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Tadpole resummations in string theory

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Abstract

While R–R tadpoles should be canceled for consistency, string models with broken supersymmetry generally have uncanceled NS–NS tadpoles. Their presence signals that the background does not solve the field equations, so that these models are in "wrong" vacua. In this Letter we investigate, with reference to some prototype examples, whether the true values of physical quantities can be recovered resumming the NS–NS tadpoles, hence by an approach that is related to the analysis based on String Field Theory by open–closed duality. We show that, indeed, the positive classical vacuum energy of a Dp-brane of the bosonic string is *exactly* canceled by the negative contribution arising from tree-level tadpole resummation, in complete agreement with Sen's conjecture on open-string tachyon condensation and with the consequent analysis based on String Field Theory. We also show that the vanishing classical vacuum energy of the SO(8192) unoriented bosonic open-string theory does not receive any tree-level corrections from the tadpole resummation. This result is consistent with the fact that this (unstable) configuration is free from tadpoles of massless closed-string modes, although there is a tadpole of the closed string tachyon. The application of this method to superstring models with broken supersymmetry is also discussed.

1. Introduction

String models with tension in the TeV region [1] are an exciting possibility for physics beyond the Standard Model (for a review, see Refs. [2,3]). This scenario was made concrete by the modern understanding of open strings and orientifold compactifications [4], and by many important subsequent developments (for a review, see Ref. [5]). In this framework, the Higgs doublet responsible for the electroweak symmetry breaking is typically identified with some massless open-string mode on D3-branes, that can acquire a negative mass squared via radiative corrections in models without supersymmetry [6,7]. The scale of electroweak symmetry breaking is essentially determined by the string tension (and/or by the radii of the compact space), and this scenario can be a natural dynamical setting where string effects play a direct role for Particle Physics.

In string models with broken supersymmetry, however, there is in general a vexing difficulty, the so-called NS–NS tadpole problem. The existence of tadpoles in the NS–NS closed-string sector signals that the assumed background metric and field configuration (typically flat spacetime with vanishing background fields) *is not* a solution of String Theory. On the other hand, it is quite difficult to construct string models with more general background metrics or non-trivial fields, so that a prescription to cure this problem proposed in Refs. [8–10] is also not easy to implement. The actual difficulty manifests itself via the emergence of infrared divergences in loop calculations of open string amplitudes.

Tadpole resummation as a possible way to overcome this problem was proposed in Ref. [11], where several examples of field theories defined in "wrong" vacua were discussed. Indeed, barring convergence issues and other subtleties, the correct value of the vacuum energy can generically be recovered by the procedure of tadpole resummation. This procedure is nonetheless quite complicated, since it requires that one add up all tree-level diagrams involving tadpole contributions.

In this Letter we propose a concrete method to implement this procedure in String Theory, showing that tadpole resummations actually lead to the correct answer for open-string tachyon condensation. To this end, we combine the boundary state formalism with some information drawn from the low-energy dy-

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namics of branes. In Section 2 the boundary state formalism for D-branes and O-planes is thus briefly reviewed. In Section 3 we study the vacuum energies (tensions) of Dp-branes for the bosonic string[1] in flat 26-dimensional spacetime as a first simple example. These Dp-branes have generally tadpoles for dilaton, graviton and tachyon modes. The conjecture of open string tachyon condensation [12] claims that they should decay to the vacuum, so that the actual vacuum energy should vanish. This conjecture has received strong support from String Field Theory [13], although the resulting mechanism appears rather complicated and rests crucially on the contributions of open string massive modes. Here we show that, rather remarkably, the phenomenon can be understood in somewhat simpler terms: in the dual closed channel a *negative* contribution originating from tree-level tadpole resummations *exactly cancels* the positive classical vacuum energy of the Dp-brane. Section 4 is devoted to a similar analysis of tadpole resummations for the D25-branes of the SO(8192) open bosonic string [14–16]. The absence of massless dilaton and graviton tadpoles makes somehow this D25-brane–O25-plane system a solution of String Theory, albeit an unstable one. We show that, indeed, in this case tree-level tadpole resummations do not produce any correction to the D25-brane tension, despite the presence of a tadpole for the tachyon. The last section is devoted to a brief discussion of the limitations of the method (originating from the actual neglect of the gravitational back reaction) and of its application to superstring models with broken supersymmetry.

