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Extension of maps to nilpotent spaces. II

M. Cencelj [a,1], A.N. Dranishnikov [b,*]

[a] IMFM, University of Ljubljana, P. O. B. 2964, SI-1001 Ljubljana, Slovenia
[b] Department of Mathematics, University of Florida, P.O. Box 118105, 358 Little Hall, Gainesville, FL 32611-8105, USA

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Abstract

Let M be a nilpotent CW-complex with finitely generated fundamental group. We give necessary and sufficient cohomological dimension theory conditions for a finite-dimensional metric compactum X so that every map $A \to M$, where A is a closed subset of X can be extended to a map $X \to M$.

This is a generalization of a result by Dranishnikov [Mat. Sb. 182 (1991)] where such conditions were found for simply-connected CW-complexes M, and Cencelj and Dranishnikov forthcoming paper [Cannad. Bull. Math.] where such conditions were found for nilpotent CW-complexes M with finitely generated homotopy groups.

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We generalize the main theorem of [4] and Theorem 7 of [2] to obtain the following theorem. We use the Kuratowski notation $X \tau M$ for the case every map from a closed subset of X to M can be extended over all of X. We recall that the cohomological dimension of a space X can be defined in these notations as follows: $\dim_G X \leqslant n$ iff and only if $X \tau K(G,n)$, where $K(G,n)$ is an Eilenberg–MacLane complex.

Theorem 1. *For any nilpotent CW-complex M with finitely generated fundamental group and finite-dimensional metric compactum X, the following are equivalent*:

(1) $X \tau M$;
(2) $X \tau SP^\infty M$;

[*] Corresponding author. Supported in part by NSF grant DMS-997109.
E-mail addresses: matija.cencelj@uni-lj.si (M. Cencelj), dranish@math.ufl.edu (A.N. Dranishnikov).
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(3) $\dim_{H_i(M)} X \leqslant i$ *for every* $i > 0$;
(4) $\dim_{\pi_i(M)} X \leqslant i$ *for every* $i > 0$.

We recall that a group G is called *nilpotent* if its lower central series $G = \Gamma^1 G \supset \Gamma^2 G \supset \cdots \supset \Gamma^k G \supset \Gamma^{k+1} G = 1$ has a finite length k called the nilpotency class of G. Here $\Gamma^2 G = [G, G]$ and $\Gamma^i G = [G, \Gamma^{i-1} G]$. The main examples of nilpotent groups are upper triangular matrix groups. The action of an upper triangular matrix group on a corresponding vector space suggest a definition of a *nilpotent action*. An action $\alpha : G \to \operatorname{Aut} H$ is called nilpotent if there is a G-invariant normal stratification $H = H_1 \supset \cdots \supset H_i \supset \cdots \supset H_n = *$ such that H_i / H_{i+1} is abelian and the induced action on H_i / H_{i+1} is trivial for all i. A topological space is called *nilpotent* if $\pi_1(X)$ is a nilpotent group and the action of $\pi_1(X)$ on the higher dimensional homotopy groups is nilpotent.

As opposed to [2, Theorem 7] we do not assume M to have all the homotopy groups finitely generated. The requirement that M has finitely generated fundamental group, however, seems to be necessary in view of [8, Example 5.2]. It can be dropped if the fundamental group is abelian.

In order to prove this theorem we first prove two propositions and a lemma.

Proposition 2. *The following conditions for an abelian group G are equivalent*:
(1) *G is p-divisible*;
(2) $\operatorname{Ext}(\mathbb{Z}_{p^\infty}, G) = 0$;
(3) $\operatorname{Ext}(\mathbb{Z}_{p^\infty}, G)$ *is p-divisible.*

Proof. This is a direct consequence of the short exact sequence

$$0 \to \lim{}^1 \operatorname{Hom}(\mathbb{Z}_{p^n}, G) \to \operatorname{Ext}(\mathbb{Z}_{p^\infty}, G) \to \widehat{G}_p \to 0,$$

where $\widehat{G}_p = \lim G/p^n G$ is the p-adic completion. □

Proposition 3. *Let G be a nilpotent group such that the abelianization $\operatorname{Ab} G$ is p-divisible. Then* $\operatorname{Ext}(\mathbb{Z}_{p^\infty}, G) = 0$.

