\alpha_s$ are therefore to be viewed as expressing the time-dependence of the averaged components of the field strength, $F^0{}_\rho F^{0\rho}$ and $F^i{}_\rho F^{j\rho}$, respectively, where the contributions from the classical radiation bath and the quantum condensate have both been taken into account. We may regard the contribution from $\alpha$ as arising purely from quantum mechanical effects. Since the electric field is orthogonal to the magnetic field in the radiation, $\sum_i E_iB_i = 0$, it also follows that $\langle F_{\mu\nu}F^{*\mu\nu}\rangle \propto \left\langle\sum_i E_iB_i\right\rangle = 0$ at the classical level. This corresponds to specifying $\beta = 0$.

We now proceed to determine the effective equation of state for the BI field. When the connected parts of the operators in the vacuum expectation values are neglected, i.e., when

$$\left\langle\left(F_{\mu\nu}F^{\mu\nu}\right)^n\left(F_{\mu\nu}F^{*\mu\nu}\right)^m\right\rangle_V = \alpha^n\beta^m, \quad \text{etc.}, \tag{8}$$

the expressions for the energy density, $\rho_{\mathrm{BI}}$, and pressure, $p_{\mathrm{BI}}$, of the BI field follow directly from Eq. (3). We find that

$$\rho_{\mathrm{BI}} = \frac{\lambda}{2}\left(\frac{1+\frac{\alpha}{2}-\frac{\alpha_t}{4}}{\sqrt{1+\frac{\alpha}{2}-\frac{\beta^2}{16}}} - 1\right), \tag{9}$$

$$p_{\mathrm{BI}} = -\frac{\lambda}{2}\left(\frac{1+\frac{\alpha}{2}-\frac{\alpha_s}{4}}{\sqrt{1+\frac{\alpha}{2}-\frac{\beta^2}{16}}} - 1\right). \tag{10}$$

Since the BI field is minimally coupled to Einstein gravity, its dynamics is determined by the conservation of its energy–momentum, $\nabla_\mu T^{\mu\nu}_{\mathrm{BI}} = 0$. For the FRW metric (4), this reduces to the ordinary differential equation:

$$\dot\rho_{\mathrm{BI}} + 3H(\rho_{\mathrm{BI}} + p_{\mathrm{BI}}) = 0, \tag{11}$$

where $H \equiv \dot a/a$ represents the Hubble expansion parameter and a dot denotes differentiation with respect to cosmic time. In the purely classical limit, $\alpha = \beta = 0$, we find that the equation of state corresponds to that of a relativistic fluid, $p_{\mathrm{BI}} = \rho_{\mathrm{BI}}/3$, as expected, and this implies that the energy density redshifts with the expansion of the universe in the standard fashion, $\rho_{\mathrm{BI}} \propto \epsilon \propto a^{-4}$.

We now consider the effects of the quantum condensate. In general, a matter degree of freedom may be viewed as phantom matter if it has positive energy density and negative pressure and if its equation of state satisfies $w \equiv p/\rho < -1$, i.e., if $p + \rho > 0$ [8]. This implies that its energy density increases with time. By comparing Eqs. (9) and (10), we deduce immediately that

$$\rho_{\mathrm{BI}} + p_{\mathrm{BI}} = \frac{\lambda}{8} \frac{\alpha_s - \alpha_t}{\sqrt{1 + \frac{\alpha}{2} - \frac{\beta^2}{16}}} \tag{12}$$

and it follows immediately that a necessary condition for the quantum BI condensate to act as phantom matter is that $\alpha_t > \alpha_s$. However, since $\epsilon$ is a semi-positive-definite quantity, we conclude from Eq. (7) that $\alpha_t < \alpha_s$ is always satisfied. Thus, the BI field cannot act as phantom matter in the context discussed here. This is a general result and is independent of the explicit time-dependence of the expectation value of the field strength.