2. Boundary states in the bosonic string

In superstrings, D-branes can be conveniently described by boundary states for the closed string in the world-sheet theory (for a review, see Ref. [17]). The technique also applies for the D-branes of the bosonic string, that despite the lack of an RR charge, bear strong similarities to their supersymmetric counterparts. For the 26-dimensional bosonic string in flat spacetime, a Dp-brane boundary state at the origin, $|B_p\rangle$, satisfies the conditions

$$\partial_\tau X^\alpha(\sigma, \tau = 0)|B_p\rangle = 0, \quad \alpha = 0, 1, \ldots, p, \tag{1}$$

$$X^i(\sigma, \tau = 0)|B_p\rangle = 0, \quad i = p+1, p+2, \ldots, 25, \tag{2}$$

where $X^\mu(\sigma, \tau)$ is a closed-string coordinate. The boundary state can be explicitly expressed as a coherent state built from the string oscillators,

$$|B_p\rangle = \left|B_p^X\right\rangle\left|B^{\mathrm{gh}}\right\rangle, \tag{3}$$

$$\left|B_p^X\right\rangle = N_p \delta^{d_\perp}(\hat{x}) \exp\left(-\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^\mu S_{\mu\nu} \tilde{\alpha}_{-n}^\nu\right)|0\rangle, \tag{4}$$

$$\left|B^{\mathrm{gh}}\right\rangle = \exp\left(\sum_{n=1}^{\infty}(c_{-n}\tilde{b}_{-n} - b_{-n}\tilde{c}_{-n})\right)|0\rangle_{\mathrm{gh}}, \tag{5}$$

where $d_\perp \equiv d - (p+1)$, $S_{\mu\nu} \equiv (\eta_{\alpha\beta}, -\delta_{ij})$, the spacetime signature is "mostly plus", and

$$X^\mu = \hat{x}^\mu + \alpha' p^\mu \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \big(\alpha_n^\mu e^{-i(\tau-\sigma)n} + \tilde{\alpha}_n^\mu e^{-i(\tau+\sigma)n}\big), \tag{6}$$

$$b_- = \sum_{n=-\infty}^{\infty} b_n e^{-i(\tau-\sigma)n}, \qquad b_+ = \sum_{n=-\infty}^{\infty} \tilde{b}_n e^{-i(\tau+\sigma)n}, \tag{7}$$

$$c_- = \sum_{n=-\infty}^{\infty} c_n e^{-i(\tau-\sigma)n}, \qquad c_+ = \sum_{n=-\infty}^{\infty} \tilde{c}_n e^{-i(\tau+\sigma)n}. \tag{8}$$

The bc-ghost contribution is determined so that the full boundary state is BRST invariant, while the normalization constant N_p for one Dp-brane is

$$N_p \equiv \frac{T_p}{2}, \quad T_p \equiv \frac{\sqrt{\pi}}{2^{(d-10)/4}} \big(4\pi^2\alpha'\big)^{\frac{d-2p-4}{4}}, \tag{9}$$

with $d = 26$ and the 26-dimensional Planck constant $\kappa = 1$, so that the single dilaton and graviton tadpole couplings in the Einstein frame are reproduced [18]. For a collection of n coincident Dp-branes the normalization factor should be multiplied by n. The amplitudes for single dilaton or graviton emission are

$$A_{\mathrm{dilaton}} = A^{\mu\nu}\epsilon_{\mu\nu}^{(\phi)} = T_p V_{p+1} \frac{d-2p-4}{2\sqrt{d-2}}, \tag{10}$$

$$A_{\mathrm{graviton}} = A^{\mu\nu}\epsilon_{\mu\nu}^{(h)} = -T_p V_{p+1} \eta^{\alpha\beta}\epsilon_{\alpha\beta}^{(h)}, \tag{11}$$

$$A^{\mu\nu} \equiv \big\langle 0; k\big|\alpha_1^\mu \tilde{\alpha}_1^\nu\big|B_p^X\big\rangle = -\frac{T_p}{2} V_{p+1} S^{\mu\nu}, \tag{12}$$

where V_{p+1} is the Dp-brane world volume and $\epsilon_{\mu\nu}^{(\phi)}$ and $\epsilon_{\mu\nu}^{(h)}$ are projection and polarization tensors for the dilaton and the graviton, respectively. These amplitudes are also determined by the effective action for a Dp-brane in the Einstein frame,

$$S_{\mathrm{D}p} = -T_p \int d^{p+1}\xi\, e^{-\frac{d-2p-4}{2\sqrt{d-2}}\phi} \sqrt{-\det g_{\alpha\beta}}, \tag{13}$$

where we are ignoring both the B-field and the gauge field on the Dp-brane, for simplicity. With this normalization factor, the open string one-loop vacuum amplitudes on Dp-branes are simply obtained as

$$\mathcal{A}_p = \frac{1}{2!}\langle B_p|D|B_p\rangle = \frac{1}{2!} V_{p+1} N_p^2 \Delta_p, \tag{14}$$