Proof. We apply induction on the nilpotency class k of G. If $k = 1$, G is an abelian group and the result follows from Proposition 2. Now assume that G is of class k, i.e., $(k+1)$st group $\Gamma^{k+1} G$ of the lower central series is trivial. Then the group $G/\Gamma^k G$ is of class $k-1$. Every short exact sequence of nilpotent groups defines the six term exact sequence for Hom and Ext (see [1]). Then the exact sequence

$$\operatorname{Ext}\bigl(\mathbb{Z}_{p^\infty}, \Gamma^k G\bigr) \to \operatorname{Ext}(\mathbb{Z}_{p^\infty}, G) \to \operatorname{Ext}\bigl(\mathbb{Z}_{p^\infty}, G/\Gamma^k G\bigr)$$

and the induction hypothesis imply that $\operatorname{Ext}(\mathbb{Z}_{p^\infty}, G) = 0$. □

We recall that for every abelian group G there exists a Bockstein family $\sigma(G)$ of abelian groups [4] such that for every metric compactum X we have

$$\dim_G X = \max_{H \in \sigma(G)} \dim_H X.$$

The family $\sigma(G)$ is a subfamily of the family $\sigma = \mathbb{Q} \cup (\bigcup_p \sigma_p)$, where $\sigma_p = \{\mathbb{Z}_p, \mathbb{Z}_{p^\infty}, \mathbb{Z}_{(p)}\}$. Here $\mathbb{Z}_{p^\infty}$ is the direct limit of the groups $\mathbb{Z}_{p^k}$ and $\mathbb{Z}_{(p)} = \{m/n;\, n$ not divisible by $p\}$ is the p-localization of the integers. The family $\sigma(G)$ is defined by the following rule: $\mathbb{Z}_{(p)} \in \sigma(G)$ if and only if $F(G)$ is not p-divisible; $\mathbb{Z}_p \in \sigma(G)$ if and only if the group G_p is not p-divisible; $\mathbb{Z}_{p^\infty} \in \sigma(G)$ if and only if $G_p \neq 0$ and G_p is p-divisible; and $\mathbb{Q} \in \sigma(G)$ if $F(G) \neq 0$. Here G_p is the p-torsion subgroup of G and $F(G) = G/Tor(G)$.

The proof of the following lemma is based on the properties of p-completion and Bockstein's inequalities. First, we recall the Bockstein inequalities:

(BI1) $\dim_{\mathbb{Z}_{p^\infty}} X \leqslant \dim_{\mathbb{Z}_p} X$,
(BI2) $\dim_{\mathbb{Z}_p} X \leqslant \dim_{\mathbb{Z}_{p^\infty}} X + 1$,
(BI3) $\dim_{\mathbb{Z}_p} X \leqslant \dim_{\mathbb{Z}_{(p)}} X$,
(BI4) $\dim_{\mathbb{Q}} X \leqslant \dim_{\mathbb{Z}_{(p)}} X$,
(BI5) $\dim_{\mathbb{Z}_{(p)}} X \leqslant \max\{\dim_{\mathbb{Q}} X, \dim_{\mathbb{Z}_{p^\infty}} X + 1\}$,
(BI6) $\dim_{\mathbb{Z}_{p^\infty}} X \leqslant \max\{\dim_{\mathbb{Q}} X, \dim_{\mathbb{Z}_{(p)}} X - 1\}$.

The p-completion of a complex M is a $\mathbb{Z}_p$-complete space $\widehat{M}_p$ together with a map $M \to \widehat{M}_p$ which induces an isomorphism of homologies $H_*(M; \mathbb{Z}_p) \to H_*(\widehat{M}_p; \mathbb{Z}_p)$.

Lemma 4. *Let M be a connected nilpotent CW-complex with finitely generated fundamental group and let X be a finite-dimensional metric compactum. If*

$$\dim_{H_i(M)} X \leqslant i \quad \textit{for every } i \geqslant 1,$$

then

$$\dim_{\pi_i(M)} X \leqslant i \quad \textit{for every } i \geqslant 1.$$

Proof. Let $\pi_n = \pi_n(M)$ and $H_n = H_n(M)$. We prove $\dim_{\pi_n} X \leqslant n$ by induction on n. As it was shown in [2], Theorem 1, $\dim_G X = \dim_{Ab\,G} X$ for a finitely generated nilpotent group G, where AbG is the abelianization of G. Since $H_1(M) = Ab\pi_1(M)$, the claim holds for $n = 1$.

