



Predictions of striking energy and angular dependence in $pp \rightarrow (pp)_{S\text{-wave}}\pi^0$ production

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Abstract

A phenomenological calculation from threshold to 800 MeV of the initial proton beam energy is presented to describe recent data on the reaction $pp \rightarrow (pp)_{S\text{-wave}}\pi^0$ with a low energy cut on the final state diproton excitation energy. A strong forward dip is obtained in the differential cross section as in the data from COSY at 800 MeV, although the absolute value of the forward cross section is too low. Earlier low energy data from Celsius are reasonably well reproduced. In the unexplored energy interval between these two experiments the model predicts a spectacular energy dependence both in the forward direction and in the angle-integrated cross section, which may be related to a delicate interplay of the $\Delta(1232)$ and NN background in different interfering partial wave amplitudes.

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Measurements of pion production cross sections in two-nucleon collisions have in a broad sense existed for a long time (for a historical reference see Ref. [1], a modern review close to threshold Ref. [2]). However, only quite recently have experiments on $NN \rightarrow NN\pi$ reactions with a cut on the final NN excitation energy opened a new chapter in comparison with theory. Restriction to only one NN partial wave (S wave) simplifies the comparison tremendously to be basically similar to the simple $NN \rightarrow d\pi$. It is clear that in this kind of experiments good resolution of momenta is essential and cooled beams give an obvious advantage, although such experiments were initiated at TRIUMF with measurements of the differential cross section and analyzing power A_y in quasifree $pn \rightarrow (pp)_{S\text{-wave}}\pi^-$ [3–5]. In these experiments a cut of ≈ 1.5 MeV was applied on the final diproton energy (37.5 MeV/ c on the canonical c.m. momentum [6]). The data agreed reasonably well with the predictions of Refs. [7,8] for the inverse quasifree absorption of negative pions on the 1S_0pp pair in ^3He .

Later differential cross sections between 310 and 425 MeV for $pp \rightarrow (pp)\pi^0$ have been obtained at Celsius both integrated

over the final momentum magnitudes (and the nucleon relative angle) and also applying a diproton energy cut of 3 MeV (53 MeV/ c momentum) [9]. In the latter case the final diproton should be rather purely S wave. An interesting feature was that above 350 MeV the slope of the angular distribution applying the energy cut was opposite as compared with the case without the cut. Normally the cross sections tend to find a maximum in the forward direction. However, the $pp \rightarrow (pp)_{S\text{-wave}}\pi^0$ cross section decreases for the decreasing reaction angle. This is in agreement with the predictions given already in Ref. [7] for pion absorption on a pp pair in the corresponding isospin situation. A similar behaviour is also seen in a very recent measurement by the ANKE Collaboration at COSY of this reaction at 800 MeV very near the forward direction [10].

The basically phenomenological model has been presented in the past in some detail for mechanisms in Refs. [7,8] (albeit for pion absorption on a bound diproton) and for the treatment of the long range free nucleon wave functions and the Coulomb interaction in Ref. [11]. The mechanisms in the production operator involve first the direct production from each nucleon with distorted initial and final pp states. This is Galilean invariant with the axial current part $\propto \mathbf{q} \cdot \boldsymbol{\sigma}$ and the corresponding recoil term (axial charge) $\omega_{\mathbf{q}}(\mathbf{p} + \mathbf{p}') \cdot \boldsymbol{\sigma}/2M$. In pion reactions the all