$$\Delta_p \equiv \frac{\pi\alpha'}{2} \int_0^\infty ds \int \frac{d_\perp^d p}{(2\pi)^{d_\perp}} e^{-\frac{\pi\alpha'}{2} p_\perp^2 s} \frac{1}{\eta(is)^{24}} \tag{15}$$

$$= \frac{\pi\alpha'}{2} \int_0^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}} \frac{1}{\eta(is)^{24}}, \tag{16}$$

where

$$D \equiv \frac{\alpha'}{4\pi} \int_0^\infty dt \int_0^{2\pi} d\varphi\, z^{L_0} \bar{z}^{\tilde{L}_0} \tag{17}$$

[1] These Dp-branes do not carry RR charges, but are nonetheless characterized as being locations of the endpoints of open strings.

is the closed string propagator operator, with $z = e^{-t}e^{i\varphi}$. The factor $1/2!$ in Eq. (14) reflects the fact that the amplitude is of second order in the tadpole insertion.

The boundary state of the O-plane, the crosscap state, is very similar. For the O25-plane, the crosscap state should satisfy the condition

$$X^\mu(\sigma,\tau)|C_{25}\rangle = X^\mu(\sigma,\pi-\tau)|C_{25}\rangle, \quad \mu = 0,1,\ldots,25. \tag{18}$$

The explicit form of the crosscap state is

$$|C_{25}\rangle = |C_{25}^X\rangle|C^{\text{gh}}\rangle, \tag{19}$$

$$|C_{25}^X\rangle = \tilde{N}_{25}\exp\left(-\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}\alpha_{-n}^\mu\eta_{\mu\nu}\tilde{\alpha}_{-n}^\nu\right)|0\rangle, \tag{20}$$

$$|C^{\text{gh}}\rangle = \exp\left(\sum_{n=1}^{\infty}(-1)^n(c_{-n}\tilde{b}_{-n} - b_{-n}\tilde{c}_{-n})\right)|0\rangle_{\text{gh}}. \tag{21}$$

The normalization constant $\tilde{N}_{25}$ is determined as $\tilde{N}_{25} = 2^{13}N_{25}$ in the same way as N_p. The Klein bottle and Möbius strip amplitudes read

$$\mathcal{K} = \frac{1}{2!}\langle C_{25}|D|C_{25}\rangle = \frac{1}{2!}V_{26}\tilde{N}_{25}^2\Delta_{25}, \tag{22}$$

$$\begin{aligned}\mathcal{M} &= \frac{1}{2!}\big(\langle B_{25}|D|C_{25}\rangle + \langle C_{25}|D|B_{25}\rangle\big) \\ &= V_{26}N_{25}\tilde{N}_{25}\tilde{\Delta}_{25},\end{aligned} \tag{23}$$

where

$$\tilde{\Delta}_{25} \equiv -\frac{\pi\alpha'}{2}\int_0^\infty ds\frac{1}{\hat{\eta}(is+1/2)^{24}}. \tag{24}$$

From Eq. (14) with $p = 25$, and Eqs. (22) and (23), one can see that $n = 2^{13} = 8192$ D25-branes, with the normalization factor $n \times N_{25}$ for D25-brane boundary state, are necessary and sufficient to cancel tadpoles in the unoriented bosonic closed string theory [14–16].

3. Tadpole resummations on Dp-branes

At the classical level, the vacuum energy density on a Dp-brane coincides with its tension, $\Lambda_p^{\text{cl}} = T_p$, and can be read from Eq. (13). The open string one-loop correction to the vacuum energy is given by Eq. (14), and contains divergences due to the tadpoles of dilaton, graviton and tachyon in Δ_p. These tadpole contributions can be exhibited expanding the integrand of Δ_p for large value of s, as

$$\Delta_p \to \frac{\pi\alpha'}{2}\int^\infty ds\frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}}\big(e^{2\pi s} + 24 + \mathcal{O}(e^{-2\pi s})\big). \tag{25}$$

The first term within brackets is the contribution of tachyon tadpoles while the second is the overall contribution from massless dilaton and graviton tadpoles. These divergences can in principle be regularized via an "infrared" cutoff on s (or "ultraviolet", from the open-channel perspective), and we shall do it implicitly in what follows.

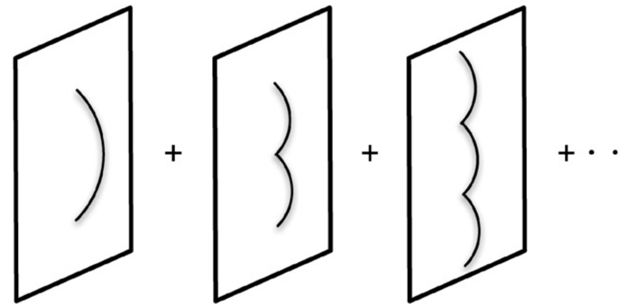

Fig. 1. Closed string bouncing on a Dp-brane.