On the other hand, in the limit where the quantum condensate dominates the classical contributions, $\alpha \gg \epsilon$, it follows from Eq. (7) that $\alpha \approx \alpha_s \approx \alpha_t$, and comparison of Eqs. (9) and (10) then implies that

$$\rho_{\mathrm{BI}} = -p_{\mathrm{BI}} = \frac{\lambda}{2} \left( \frac{1 + \frac{\alpha}{4}}{\sqrt{1 + \frac{\alpha}{2} - \frac{\beta^2}{16}}} - 1 \right), \tag{13}$$

$$w_{\mathrm{BI}} \equiv \frac{p_{\mathrm{BI}}}{\rho_{\mathrm{BI}}} = -1. \tag{14}$$

In other words, in the limit where the classical contribution is negligible, the BI field *behaves precisely as an effective cosmological constant*. This is remarkable, given that the initial time dependence of the quantum contributions, $\alpha$ and $\beta$, has not been specified in the analysis. For consistency with Eq. (11), we require that the ratio

$$\frac{1 + \frac{\alpha}{4}}{\sqrt{1 + \frac{\alpha}{2} - \frac{\beta^2}{16}}} \tag{15}$$

be time-independent and this imposes a restriction on the functional form of the parameters $\{\alpha_s, \alpha_t, \beta\}$, but does not necessarily imply that they should be time-independent themselves. We further require that

$$\frac{\alpha}{2} > -1 + \frac{\beta^2}{16} \tag{16}$$

and, in the limit where $8\alpha \to -16 + \beta^2$ or $\alpha \to +\infty$, both the energy density and pressure diverge. It also follows from the identity

$$\left(1 + \frac{\alpha}{4}\right)^2 - \left(1 + \frac{\alpha}{2} - \frac{\beta^2}{16}\right) = \frac{\alpha^2 + \beta^2}{16} \geqslant 0, \tag{17}$$

that the energy density is positive (negative) for positive (negative) coupling parameter, $\lambda$.

The question that now arises, therefore, is whether further constraints can be imposed on the time-dependent parameters from cosmological considerations. If the BI condensate is to act as a viable dark energy candidate, it can only be starting to dominate the energy density of the universe at the present epoch. This implies that the parameters $\{\alpha, \beta\}$ should be evolving in such a way that they are much less then unity today, otherwise the coupling parameter, $\lambda$, would have to be severely fine-tuned. Consequently, it is natural to Taylor expand Eqs. (9) and (10) to first-order in these parameters and we deduce that the effective equation of state in this limit is given by

$$w_{\mathrm{BI}} \approx -1 + \frac{128\epsilon}{96\epsilon + 3\beta^2}. \tag{18}$$

Eq. (18) illustrates how the classical contribution to the field strength, $\epsilon$, pushes the equation of state away from that of a cosmological constant. If the quantum condensate contribution to the energy density redshifts more slowly than the classical sector as the universe expands, the behavior of the BI field will gradually approach that of a cosmological constant as time proceeds. Consistency with observations requires that the second term on the right-hand side of Eq. (18) should not exceed 0.26 by the present epoch [3,7]. It is interesting that the equation of state (18) is independent of $\alpha$ at this level of approximation.

To summarize, therefore, we have considered the possible cosmological implications of including ultraviolet quantum effects on Born–Infeld matter degrees of freedom that generate a condensate similar to that of the gluon condensate in QCD. In a FRW background, such effects introduce time-dependent corrections to the energy and pressure of the BI field, thereby altering its equation of state away from that of a classical radiation fluid. Since, in general, the determination of the time-dependence of such corrections is highly involved, we have invoked a phenomenological approach by investigating the conditions that should be satisfied by the quantum condensate if it is to act as a viable candidate for dark energy or phantom matter. It was found that such a field cannot act as phantom matter unless it has negative energy density. Such a no-go result follows even though the precise time-dependence of the quantum corrections is unknown.

On the other hand, we note that although negative energy densities may appear to be unphysical, spatial distributions of negative energy sources have recently been investigated in detail in Ref. [21], where it was shown by specific example that there exist distributions that are indeed allowed.[1] Similarly, the consideration of the Casimir energy in various models (see, for instance, Ref. [22]) shows that quantum corrections to the energy density may be negative.

In the classical limit, the BI field behaves as a relativistic perfect fluid. Surprisingly, however, in the opposite limit where the classical contribution is negligible relative to the quantum condensate, the field acts as an effective cosmological constant. There exists an intermediate regime of parameter space where the field may act as a dark energy source under the very weak condition that the classical sector redshifts with the expansion of the universe more rapidly than that of the quantum condensate.

