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Asymptotic Synchronization of a Class of Neural Networks with Reaction-Diffusion Terms and Time-Varying Delays

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Abstract—In this paper, the problem of asymptotic synchronization for a class of neural networks with reaction-diffusion terms and time-varying delays is investigated. Using the drive-response concept, a control law is derived to achieve the state synchronization of two identical neural networks with reaction-diffusion terms. Moreover, we derive a sufficient asymptotic synchronization condition for the neural networks with reaction-diffusion terms if reaction-diffusion terms satisfy a weaker condition. The synchronization condition is easy to verify and relies on the connection matrix in the driven networks and the suitable designed controller gain matrix in the response networks. © 2006 Elsevier Ltd. All rights reserved.

Keywords—Asymptotic synchronization, Neural networks, Reaction-diffusion terms, Lyapunov functional, Time-varying.

1. INTRODUCTION

In the past few years, there has been increasing interest in the potential applications of the dynamics of artificial neural networks in many areas [1–10]. In such applications, analysis of the equilibrium points is a prerequisite. Thus, different types of neural networks with or without time delays have been widely investigated and many stability criteria have been obtained [8–23]. Nowadays, some authors pay attention to the exponential synchronization of neural networks [24–26]. Chaos synchronization [27,28] has been investigated for a decade, for which many effective methods have been presented [24–37]. In 1990, Pecora and Carroll [27] addressed the synchronization of chaotic systems using a drive-response conception. The idea is to use the output of the drive system to control the response system so that they oscillate in a synchronized manner. Recently, the synchronization of coupled chaotic systems has received considerable

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attention in the last decade due to its potential applications in creating secure communication systems [34–37].

As we know, Hopfield neural networks, cellular neural networks and bidirectional associative memory networks can exhibit some complicated dynamics and even chaotic behaviors if the network's parameters and time delays are appropriately chosen [38,39]. However, there are few studies in the synchronization issue for a class of neural networks with time-varying delays. This work, inspired by the above works, addresses the synchronization problem of a class of chaotic neural networks with reaction-diffusion terms and time-varying delays. This class of chaotic neural networks unifies several well-known neural networks, such as Hopfield neural networks, cellular neural networks.

The aim of this paper is to further develop criteria for the synchronization problem for a class of chaotic neural networks with reaction-diffusion terms and time-varying delays. More precisely, in this paper, the synchronization for this class of chaotic neural networks is studied based on the Lyapunov stability theory, and a sufficient condition for the asymptotic synchronization of the neural networks is derived. The condition for asymptotic synchronization is in the form of a few algebraic inequalities, which is very convenient to verify.

2. SOME CRITERIA FOR ASYMPTOTIC SYNCHRONIZATION

2.1. A Class of Neural Networks with Reaction-Diffusion Terms

In the sequence, we will study a class of neural networks with time-varying delays described by the following differential equations:

$$\begin{aligned}\frac{\partial u_i(t,x)}{\partial t} &= \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right) - a_i u_i(t,x) \\ &+ \sum_{j=1}^{n} w_{ij} g_j(u_j(t,x)) + \sum_{j=1}^{n} h_{ij} g_j(u_j(t-\tau_j(t),x)) + J_i,\end{aligned} \tag{1}$$

for $i \in \{1,2,\dots,n\}$, $t > 0$ where $x = (x_1, x_2, \dots, x_l)^\top \in \Omega \subset R^l$, Ω is a bounded compact set with smooth boundary $\partial\Omega$ and $\operatorname{mes}\Omega > 0$ in space R^l; $u_i(t,x)$ is the state of the i^{th} unit at time t; $g_i(\cdot)$ denotes the signal functions of the i^{th} neurons at time t and in space x; J_i denotes the external inputs on the i^{th} neurons; $a_i > 0$ is constant; $\tau_j(t)$, $j = 1,\dots,n$, are time-varying delays of the neural network satisfying $0 \le \tau_j(t) \le \tau^*$ and $0 \le \dot{\tau}_j(t) = \sigma < 1$; w_{ij} and h_{ij} stand for the weights of neuron interconnections. Smooth functions $D_{ik} = D_{ik}(t,x,u) \ge 0$ correspond to the transmission diffusion operators along the i^{th} neurons.