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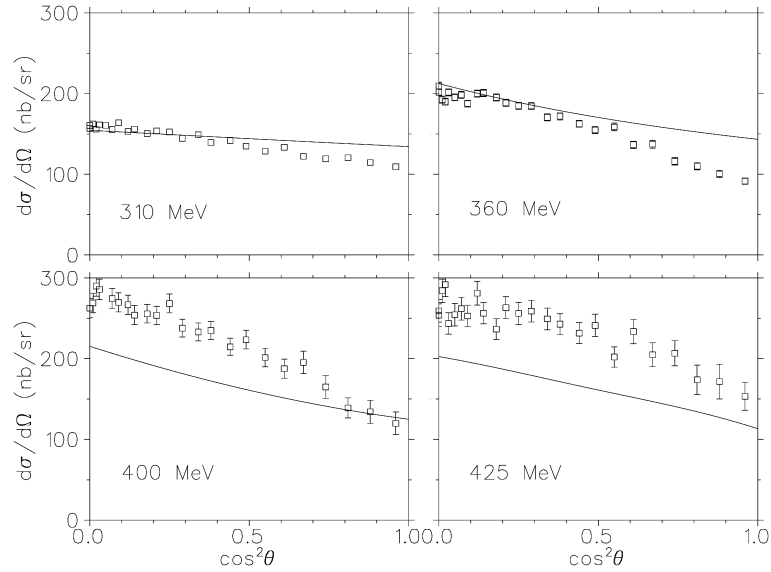


Fig. 1. The model vs. the Celsius data [9] at various energies with a cut of 3 MeV in the relative final pp energy to constrain them to the S wave.

important $\Delta(1232)$ resonance is treated as excitation of a ΔN intermediate state by coupled channels with a transition potential including pion and ρ meson exchange. This covers pion rescattering in the $\pi N p_{3/2}$ partial wave. Following Ref. [8] pion s -wave rescattering from the second nucleon is parameterized by fitting free πN scattering. For the corresponding propagator I assume the intermediate pion to be on the energy shell, if it is emitted by the Δ and off shell (as in e.g. Ref. [12]), if emitted by a nucleon.¹ A monopole form factor with the cut-off mass 550 MeV is also included in the exchange. Further, the heavy meson exchange effect suggested in Refs. [14,15] to account for the missing $pp \rightarrow (pp)\pi^0$ threshold strength [16] is used.² The latter is implemented as in Ref. [8] fitting the data at a single energy 290 MeV and the agreement with data is very good up to $\eta = q_{\max}/m_\pi \approx 0.6$, i.e. in the range where the S -wave production is by far dominant [8].

The interactions thus fixed will be applied at higher energies with a constraint of the two final state protons being in the 1S_0 state. Having only one NN final state simplifies the theoretical treatment significantly to resemble the reaction $pp \rightarrow d\pi^+$, although the long-range behaviour of the free protons requires some extra care [11]. When the laboratory energy increases above, say, 350 MeV, the final state nucleons will not remain in the S wave, if the whole phase space is included, and the results of the present calculation involving only that should fall below any such experiment as seen e.g. in Ref. [8]. With a cut on the final diproton excitation energy the validity range is increased and only the limitations of the model itself will eventually make it fail. In experiments this cut is the way to single out S -wave final nucleons.

¹ On chiral perturbation arguments Ref. [13] suggests putting the pion on the energy shell also in this case.

² As another possibility to account for this at least partially off-shell pion rescattering has been proposed [17,18]. In the light of Ref. [13] this may be questioned.

It will be seen that due to cancellations and interferences the results are even excessively sensitive to the amplitudes. However, in this Letter the aim is not to try a detailed fine tuning by varying interactions and phases more than done above in fixing the model at threshold or to estimate possible P -wave contamination in the final NN system. Also the effect of the ΔN admixture in the final state noted in Ref. [19] is minimized by the energy cut and in particular does not give an explicitly energy dependent effect. Some possibly missing d -wave pion strength (as indicated by the somewhat weaker angular dependence than seen in the Celsius data in Fig. 1) could be obtained by including explicit d -wave rescattering. These adjustments could hardly affect the main results of this communication, namely spectacular variation of the forward cross section as a function of energy in the experimentally uncharted region between the Celsius and COSY data.