Let $\dim_{\pi_i(M)} X \leqslant i$ hold for all $i < n$. For the group π_n ($n \geqslant 2$) there is a short exact sequence

$$0 \to \left(\bigoplus_{p \text{ prime}} G_p^n\right) \to \pi_n \to F(\pi_n) \to 0,$$

where G_p^n is the Sylow p-subgroup of π_n and $F(\pi_n)$ is torsion-free. Therefore it suffices to show $\dim_{F(\pi_n)} X \leqslant n$ and $\dim_{G_p^n} X \leqslant n$.

Let us first show that $F(\pi_n) \neq 0$ implies $\dim_{\mathbb{Q}} X \leqslant n$. If π_i, $i < n$, are torsion groups, the Hurewicz theorem modulo the generalized Serre class of torsion groups implies $F(H_n) \neq 0$ and hence $\dim_{\mathbb{Q}} X \leqslant n$. If, however, at least one of the groups π_i is not a torsion group, then by the same Hurewicz theorem we obtain $F(H_j) \neq 0$ for some $j < n$. Therefore, $\mathbb{Q} \in \sigma(F(H_j))$ and $\dim_{\mathbb{Q}} X \leqslant \dim_{H_j} X \leqslant j < n$.

Let p be a prime number. We consider the case when $F(\pi_n)$ is not p-divisible. In that case $\mathbb{Z}_{(p)} \in \sigma(F(\pi_n))$. We show that $\dim_{\mathbb{Z}_{(p)}} X \leqslant n$.

The Bockstein inequalities imply the following alternative [6]:
either $\dim_{\mathbb{Z}_{(p)}} X = \dim_{\mathbb{Q}} X$ or $\dim_{\mathbb{Z}_{(p)}} X = \dim_{\mathbb{Z}_{p^\infty}} X + 1$.

We may assume that all groups H_i, $1 \leqslant i < n$, are p-divisible without p-torsions. Otherwise, $\mathbb{Z}_p \in \sigma(H_i)$ or $\mathbb{Z}_{p^\infty} \in \sigma(H_i)$ and we have $\dim_{\mathbb{Z}_p} X \leqslant \dim_{H_i} X \leqslant i < n$ or $\dim_{\mathbb{Z}_{p^\infty}} \leqslant \dim_{H_i} X \leqslant i < n$. In view of the inequality (BI2), in both cases we have $\dim_{\mathbb{Z}_{p^\infty}} +1 \leqslant n$. Then the inequality $\dim_{\mathbb{Q}} X \leqslant n$ and the above alternative imply that $\dim_{\mathbb{Z}_{(p)}} X \leqslant n$.

Because of induction assumption, similarly we may assume that all groups π_i, $1 < i < n$, are p-divisible and without p-torsions.

Since M is a nilpotent CW-complex its p-completion $\widehat{M}_p$ exists [1]. Our assumptions, the propositions and the exact sequence

$$0 \to \operatorname{Ext}(\mathbb{Z}_{p^\infty}, \pi_i) \to \pi_i\big(\widehat{M}_p\big) \to \operatorname{Hom}(\mathbb{Z}_{p^\infty}, \pi_{i-1}) \to 0$$

imply $\pi_i(\widehat{M}_p) = 0$ for $i < n$. Here we used the fact that $\operatorname{Hom}(\mathbb{Z}_{p^\infty}, \pi_1) = 0$ which follows from the equality $G_p^1 = 0$. The latter follows from the absence of p-torsions in H_1 and a property of nilpotent groups (see [2, Proposition 2]).