The boundary conditions and initial conditions are given by

$$\frac{\partial u_i}{\partial n} := \left(\frac{\partial u_i}{\partial x_1}, \frac{\partial u_i}{\partial x_2}, \dots, \frac{\partial u_i}{\partial x_l}\right)^\top = 0, \qquad i = 1,2,\dots,n, \tag{2}$$

and

$$u_i(s,x) = \phi_i(s,x), \qquad s \in [-\tau^*, 0], \quad i = 1,2,\dots,n, \tag{3}$$

where $\phi_i(s,x)$ $(i = 1,2,\dots,n)$ are bounded and continuous on $[-\tau^*, 0] \times \Omega$.

We assume that the activation functions satisfy the following properties.

(H_1) The neurons activation functions $g_i(\cdot)$ $(i = 1,2,\dots,n)$ are Lipschitz-continuous, that is, there exist constants $L_i > 0$ such that

$$|g_i(\xi_1) - g_i(\xi_2)| \le L_i|\xi_1 - \xi_2|,$$

for all $\xi_1, \xi_2 \in R$.

It has been demonstrated that if the system's matrices W and H as well as the delay parameters are suitably chosen, then system (1) with $D_{ik} = D_{ik}(t, x, u) = 0$ will display a chaotic behavior [20,37,38]. However, strictly speaking, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields. So we must consider that the activations vary in space as well as in time. Therefore, herein we are concerned with the synchronization problem of this class of chaotic neural networks.

2.2. Asymptotic Synchronization Problem

Chaos dynamics are extremely sensitive to initial conditions. Even infinitesimal changes in the initial condition will lead to an asymptotic divergence of orbits. In order to observe the synchronization behavior in this class of neural networks, we have two time-varying neural networks with reaction-diffusion terms where the drive system with state variable denoted by $u_i(t,x)$ drives the response system having identical dynamical equations denoted by state variable $\tilde{u}_i(t,x)$. However, the initial condition on the drive system is different from that of the response system. Therefore, the reaction-diffusion neural networks with drive are described by the following equations:

$$\begin{aligned}\frac{\partial \tilde{u}_i(t,x)}{\partial t} &= \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial \tilde{u}_i(t,x)}{\partial x_k}\right) - a_i\tilde{u}_i(t,x) + \sum_{j=1}^{n} w_{ij}g_j(\tilde{u}_j(t,x)) \\ &+ \sum_{j=1}^{n} h_{ij}g_j(\tilde{u}_j(t-\tau_j(t),x)) + I_i + v_i(t), \qquad i = 1,2,\ldots,n,\end{aligned} \tag{4}$$

where $v_i(t)$ denotes the external control input that will be appropriately designed for a certain control objective and the initial condition is given as follows:

$$\tilde{u}_i(s,x) = \varphi_i(s,x), \qquad s \in [-\tau^*, 0], \quad i = 1,2,\ldots,n, \tag{5}$$

where $\varphi_i(s,x)$ $(i = 1,2,\ldots,n)$ are bounded and continuous on $[-\tau^*, 0] \times \Omega$.

Definition 1. *System (1) and the uncontrolled system (4) (i.e., $v \equiv 0$ in (4)) are said to be asymptotically synchronized, if the following equation:*

$$\lim_{t\to\infty} |u_i(t,x) - \tilde{u}_i(t,x)| = 0, \qquad \forall\, t \geq 0, \quad i = 1,2,\ldots,n,$$

holds.

Asymptotic Synchronization Problem. The asymptotic synchronization problem considered here is to determine the control input v_i associated with the state-feedback for the purpose of asymptotically synchronizing the two identical chaotic neural networks with the same system's parameters but differences in the initial conditions.