One specific but relevant detail concerning the role of the Δ should still be mentioned. The reactions $pp \rightarrow d\pi^+$ and $pp \rightarrow np\pi^+$ are dominated by the Δ causing a wide peak in the cross section at the nominal mass of the Δ excitation around 600 MeV (lab). However, in the present case with a final 1S_0 nucleon pair the initial state 1D_2 coupled to an S -wave ΔN is not possible, but the Δ is excited at least in a p wave. Because of the centrifugal energy the Δ excitation should be somewhat suppressed. Further, in Ref. [20] it was shown that the ΔN energies with different angular momenta appeared as a rotational series $M_N + M_\Delta + 40L_{\Delta N}(L_{\Delta N} + 1)$ MeV corresponding to an effective distance of about 1 fm and giving a good representation of the isospin one “dibaryons”. Therefore, in the present case the Δ should appear shifted to the laboratory energy of about 800 MeV, i.e. the energy of the recent COSY/ANKE experiment [10]. However, actually the calculation does not indicate a peaking at this energy but rather in the proximity of the nominal Δ mass. This means that the peaking is actually not exclusively due to Δ dynamics but the oscillatory NN wave functions play also a decisive role.

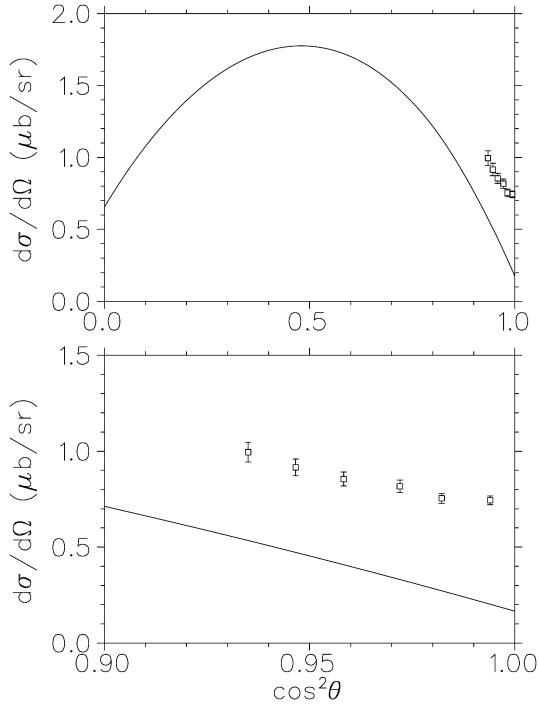


Fig. 2. The model results at 800 MeV. The upper panel shows the whole angular range, while the lower shows the forward range relevant to the COSY/ANKE experiment [10], from which the data are taken.

Fig. 1 shows the calculation compared with a representative selection (four of six energies with the best error limits) of the Celsius differential cross section data [9], which are constrained to the final $^1S_0 pp$ state by a cut off $E_{pp} \leq 3$ MeV on the diproton energy. The trends are very similar and also the agreement in the absolute magnitude is still quite reasonable in this energy range, considering that the highest maximum final pion momentum is even twice that of Ref. [16]. In particular, the cross section distinctly gets a minimum in the forward direction. This is in contrast to most other situations, e.g. the same cross sections without the cut [9].

In the recent data [10] from COSY measured at 800 MeV in the near-forward direction it was found an extraordinarily strong angular dependence with the cross section dropping down by 30% in the interval where $\cos\theta$ changed only from 0.97 to 1. This steep dip is rather unexpected even in the light of the previous Celsius results showing the minimum in the forward direction. As seen in Fig. 2 such a very steep descent is also obtained by the present model, although the absolute scale is too low. However, since the cross section drops by an order of magnitude, it is clear that there is extremely strong cancellation in the forward direction between different partial wave amplitudes (three important ones discussed below) and so a relatively minor change in a single partial wave may cause a large change in the cross section. Also another minimum is predicted at 90° and a maximum at about 50° .

With such a strong variation and interference it is also relevant to divide the cross section explicitly into partial waves to find the important ones. Due to parity and angular momentum conservation the 1S_0 final nucleon state is only possible for