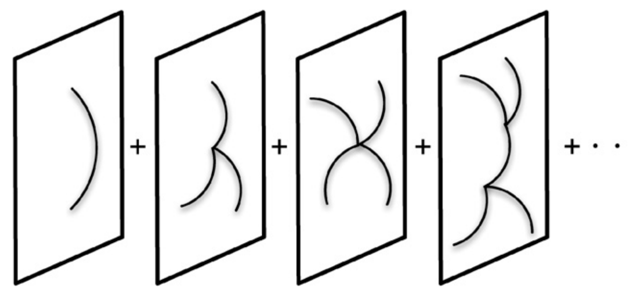

Fig. 2. Summation of tree-level contributions.

It is important to recognize that, in addition to the cylinder amplitude, the simplest contribution in the closed-string picture, there are many other three-level contributions with closed string contact interactions, as shown in Figs. 1 and 2. There are also closed-string one-loop and higher-loop contributions, that we neglect in the present calculations. As we stressed above, the existence of the contact interactions between dilaton/graviton and D-branes can be simply inferred from effective action of Eq. (13).

The effect of the two point contact interaction, that can be understood as a process in which a closed string bounces off the D-brane at the origin, can be accounted for inserting the operator

$$\hat{M} \equiv \int d^dx\,\delta^{d_\perp}(x)|\tilde{B}_p(x)\rangle(-T_p)\langle\tilde{B}_p(x)|, \tag{26}$$

where

$$|\tilde{B}_p(x)\rangle = |\tilde{B}_p^X(x)\rangle|B^{\text{gh}}\rangle, \tag{27}$$

$$|\tilde{B}_p^X(x)\rangle \equiv \frac{1}{T_p}\delta^{p+1}(\hat{x}-x)|B_p^X\rangle. \tag{28}$$

The state $|\tilde{B}_p(x)\rangle$ is essentially a Dp-brane boundary state with a different normalization, with the position of the closed string on the Dp-brane fixed at the generic point x, and indeed the integration in the definition of $\hat{M}$ is over the Dp-brane world volume. The overall normalization of the operator $\hat{M}$ is such that the coupling constant of the dilaton two-point contact interaction coincides with that present in the effective field theory of Eq. (13).

The "one-bounce" contribution to the Dp-brane vacuum energy of Fig. 1 is thus

$$A_1 = \frac{1}{2!}\langle B_p|D\hat{M}D|B_p\rangle = \frac{1}{2!}\int d^d x\, \delta^{d_\perp}(x)\big\langle B_p\big|D\big|\tilde{B}_p(x)\big\rangle(-T_p)\big\langle\tilde{B}_p(x)\big|D\big|B_p\big\rangle. \tag{29}$$

The quantity $\langle\tilde{B}_p(x)|D|B_p\rangle$ can be calculated in the same way as the cylinder amplitude:

$$\big\langle\tilde{B}_p(x)\big|D\big|B_p\big\rangle = \big\langle B_p\big|D\big|\tilde{B}_p(x)\big\rangle = \frac{N_p^2}{T_p}\frac{\pi\alpha'}{2}\int_0^\infty ds\,\frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}}\,e^{-\frac{x_\perp^2}{2\pi\alpha' s}}\,\frac{1}{\eta(is)^{24}}. \tag{30}$$

Therefore,

$$A_1 = \frac{1}{2!}V_{p+1}N_p^2\left(\frac{N_p}{T_p}\right)^2(-T_p)\Delta_p^2. \tag{31}$$

The "two-bounce", "three-bounce" and all the other amplitudes of this type can then be computed in the same way. The complete "two-point function" is then obtained summing all these contributions as depicted in Fig. 1.

$$A_p^{(2)} = \frac{1}{2!}\big\{\langle B_p|D|B_p\rangle + \langle B_p|D\hat{M}D|B_p\rangle + \langle B_p|D\hat{M}D\hat{M}D|B_p\rangle + \cdots\big\}, \equiv \frac{1}{2!}\langle B_p|D_M|B_p\rangle = \frac{1}{2!}V_{p+1}N_p^2\frac{\Delta_p}{1+T_p(N_p/T_p)^2\Delta_p}. \tag{32}$$