3. MAIN RESULTS

3.1. Controller Design

Let us define the synchronization error signal $e_i(t,x) = u_i(t,x) - \tilde{u}_i(t,x)$, where $u_i(t,x)$ and $\tilde{u}_i(t,x)$ are the i^{th} state variable of the drive and response neural networks, respectively. Therefore, the error dynamics between (1) and (4) can be expressed by

$$\begin{aligned}\frac{\partial e_i(t,x)}{\partial t} &= \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i(t,x)}{\partial x_k}\right) - a_ie_i(t,x) \\ &+ \sum_{j=1}^{n} w_{ij}\left[g_j\left(e_j(t,x) + \tilde{u}_j(t,x)\right) - g_j\left(\tilde{u}_j(t,x)\right)\right] - v_i(t) \\ &+ \sum_{j=1}^{n} h_{ij}\left[g_j\left(e_j(t-\tau_j(t),x) + \tilde{u}_j(t-\tau_j(t),x)\right) - g_j\left(\tilde{u}_j(t-\tau_j(t),x)\right)\right],\end{aligned} \tag{6}$$

for $i = 1, 2, \dots, m$. If the state variables of the drive system are used to drive the response system, then the control input vector with state feedback is designed as follows:

$$\begin{bmatrix} v_1(t) \\ \vdots \\ v_n(t) \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} M_{1j}(u_j(t,x) - \tilde{u}_j(t,x)) \\ \vdots \\ \sum_{j=1}^{n} M_{nj}(u_j(t,x) - \tilde{u}_j(t,x)) \end{bmatrix} = \begin{bmatrix} M_{11} & \cdots & M_{1n} \\ \vdots & \vdots & \vdots \\ M_{n1} & \cdots & M_{nn} \end{bmatrix} \begin{bmatrix} u_1(t,x) - \tilde{u}_1(t,x) \\ \vdots \\ u_n(t,x) - \tilde{u}_n(t,x) \end{bmatrix}, \tag{7}$$

where $M = (M_{ij})_{n\times n} \in R^{n\times n}$ is the controller gain matrix and will be appropriately chosen for asymptotically synchronizing both drive system and response system.

3.2. Asymptotic Synchronization

The asymptotic synchronization problem of systems (1) and (4) can be solved if the controller gain matrix is suitably designed. The asymptotic synchronization condition is established in the following main theorem.

THEOREM 1. *For these drive-response neural networks (1) and (4) which satisfy boundary condition (2) and initial condition (3) and assumption* (H_1), *if the controller gain matrix M in (7) is real symmetric and positive definite, and there exist $\lambda_i > 0$ satisfying*

$$\begin{aligned} 2a_i + 2\sum_{j=1}^{n} |M_{ij}| + 2\sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i}|M_{ji}| &> \sum_{j=1}^{n} L_j|w_{ij}| \\ + L_i \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i}|w_{ji}| + \sum_{j=1}^{n} L_j|h_{ij}| &+ L_i \frac{1}{1-\sigma} \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i}|h_{ji}|, \end{aligned} \tag{8}$$

for $i, j = 1, 2, \dots, n$, then the asymptotic synchronization of systems (1) and (4) is obtained.

PROOF. For convenience, let $e_i = e_i(t,x)$. In order to confirm that the origin of (6) is globally asymptotically synchronized, we consider the following Lyapunov function:

$$V(t) = \int_{\Omega} \sum_{i=1}^{n} \lambda_i \left[e_i^2 + \frac{1}{1-\sigma} \sum_{j=1}^{n} L_j|h_{ij}| \int_{t-\tau_j(t)}^{t} e_j^2(s,x)\, ds \right] dx. \tag{9}$$

Evaluating the time derivative of V along the trajectory of (6) gives

$$\begin{aligned} \dot{V}(t) &\le \int_{\Omega} \sum_{i=1}^{n} \lambda_i \left[2e_i \frac{\partial e_i}{\partial t} + \frac{1}{1-\sigma} \sum_{j=1}^{n} L_j|h_{ij}|e_j^2(t,x) - \sum_{j=1}^{n} L_j|h_{ij}|e_j^2(t-\tau_j(t),x) \right] dx \\ &\le \int_{\Omega} \sum_{i=1}^{n} \lambda_i \left[2e_i \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{ik} \frac{\partial e_i}{\partial x_k} \right) - 2a_i e_i^2 + 2|e_i| \sum_{j=1}^{n} L_j|w_{ij}|\,|e_j| \right. \\ &+ 2|e_i| \sum_{j=1}^{n} L_j|h_{ij}|\,|e_j(t-\tau_j(t),x)| - 2|e_i| \sum_{j=1}^{n} |M_{ij}|\,|e_j| + \frac{1}{1-\sigma} \sum_{j=1}^{n} L_j|h_{ij}|e_j^2(t,x) \\ &\left. - \sum_{j=1}^{n} L_j|h_{ij}|e_j^2(t-\tau_j(t),x) \right] dx \end{aligned} \tag{10}$$