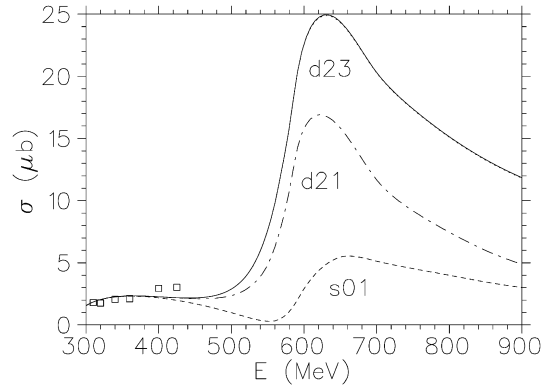


Fig. 3. The cumulative sum of the contributions to the overall cross section from different partial wave amplitudes: solid s_{01} , dashed d_{21} , dotted d_{23} . The solid curve denoting the total integrated cross section including also the g -wave pions is indistinguishable from the dotted one. The data are obtained by integrating the fits of Ref. [9].

even pion angular momentum l_π and initial nucleon states with $L = J \pm 1$. As can be seen in Fig. 3 all partial wave amplitudes s_{01} , d_{21} and d_{23} (in the notation $l_\pi J_L$) are about equally important at 800 MeV. In this figure the contribution of each partial wave to the angle integrated cross section is presented as a function of the incident laboratory energy. The g -wave pions contribute negligibly. The low-energy Celsius data [9] are reasonably well reproduced. Unfortunately there are no comparable data at other energies.

Further, a drastic energy dependence between 500 and 800 MeV is found. Since such a rapid variation is not found without the cut, this may be related to the constraint of small energy of the final state diproton. Namely in that case the phase space integral is very limited so that the angle integrated cross section is nearly just a sum of the squared reaction matrix elements taken with the single maximal pion momentum. With this kinematics the role of the d -wave pions is emphasized. Also the contribution of a single partial wave can reach even zero as seems to happen for the s_{01} pions. In an incoherent integral over a wide energy range this would be highly unlikely even for a single partial wave. Therefore, the phase space integral over the whole phase space would probably have a moderating effect in sharp changes and oscillations.

For orientation of the most imminent further experiments at COSY [21], Fig. 4 presents the energy dependence of the forward cross section and its slope. The theoretical slope is defined as the difference of the cross section at $\cos^2\theta = 0.9$ and $\cos^2\theta = 1$ divided by 0.1 (i.e. approximately minus the derivative with respect to $\cos^2\theta$). For the COSY data [10] the extremes of $\cos^2\theta$ were used and for the Celsius the fits published in Ref. [9]. If possible, the energy dependence is even more dramatic in this quantity just in the experimentally uncharted region and the role of the Δ is extremely prominent.

Here actual interference of all different partial waves is possible and apparently at about 550 and 700 MeV the forward amplitude changes its sign producing the small minima. Also, because the forward cross section may be an order of magnitude smaller than the bulk of the cross section as in Fig. 2, due to destructive interferences the absolute detailed prediction may not

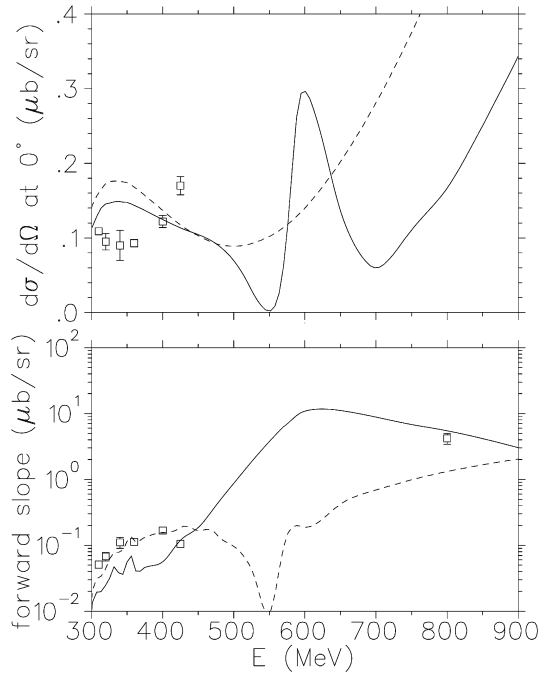


Fig. 4. Predictions for the forward cross section and its slope defined as in the text. The data are from the fits of Ref. [9] and from Ref. [10] (800 MeV). The forward cross section at 800 MeV would be outside the figure at 700 nb/sr. The dashed curve shows the result of a purely nucleonic calculation. For the slope it presents the absolute value (the slope is opposite to the full result above 550 MeV).

be exactly correct, but still violent energy and angular variations are expected. Certainly the expected behaviour of the cross section is sharper than that in the widely studied $pp \rightarrow d\pi^+$ both as a function of energy and angle. From the smooth curve of the purely nucleonic result (dashed curve) is clear that the basic origin of the peaking is the presence of the Δ . Although not shown, the strengths of the individual d -wave contributions in Fig. 3 are dominated by the Δ . Without it they would be monotonously increasing and about an order of magnitude smaller in the peak region.