The result is a geometric series involving the regularized Δ_p. Notice that, in the limit that the "infrared" regulator for Δ_p is removed, a simple finite value obtains for the two-point function:

$$A_p^{(2)} = \frac{1}{2!}V_{p+1}T_p, \tag{33}$$

so that the corresponding contribution to the vacuum energy density on a Dp-brane is

$$\Lambda_p^{(2)} = -\frac{A_p^{(2)}}{V_{p+1}} = -\frac{1}{2!}T_p. \tag{34}$$

Let us now turn to the contributions depicted in Fig. 2. There is a dilaton three-point contact interaction with the Dp-brane, that again can be deduced from Eq. (13). In the boundary state formalism this interaction could be represented via an operator acting on three states:

$$\hat{M}^{(3)} \equiv \frac{1}{3!}T_p\left(\frac{N_p}{T_p}\right)^3\int d^d x\,\delta^{d_\perp}(x) \times \big|\tilde{B}_p(x)\big\rangle\big|\tilde{B}_p(x)\big\rangle\big|\tilde{B}_p(x)\big\rangle. \tag{35}$$

The legs of the resulting "three-point function" should be *full* "two-point functions", namely,

$$A_p^{(3)} = \frac{1}{3!}T_p\int d^d x\,\delta^{d_\perp}(x)\big\langle B_p\big|D_M\big|\tilde{B}_p(x)\big\rangle \times\big\langle B_p\big|D_M\big|\tilde{B}_p(x)\big\rangle\big\langle B_p\big|D_M\big|\tilde{B}_p(x)\big\rangle, \tag{36}$$

where 1/3! is a symmetry factor.[2]

A straightforward calculation gives

$$\big\langle B_p\big|D_M\big|\tilde{B}_p(x)\big\rangle = \frac{1}{1+T_p(N_p/T_p)^2\Delta_p}\big\langle B_p\big|D\big|\tilde{B}_p(x)\big\rangle, \tag{37}$$

and therefore

$$A_p^{(3)} = \frac{1}{3!}T_pV_{p+1}\left(\frac{1}{1+T_p(N_p/T_p)^2\Delta_p}\cdot\frac{N_p^2}{T_p}\Delta_p\right)^3 = \frac{1}{3!}V_{p+1}T_p. \tag{38}$$

The contribution to the vacuum energy density on a Dp-brane is thus

$$\Lambda_p^{(3)} = -\frac{A_p^{(3)}}{V_{p+1}} = -\frac{1}{3!}T_p. \tag{39}$$

One can continue this calculation along similar lines for the "four-point function", and in fact to all orders. The end result is very similar to the example of the scalar field with exponential potential of Ref. [11]:

$$A_p^{\text{tree}} \equiv A_p^{(2)} + A_p^{(3)} + A_p^{(4)} + \cdots = V_{p+1}T_p\sum_{n=1}^\infty\frac{1}{n(n+1)} = V_{p+1}T_p, \tag{40}$$

since the series can be easily shown to add up to one. Therefore, the total contribution to the vacuum energy density on a Dp-brane arising from the tree-level tadpole resummation is simply

$$\Lambda_p^{\text{tree}} = -A_p^{\text{tree}}/V_{p+1} = -T_p, \tag{41}$$

and exactly cancels the classical energy density $\Lambda_p^{\text{cl}} = T_p$.

4. Tadpole resummations on a D25-brane in the unoriented SO(8192) theory

The tree-level tadpole resummation of the previous section is essentially governed by the fact that the open string one-loop vacuum amplitude, Δ_p, contains a divergent massless contribution. It is therefore interesting to verify that the contribution vanishes in the absence of massless tadpoles, even if a tachyon tadpole is present. In this section we thus examine the tadpole resummation in the SO(8192) theory, where massless tadpoles are canceled as a result of the cooperative action of the D25-branes and an O25-plane.

There are three types of open-string one-loop contributions to the vacuum energy, associated to the cylinder, Möbius strip

[2] Its origin is as follows: the vertex carries a 1/3!, the third order of the tadpole insertion brings about one more 1/3!, while the number of contractions gives rise to a factor 3!.

and Klein bottle amplitudes:

$$\mathcal{A} = \frac{1}{2!}\langle B_{25}|D|B_{25}\rangle = \frac{1}{2!}V_{26}N_{25}^2\Delta_{25}, \tag{42}$$

$$\mathcal{M} = \frac{1}{2!}\big(\langle B_{25}|D|C_{25}\rangle + \langle C_{25}|D|B_{25}\rangle\big) = \frac{1}{2!}\cdot 2\langle C_{25}|D|B_{25}\rangle = V_{26}N_{25}\tilde{N}_{25}\tilde{\Delta}_{25}, \tag{43}$$

$$\mathcal{K} = \frac{1}{2!}\langle C_{25}|D|C_{25}\rangle = \frac{1}{2!}V_{26}\tilde{N}_{25}^2\Delta_{25}. \tag{44}$$