This is potentially interesting because it suggests a mechanism for addressing the fine-tuning problem associated with dark energy cosmology. This is the problem of understanding why the dark energy has such a low density relative to the Planck scale. A possible scenario that could be considered is the case where the classical sector dominates the energy density of the BI field shortly after the condensate forms in the very early universe and remains the dominant contribution until a relatively recent epoch is reached that corresponds to a redshift of $z \approx \mathcal{O}(\text{few})$. In this case, the BI field may act as a relativistic degree of freedom for most of the history of the universe without violating the observational limits imposed by primordial nucleosynthesis and CMB constraints. If we further assume that equipartition holds in the early universe, the energy density of the BI field would remain comparable to that of the background CMB during this time. However, if the time dependence of the quantum parameters then changed so that these quantities redshifted more slowly than the classical contribution, the BI field would transform into a cosmological constant, but with a sufficiently low energy density at the present epoch. It would soon come to dominate the universe as its energy density remained approximately constant.

In order to proceed further, a number of unresolved questions should be addressed. In particular, the action (1) contains a string coupling constant within the field strength $F_{\mu\nu}$. It is important, therefore, to consider the energy scales associated with the condensates that we have considered. The coupling constant of the BI theory is predicted by string theory to be much higher than the observable, present-day dark energy. However, the BI theory arises from higher-dimensional string theory as an effective D-brane theory with an associated brane tension, and since we have considered the case where the quantum contributions $\alpha$ and $\beta$ are of the order unity or less, the only scale that arises is that of the brane tension. In the standard D-brane setting, it is expected that the bulk cosmological constant and brane tension should both be of the order of the Planck scale. On the other hand, in warped compactification models, such as the Randall–Sundrum scenarios [23], the (higher-dimensional) Planck scale might be considerably lower. Moreover, it has been argued that the confinement of QCD might be dual to the Higgs mechanism [24]. In this case, small values for the parameters $\alpha$ and $\beta$ would arise naturally since they would be determined by the ratio of the confinement or Higgs scale with respect to the four-dimensional Planck scale. Alternatively, it would be interesting to consider mechanisms such as renormalization group screening to reduce the magnitude of the induced cosmological constant. Of course, this very important question of physical energy scales can only be addressed concretely after a direct calculation. We propose that the scenario we have outlined above provides strong motivation for considering the field theoretic issues that are involved in determining the explicit time-dependence of the condensate vacuum expectation values in an expanding FRW universe.

Finally, we remark that the quantum effects associated with CFT matter may also be accounted for by including the contributions due to the conformal anomaly. These take the general form

$$T = b\left(F + \frac{2}{3}\Box R\right) + b'G + b''\Box R, \tag{19}$$

where $F$ is the square of four-dimensional Weyl tensor and $G$ is the Gauss–Bonnet invariant. If there are $N$

[1] In our case, we note that in the somewhat unlikely event that the classical contribution is absent, the ansatz $\alpha_t = 3\alpha$ and $\alpha_s = \alpha/3$ could be made and this would yield phantom-like behavior for the BI condensate. However, a definitive conclusion can only be drawn after a complete calculation has been performed.

scalar, $N_{1/2}$ spinor, $N_1$ vector, $N_{\mathrm{HD}}$ higher derivative conformal scalars, and $N_2$ ($=0$ or 1) graviton fields present, then $b$, $b'$ and $b''$, are given by

$$b = \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{\mathrm{HD}}}{120(4\pi)^2},$$
$$b' = -\frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{\mathrm{HD}}}{360(4\pi)^2},$$
$$b'' = 0, \tag{20}$$

respectively. The contributions to the energy density and pressure due to the conformal anomaly have been found explicitly when the metric has the FRW form (4) [19]. Specifically, for the case of pure de Sitter space, it was found that

$$\rho_A = -p_A = -6b'H^4. \tag{21}$$

It is remarkable, that for higher derivative conformal scalar the quantum CFT energy density becomes negative as well. It then follows that the (nearly de Sitter) Friedman equation is given by

$$H^2 = \frac{8\pi}{3m_{\mathrm{P}}^2}\left\{\frac{\lambda}{6}\left(\frac{1+\frac{\alpha}{4}}{\sqrt{1+\frac{\alpha}{2}-\beta^2}} - 1\right) - 6b'H^4\right\}, \tag{22}$$

where $m_{\mathrm{P}}$ is the Planck mass and we have neglected any classical effects. (For the exact expression of the CFT energy density in an arbitrary FRW spacetime, see [19].)

Hence, we deduce that BI quantum effects combined with the vacuum polarization which arises due to CFT matter contributions also serve as an effective cosmological constant with a negative equation of state and may in principle serve as a mechanism for realizing an accelerated phase of cosmic expansion.

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