In summary, a phenomenological model calculation is performed for the reaction $pp \rightarrow (pp)_{s\text{-wave}}\pi^0$. Partly the aim has been to provide some predictions in anticipation of experiments at COSY. However, the finding of extreme energy and angular dependencies may have also wider interest and applications in other similar reactions in attempts to extract information on reaction matrix elements. The constraint of a small relative momentum for the final state protons seems to favour this strong variation of the cross section in particular in the forward direction but also in the angle-integrated cross section. Obviously this cut also tends to stress higher pion waves than cross sections integrated over all possible momenta. Such a strict constraint may be a way to get hands on the squared reaction matrix elements at (nearly) a single momentum choice.

Already the experimental finding of opposite slopes of $d\sigma/d\Omega$ with and without the cut on the final pp excitation energy is suggestive of physical differences. Calculations of total production cross sections (integrated over all phase space) show that already at 400 MeV the amplitudes $^1D_2 \rightarrow ^3P_2s$,

$^3P_1 \rightarrow ^3P_0p$ and $^3F_3 \rightarrow ^3P_2p$ are each as large as $^3P_0 \rightarrow ^1S_0p$ with also sizable contributions from $^3P_2 \rightarrow ^3P_1p$, $^3P_2 \rightarrow ^3P_2p$ and $^3F_2 \rightarrow ^3P_2p$. The 1S_0s part of the cross section would then be only about one sixth of the total in line with the phenomenological fits of Ref. [9]. Apparently this complexity makes a detailed partial wave analysis improbable. However, with a cut, due to the additional simplicity of the spin structure of the final 1S_0 state, such an analysis is amenable with measurements of also spin observables. This, in turn, would act as a strong constraint on any modelling of the reaction over the whole phase space range.

A plausible scenario for this kind of interferences could be the following: As in Refs. [8,11] the s -wave production is governed by the axial charge terms at threshold, while higher in energy the axial current competes. With the energy cut this term (with a direct proportionality to q and getting a large portion of its strength from the ΔN admixture) can even take over as indicated by the deep minimum at 550 MeV in Fig. 3. If the d_{21} is destructive with the above low energy amplitude (and d_{23} constructive), their magnitudes at 550 MeV would be rather suitable for the total forward amplitude to become small, too. Then, just above this energy the s_{01} and d_{21} waves would become constructive building up a steep rise, while the destructive d_{23} is still smaller than the combination of these two. The maximum of the d_{23} contribution is at 700 MeV, 100 MeV higher than d_{21} , and there it takes over cancelling the two at this energy and producing after a steep drop the distinct minimum. Finally, well above the Δ the NN takes over with the trend shown by the dashed line in Fig. 4. Of course, at high energies other resonances could be important, which are not included in the present model.

The discrepancy with the COSY data in the forward direction may be due to overly delicate destructive interference, which a minor change in just one of the amplitudes might moderate. By some exploratory model variations it was not possible to significantly improve the situation. Making pion s -wave rescattering somewhat stronger actually decreased the forward cross section. Change of the size of the energy cut within the experimental precision has no significant effect. An intriguing possibility could be a need for explicit pion d -wave rescattering possibly involving the $N(1520)\frac{1}{2}^-$ resonance.

It would certainly be interesting to extend experiments both to larger angles to check whether the model gives the total normalization reasonably and also to the unexplored energies accessible at COSY, where extreme energy dependence shown in Figs. 3 and 4 is predicted. Further details with model dependence and spin observables will be studied in a forthcoming paper [22].

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