The divergences due to the tadpoles emerge from

$$\Delta_{25} \to \frac{\pi\alpha'}{2}\int^{\infty} ds\,\big(e^{2\pi s} + 24 + \mathcal{O}\big(e^{-2\pi s}\big)\big), \tag{45}$$

$$\tilde{\Delta}_{25} \to \frac{\pi\alpha'}{2}\int^{\infty} ds\,\big(e^{2\pi s} - 24 + \mathcal{O}\big(e^{-2\pi s}\big)\big). \tag{46}$$

Notice that the tachyon contributions have the same sign, while the massless dilaton/graviton contributions have opposite signs, in these two quantities. One can define two divergent quantities related, respectively, to the tachyon and dilaton/graviton tadpoles, as

$$\Delta_T \equiv \frac{\pi\alpha'}{2}\int_0^{\infty} ds\, e^{2\pi s}, \tag{47}$$

$$\Delta \equiv \frac{\pi\alpha'}{2}\int_0^{\infty} ds 24, \tag{48}$$

that can be regulated introducing an "infrared" cutoff on s. One can then write

$$\Delta_{25} \to \Delta_T + \Delta + \text{finite}, \tag{49}$$

$$\tilde{\Delta}_{25} \to \Delta_T - \Delta + \text{finite}. \tag{50}$$

In order to obtain the full "two-point functions", "bounce effects" on both D25-branes and the O25-plane should be included. The "bounce effect" on the O25-plane can be accounted for inserting the operator

$$\hat{M}_O = \int d^{26}x\,\big|\tilde{C}_{25}(x)\big\rangle(+\tilde{T}_{25})\big\langle\tilde{C}_{25}(x)\big|, \tag{51}$$

that is obtained in the same way as the operator $\hat{M}$, where

$$\big|\tilde{C}_{25}(x)\big\rangle \equiv \frac{1}{\tilde{T}_{25}}\delta^{26}(\hat{x} - x)\big|C_{25}(x)\big\rangle \tag{52}$$

and $\tilde{T}_{25} \equiv \tilde{N}_{25}/2$ is the tension of O25-plane.

The full "two-point function" with both edges on D25-branes and without O25-bounces was already given in Eq. (32), and is

$$A^{(2)}_{\text{zero-O}} = \frac{1}{2!}\langle B_{25}|D_M|B_{25}\rangle = \frac{1}{2!}V_{26}N_{25}^2\frac{\Delta_{25}}{1 + T_{25}(N_{25}/T_{25})^2\Delta_{25}}, \tag{53}$$

where N_{25} and T_{25} include the number of D25-branes, $n = 2^{13} = 8192$. Therefore, $\tilde{N}_{25} = N_{25}$ and $\tilde{T}_{25} = T_{25}$.

The "two-point function" with both edges on D25-branes with one O25-bounce but without D25-bounces is

$$\frac{1}{2!}\langle B_{25}|D\hat{M}_O D|B_{25}\rangle = \frac{1}{2!}\int d^{26}x\,\big\langle B_{25}\big|D\big|\tilde{C}_{25}(x)\big\rangle(+\tilde{T}_{25})\big\langle\tilde{C}_{25}(x)\big|D\big|B_{25}\big\rangle = \frac{1}{2!}V_{26}N_{25}^2\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\hat{\Delta}_{25}. \tag{54}$$

There are two contributions to the "two-point function" with both edges on D25-branes with one O25-bounce and one D25-bounce:

$$\frac{1}{2!}\frac{1}{2}\langle B_{25}|D\hat{M}_O D\hat{M} D|B_{25}\rangle \quad\text{and}\quad \frac{1}{2!}\frac{1}{2}\langle B_{25}|D\hat{M} D\hat{M}_O D|B_{25}\rangle, \tag{55}$$

where the overall $1/2$ is a symmetry factor, so that

$$\frac{1}{2!}\left(\frac{1}{2}\langle B_{25}|D\hat{M}_O D\hat{M} D|B_{25}\rangle + \frac{1}{2}\langle B_{25}|D\hat{M} D\hat{M}_O D|B_{25}\rangle\right) = \frac{1}{2!}V_{26}N_{25}^2\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(-T_{25})\Delta_{25}. \tag{56}$$