From the Hurewicz theorem we obtain $\pi_n(\widehat{M}_p) = H_n(\widehat{M}_p)$. This group is $\pi_n(\widehat{M}_p) = \operatorname{Ext}(\mathbb{Z}_{p^\infty}, \pi_n)$ and its p-divisibility would imply that it is the trivial group. Since $F(\pi_n)$ is not p-divisible and the group $\widehat{F}(\pi_n) = \operatorname{Ext}(\mathbb{Z}_{p^\infty}, F(\pi_n))$ is without torsion, the exactness property of Ext and Hom [1, p. 169], implies that $\operatorname{Ext}(\mathbb{Z}_{p^\infty}, \pi_n) = \pi_n(\widehat{M}_p)$ is not a p-torsion group.

Therefore $H_n(\widehat{M}_p) \otimes \mathbb{Z}_{p^\infty} \neq 0$ and by the universal coefficient theorem $H_n(\widehat{M}_p; \mathbb{Z}_{p^\infty}) \neq 0$.

One of the main properties of the p-completion $M \mapsto \widehat{M}_p$ is that it induces an isomorphism of homology with coefficients in $\mathbb{Z}_p$. With exact sequences

$$0 \to \mathbb{Z}_{p^k} \to \mathbb{Z}_{p^{k+1}} \to \mathbb{Z}_p \to 0$$

and induction we can prove that the p-completion induces an isomorphism in homology with coefficients in $\mathbb{Z}_{p^n}$ for arbitrary n. Since the tensor product and homology commute with the direct limit the p-completion induces also an isomorphism in homology with coefficients in $\mathbb{Z}_{p^\infty}$.

Therefore $H_n(M; \mathbb{Z}_{p^\infty}) \neq 0$. Since H_{n-1} has no p-torsion this implies $H_n \otimes \mathbb{Z}_{p^\infty} \neq 0$. Thus and $\dim_{\mathbb{Z}_{(p)}} X \leqslant n$.

Thus, we proved the inequality $\dim_{\mathbb{Z}_{(p)}} X \leqslant n$ for all p for which $F(\pi_n)$ is p-divisible. Since the Bockstein family $\sigma(F(\pi_n))$ consists of all such p's, we proved the inequality $\dim_{F(\pi_n)} X \leqslant n$.

To perform the induction step we still have to prove the inequalities $\dim_{G_p^n} X \leqslant n$ for all p. When $F(\pi_n)$ is not p-divisible we have shown $\dim_{G_p^n} X \leqslant \dim_{\mathbb{Z}_p} X \leqslant \dim_{\mathbb{Z}_{(p)}} X \leqslant n$.

Assume now $F(\pi_n)$ is p-divisible. We consider two cases:

(1) G_p^n is not p-divisible. In this case $\sigma(G_p^n) = \{\mathbb{Z}_p\}$ and we have to show the inequality $\dim_{\mathbb{Z}_p} X \leqslant n$. Like above we can assume that all groups π_i, H_i, $1 < i \leqslant n-1$, have no

p-torsion and are p-divisible and that π_1 and H_1 are torsion groups, but without p-torsion. From the exact sequence

$$0 \to \mathrm{Ext}(\mathbb{Z}_{p^\infty}, \pi_i) \to \pi_i\big(\widehat{M}_p\big) \to \mathrm{Hom}(\mathbb{Z}_{p^\infty}, \pi_{i-1}) \to 0$$

and Propositions 2 and 3 we obtain $\pi_i(\widehat{M}_p) = 0$ for $0 < i < n$. Since G_p^n is not p-divisible Proposition 2 and the exactness property imply that the group

$$\pi_n\big(\widehat{M}_p\big) = \mathrm{Ext}(\mathbb{Z}_{p^\infty}, \pi_n) = \mathrm{Ext}\big(\mathbb{Z}_{p^\infty}, G_p^n\big)$$

is not trivial and is not p-divisible.

Thus the Hurewicz theorem implies $H_i(\widehat{M}_p) = 0$ for $0 < i < n$ and the group $H_n(\widehat{M}_p)$ is not p-divisible. Therefore $H_n(\widehat{M}_p) \otimes \mathbb{Z}_p \neq 0$ and $H_n(\widehat{M}_p; \mathbb{Z}_p) \neq 0$. From the main properties of the p-completion we obtain $H_n(M; \mathbb{Z}_p) \neq 0$ and since H_{n-1} is without p-torsion, $H_n \otimes \mathbb{Z}_p \neq 0$. Therefore $\mathbb{Z}_p \in \sigma(H_n)$ or $\mathbb{Z}_{(p)} \in \sigma(H_n)$. In both cases we have $\dim_{\mathbb{Z}_p} X \leqslant n$ and $\dim_{G_p^n} X \leqslant n$.