These results can be understood in terms of Feynman rules for the "propagator" and the "mass insertion". The "propagators" $\langle B|D|B\rangle$ and $\langle C|D|C\rangle$ give Δ_{25}, and the "propagators" $\langle B|D|C\rangle$ and $\langle C|D|B\rangle$ give $\hat{\Delta}_{25}$. On the other hand, the "mass insertions" determined by $\hat{M}$ and $\hat{M}_O$ give $(N_{25}/T_{25})^2(-T_{25})$ and $(N_{25}/T_{25})^2(+T_{25})$, respectively. Certain symmetry factors should be included, and there is also an overall factor $(1/2!)V_{26}N_{25}^2$. It is then straightforward to compute the "two-point function" with both edges on D25-branes, with one O25-bounce and full D25-bounce:

$$A^{(2)}_{\text{one-O}} = \frac{1}{2!}V_{26}N_{25}^2\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\hat{\Delta}_{25} \times\left(1 + \left(\frac{N_{25}}{T_{25}}\right)^2(-T_{25})\Delta_{25} + \left(\left(\frac{N_{25}}{T_{25}}\right)^2(-T_{25})\Delta_{25}\right)^2 + \cdots\right) = \frac{1}{2!}V_{26}N_{25}^2\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\hat{\Delta}_{25} \times\frac{1}{1 + T_{25}(N_{25}/T_{25})^2\Delta_{25}}. \tag{57}$$

The "two-point functions" with both edges on D25-branes with two O25-bounces and full D25-bounce are more complicated. The contribution without D25-bounces is

$$\frac{1}{2!}\langle B_{25}|D\hat{M}_O D\hat{M}_O D|B_{25}\rangle = \frac{1}{2!}V_{26}N_{25}^2\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\Delta_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\hat{\Delta}_{25}. \tag{58}$$

The next contributions with one D25 bounce have three different forms:

$$\frac{1}{3}\frac{1}{2!}\langle B_{25}|D\hat{M}_O D\hat{M}_O D\hat{M} D|B_{25}\rangle = \frac{1}{3}\frac{1}{2!}V_{26}N_{25}^2\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\Delta_{25}\left(\frac{N_{25}}{T_{25}}\right)^2 \times(+T_{25})\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(-T_{25})\Delta_{25}, \tag{59}$$

$$\frac{1}{3}\frac{1}{2!}\langle B_{25}|D\hat{M}_O D\hat{M} D\hat{M}_O D|B_{25}\rangle = \frac{1}{3}\frac{1}{2!}V_{26}N_{25}^2\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2 \times(-T_{25})\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\hat{\Delta}_{25}, \tag{60}$$

$$\frac{1}{3}\frac{1}{2!}\langle B_{25}|D\hat{M} D\hat{M}_O D\hat{M}_O D|B_{25}\rangle = \frac{1}{3}\frac{1}{2!}V_{26}N_{25}^2\Delta_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(-T_{25})\hat{\Delta}_{25}\left(\frac{N_{25}}{T_{25}}\right)^2 \times(+T_{25})\Delta_{25}\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\hat{\Delta}_{25}. \tag{61}$$

Although the first and the third of these coincide, the second is different. Taking only the dominant divergence due to the tachyon tadpoles, so that $\Delta_{25}\to\Delta_T$ and $\hat{\Delta}_{25}\to\Delta_T$, the sum of the above three contributions becomes

$$\frac{1}{2!}V_{26}N_{25}^2\Delta_T\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\Delta_T\left(\frac{N_{25}}{T_{25}}\right)^2 \times(+T_{25})\hat{\Delta}_T\left(\frac{N_{25}}{T_{25}}\right)^2(-T_{25})\Delta_T. \tag{62}$$

It is then straightforward to calculate the contribution from two or more D25-bounces, and finally the result with full D25-bounce becomes

$$A^{(2)}_{\text{two-O}} = \frac{1}{2!}V_{26}N_{25}^2\Delta_T\left(\frac{N_{25}}{T_{25}}\right)^2 \times(+T_{25})\Delta_T\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\Delta_T \times\left(1+\left(\frac{N_{25}}{T_{25}}\right)^2(-T_{25})\Delta_T \times+\left(\left(\frac{N_{25}}{T_{25}}\right)^2(-T_{25})\Delta_T\right)^2+\cdots\right) = \frac{1}{2!}V_{26}N_{25}^2\Delta_T\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\Delta_T\left(\frac{N_{25}}{T_{25}}\right)^2 \times(+T_{25})\Delta_T\frac{1}{1+T_{25}(N_{25}/T_{25})^2\Delta_T}. \tag{63}$$

It is now straightforward to obtain the contribution with full O25 and D25 bounces:

$$A^{(2)}_{\text{full}} = \frac{1}{2!}V_{26}N_{25}^2\frac{\Delta_T}{1+T_{25}(N_{25}/T_{25})^2\Delta_T} \times\left(1+\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\Delta_T +\left(\left(\frac{N_{25}}{T_{25}}\right)^2(+T_{25})\Delta_T\right)^2+\cdots\right) \tag{64}$$

$$= \frac{1}{2!}V_{26}N_{25}^2\frac{\Delta_T}{1+T_{25}(N_{25}/T_{25})^2\Delta_T} \times\frac{1}{1-T_{25}(N_{25}/T_{25})^2\Delta_T}. \tag{65}$$

Recalling that Δ_T is a divergent quantity, one can thus see that the "two-point function" with both edges on the D25-brane (cylinder with bounces), vanishes. The other two types of "two-point functions", with both edges on the O25-plane (Klein bottle with bounces) and with one edge on the D25-brane and the other on the O25-plane (Möbius strip with bounces), are exactly identical and thus also vanish. Since all "two-point function" vanish, the "three-point functions" and in fact all higher-point functions vanish, as can be simply understood by the arguments in the previous section. In conclusion, there is no contribution to the vacuum energy resulting from tadpole resummation, despite the presence of the tachyon tadpole.

If we neglect the divergence introduced by tachyon tadpoles (or if we define it by analytic continuation as $\Delta_T=-\alpha'/4$), letting $\Delta_{25}\to\Delta$ and $\hat{\Delta}_{25}\to-\Delta$, the contribution of the cylinder with bounces equals the contribution of the Klein bottle with bounces, while the contribution of cylinder with bounces equals $-1/2$ of the contribution of Möbius strip with bounces. This is easily understood, since the replacement of one D25-brane boundary state to one O25-place crosscap state gives rise to a sign change, due to $\Delta_{25}\to\hat{\Delta}_{25}$. Therefore, again, all contributions to the vacuum energy arising from tadpole resummations add up to a vanishing result, just like the tadpole contributions, in the SO(8192) theory.

5. Conclusions

It is interesting that these simple calculations give the result expected by Sen's conjecture of open-string tachyon condensation from a closed-channel perspective. The calculations combine the boundary state formalism with some information drawn from the low-energy effective field theory. It would be interesting to try and extend this method to closed-string field theory, since there is no proof that this method gives the exact result. The key problems to be considered are the following.

In open-string tachyon condensation in String Field Theory, the tachyon potential is obtained integrating out the open string massive modes (see Ref. [19] for a review). The actual numerical calculation is based on the level truncation approximation, and the obtained numerical value of the vacuum energy is indeed very close to zero. On the other hand, the role of the open string massive modes is not evident in our method, where infrared divergences due to closed string tadpoles are important. It is natural that an infinite number of open string modes are required, since we are making explicit use of open–closed string duality and we are tracking the tadpoles of low-lying closed string states, the dilaton, the graviton and the closed-string tachyon.

We have accounted for the propagation of closed strings in a rigid spacetime perpendicular to the D-branes. In particular, we have been considering a flat spacetime, which should not be a good approximation in absolute terms, since the very existence of D-branes is known to lead to a back reaction on the spacetime geometry. Indeed, it was shown in Ref. [20] that in string models with broken supersymmetry without tachyons, the dilaton tadpole curves the original flat Minkowski background, leading to a sort of spontaneous compactification. The effect of this gravitational back reaction is clearly not included in our calculation, but is similarly not included in the analyses based on String Field Theory. The gravitational back reaction may change the contribution of the tadpole resummation. Other subtleties resulting from the inclusion of gravity in the tadpole resummation in field theory were discussed in Ref. [11]. It is not clear whether or not these problems are overcome in our method.

In spite of these problems, it is straightforward to apply this method to superstring models with "brane supersymmetry breaking" [21,22], i.e., broken supersymmetry on branes with no tachyon but dilaton tadpoles. For instance, in the USp(32) Sugimoto model [21] all "two-point functions" can be computed exactly as in SO(8192) model, and the final result is

$$A^{(2)}_{\text{full}} = \frac{1}{2!} V_{10} N_9^2 \frac{\Delta_{\text{NS}}}{1 + T_9 (N_9/T_9)^2 \Delta_{\text{NS}}} \times \frac{1}{1 + T_9 (N_9/T_9)^2 \Delta_{\text{NS}}}, \tag{66}$$

where N_9 is the normalization factor of the boundary state of D9-brane, T_9 is the tension of the D9-brane and Δ_{NS} includes only a divergence due to massless dilaton/graviton tadpoles (since there is no tachyon), defined in the same way as Δ_{25}. Therefore, all "two-point functions" vanish, so that there is no correction to the vacuum energy from tadpole resummation. It would be interesting to apply this method to the calculation of other physical quantities, and for instance to the masses of scalar fields.

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