(2) $G_p^n \neq 0$ is p-divisible. Then the group π_n is p-divisible.
Since $\sigma(G_p^n) = \{\mathbb{Z}_{p^\infty}\}$, we have to show that $\dim_{\mathbb{Z}_{p^\infty}} X \leqslant n$. We obtain this directly if H_n has p-torsion elements, so assume H_n has no p-torsion. Again we can assume also that all the groups π_i, H_i, $1 \leqslant i \leqslant n-1$, are without p-torsion. Therefore the exact sequence

$$0 \to \mathrm{Ext}(\mathbb{Z}_{p^\infty}, \pi_i) \to \pi_i\big(\widehat{M}_p\big) \to \mathrm{Hom}(\mathbb{Z}_{p^\infty}, \pi_{i-1}) \to 0$$

implies $\pi_n(\widehat{M}_p) = 0$ and the group $\pi_{n+1}(\widehat{M}_p)$ maps epimorphically onto $\mathrm{Hom}(\mathbb{Z}_{p^\infty}, \pi_n)$. The latter group includes the p-adic integers $\widehat{\mathbb{Z}}_p = \lim_{\leftarrow} \mathbb{Z}_{p^n}$ since $\mathrm{Hom}(\mathbb{Z}_{p^\infty}, \mathbb{Z}_{p^\infty}) \cong \widehat{\mathbb{Z}}_p$. Therefore $\mathrm{Hom}(\mathbb{Z}_{p^\infty}, \pi_n)$ is not a p-torsion group and since $\mathbb{Z}_{p^\infty}$ is divisible, the group $\mathrm{Hom}(\mathbb{Z}_{p^\infty}, \pi_n)$ contains $\mathrm{Hom}(\mathbb{Z}_{p^\infty}, \mathbb{Z}_{p^\infty})$ which is not p-divisible, as a direct summand. Thus the group $\pi_{n+1}(\widehat{M}_p) = H_{n+1}(\widehat{M}_p)$ is neither a p-torsion group nor p-divisible. Therefore $H_{n+1}(\widehat{M}_p) \otimes \mathbb{Z}_{p^\infty} \neq 0$ and $H_{n+1}(\widehat{M}_p; \mathbb{Z}_{p^\infty}) \neq 0$. This implies $H_{n+1}(M; \mathbb{Z}_{p^\infty}) \neq 0$ and since by assumption H_n has no p-torsion elements the universal coefficient theorem gives $H_{n+1} \otimes \mathbb{Z}_{p^\infty} \neq 0$ which in turn implies $\dim_{\mathbb{Z}_{(p)}} X \leqslant n+1$.

If all the groups π_i, $1 \leqslant i \leqslant n-1$, are torsion groups, the Hurewicz theorem modulo the generalized Serre class of nilpotent torsion groups without p-torsion implies that H_n has p-torsion and thus $\dim_{\mathbb{Z}_{p^\infty}} X \leqslant n$. If, however, $F(\pi_i) \neq 0$ for some i, $1 \leqslant i \leqslant n-1$, we obtain $\dim_{\mathbb{Q}} X \leqslant i \leqslant n-1$. Bockstein's inequality (BI6) then implies $\dim_{\mathbb{Z}_{p^\infty}} X \leqslant n$. □

The nilpotency of M is essential in Lemma 4. If one takes a non nilpotent space $M = \mathbb{R}P^2$, the hypothesis of the lemma turns into the inequality $\dim_{\mathbb{Z}_2} X \leqslant 1$ but the conclusion turns into sequence of inequalities $\dim_{\mathbb{Z}_2} X \leqslant 1$, $\dim_{\mathbb{Z}} X \leqslant 2, \ldots, \dim_{\pi_n(S^2)} X \leqslant n, \ldots$. Since for all n there are n-dimensional compact spaces X with $\dim_{\mathbb{Z}_2} X \leqslant 1$ (see [6]), the hypothesis of the lemma does not imply the conclusion.

The requirement that $\pi_1(M)$ is finitely generated can be dropped when the fundamental group $\pi_1(M)$ is abelian. The general case is less clear because of the existence of a nilpotent group N with $Ab\,N = \mathbb{Q} \oplus \mathbb{Q}$ and $[N, N] = \mathbb{Z}_{p^\infty}$ (see [7, p. 28]). This group seems to give a counterexample to the implication $\dim_{Ab\,N} X \leqslant 1 \Rightarrow \dim_N X \leqslant 1$.

We recall that the Postnikov tower for a nilpotent space M [7] is an inverse system $E_1 \leftarrow E_2 \leftarrow \cdots \leftarrow E_n \leftarrow \cdots$ with bonding maps $p_{n+1} : E_{n+1} \to E_n$ whose fibers are $K(\pi_{n+1}(M), n+1)$ together with maps $\alpha_n : M \to E_n$ such that $p_{n+1}\alpha_{n+1} = \alpha_n$, α_n induces an isomorphism of homotopy groups $\pi_i(M) \to \pi_i(E_n)$ for $i \leqslant n$, $E_1 = K(\pi_1(M), 1)$ and every map $p_{n+1} : E_{n+1} \to E_n$ is the composite of principal fibrations

$$E_{n+1} = Y_c \xrightarrow{q_c} Y_{c_1} \to \cdots \to Y_1 \xrightarrow{q_1} Y_0 = E_n,$$

where the fibre of q_i is an Eilenberg–MacLane space $K(G_i, k)$ and q_i is induced by a map $\kappa_i : Y_{i-1} \to K(G_i, k+1)$, where

$$G_i = \Gamma^i \pi_k(M)/\Gamma^{i+1}\pi_k(M)$$

comes from the stratification of $\pi_k(M)$ under the nilpotent action of $\pi_1(M)$ on $\pi_k(M)$.

Recall that the Eilenberg–MacLane complex $K(G_i, k)$ is the free abelian topological group $G = FA(M(G_i, k))$ generated by the Moore space $M(G_i, k)$ [3]. Hence we may assume that $K(G_i, k+1) = BG$ is the Milnor classifying space for the topological group G and the map q_i is induced by the universal locally trivial G-bundle $\nu : EG \to BG$. Therefore we may assume that all spaces E_n in the Postnikov tower are CW complexes and all maps p_{n+1} are projections of fibre bundles.

The following is proved in [5, Assertion 7].

Proposition 5. *Let $f : E \to B$ be a locally trivial fibration with a fiber F, and assume that F, B are CW complexes. Suppose that $X\tau B$ and $X\tau F$ for some compactum X. Then $X\tau E$.*

Proof of theorem. The implications

$$X\tau M \Rightarrow X\tau SP^i M \Rightarrow \dim_{H_i(M)} X \leqslant i \text{ for every } i \geqslant 1$$

are proved in [4] without any assumption on $\pi_1(M)$. Our lemma proves that $\dim_{H_i(M)} X \leqslant i$, $\forall i \geqslant 1 \Rightarrow \dim_{\pi_i(M)} X \leqslant i$, $\forall i \geqslant 1$.

Let $K(\pi_1, 1) \leftarrow E_2 \leftarrow \cdots \leftarrow E_n \leftarrow \cdots$ be the Postnikov tower of M. By definition $\dim_{\pi_i} X \leqslant i$ means $X\tau K(\pi_i, i)$. Every map in the Postnikov tower of M is a composition of principal fibre bundle projections therefore by induction, Proposition 5 and conditions $X\tau K(\pi_i, i)$ it follows $X\tau E_n$ for every n. Let $\dim X = m$ and let N_m be the CW-complex homotopy equivalent to E_m which is obtained from M by attaching cells of dimension $\geqslant m+2$. Then $X\tau E_m$ implies $X\tau N_m$ by the Homotopy Extension Theorem. Since every map $f : X \to B^k$ of an m-dimensional space to the k-dimensional ball, $k > m$, can be pushed to the boundary, i.e., there is a map $g : X \to \partial B^k$ with $g|_{f^{-1}(\partial B^k)} = f|_{f^{-1}(\partial B^k)}$, the property $X\tau N_m$ implies the property $X\tau M$ for a compact m-dimensional space X. □

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