$$\le \int_\Omega \sum_{i=1}^n \lambda_i \left[2e_i \sum_{k=1}^l \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i}{\partial x_k}\right) - 2a_i e_i^2 + \sum_{j=1}^n L_j|w_{ij}|\left(e_i^2+e_j^2\right)\right.$$
$$+ \sum_{j=1}^n L_j|h_{ij}|\left(e_i^2 + e_j^2(t-\tau_j(t),x)\right) - 2\sum_{j=1}^n |M_{ij}|\left(e_i^2+e_j^2\right)$$
$$\left. + \frac{1}{1-\sigma}\sum_{j=1}^n L_j|h_{ij}|e_j^2(t,x) - \sum_{j=1}^n L_j|h_{ij}|e_j^2(t-\tau_j(t),x)\right] dx \qquad (10)(\text{cont.})$$
$$= \int_\Omega \sum_{i=1}^n \lambda_i \left\{2e_i \sum_{k=1}^l \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i}{\partial x_k}\right) + \left[-2a_i + \sum_{j=1}^n L_j|w_{ij}| + \sum_{j=1}^n \frac{\lambda_j}{\lambda_i}L_i|w_{ji}|\right.\right.$$
$$\left.\left. + \sum_{j=1}^n L_j|h_{ij}| - 2\sum_{j=1}^n |M_{ij}| - 2\sum_{j=1}^n \frac{\lambda_j}{\lambda_i}|M_{ji}| + \frac{1}{1-\sigma}\sum_{j=1}^n \frac{\lambda_j}{\lambda_i}L_i|h_{ji}|\right] e_i^2\right\} dx.$$

On the other hand, we have

$$\int_\Omega \sum_{i=1}^n 2\lambda_i e_i \sum_{k=1}^l \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial e_i}{\partial x_k}\right) dx$$
$$= \sum_{i=1}^n \lambda_i \int_\Omega e_i \nabla \left(D_{ik}\frac{\partial e_i}{\partial x_k}\right)_{k=1}^l dx$$
$$= \sum_{i=1}^n \lambda_i \int_\Omega \nabla \left(e_i D_{ik}\frac{\partial e_i}{\partial x_k}\right)_{k=1}^l dx - \sum_{i=1}^n \lambda_i \int_\Omega \left(D_{ik}\frac{\partial e_i}{\partial x_k}\right)_{k=1}^l \nabla e_i \, dx \qquad (11)$$
$$= \sum_{i=1}^n \lambda_i \int_{\partial\Omega} \left(e_i D_{ik}\frac{\partial e_i}{\partial x_k}\right)_{k=1}^l dx - \sum_{i=1}^n \lambda_i \sum_{k=1}^l \int_\Omega D_{ik}\left(\frac{\partial e_i}{\partial x_k}\right)^2 dx$$
$$\le -\sum_{i=1}^n \lambda_i \sum_{k=1}^l \int_\Omega D_{ik}\left(\frac{\partial e_i}{\partial x_k}\right)^2 dx.$$

Hence, substituting (11) into (10), it follows that

$$\dot V(t) \le -\sum_{i=1}^n \lambda_i \sum_{k=1}^l \int_\Omega D_{ik}\left(\frac{\partial e_i}{\partial x_k}\right)^2 dx + \int_\Omega \sum_{i=1}^n \lambda_i \left\{-2a_i + \sum_{j=1}^n L_j|w_{ij}| + \sum_{j=1}^n \frac{\lambda_j}{\lambda_i}L_i|w_{ji}|\right.$$
$$\left. + \sum_{j=1}^n L_j|h_{ij}| - 2\sum_{j=1}^n |M_{ij}| - 2\sum_{j=1}^n \frac{\lambda_j}{\lambda_i}|M_{ji}| + \frac{1}{1-\sigma}\sum_{j=1}^n \frac{\lambda_j}{\lambda_i}L_i|h_{ji}|]e_i^2\right\} dx \qquad (12)$$
$$< 0.$$

Now, by a standard Lyapunov-type theorem in functional differential equations, see, e.g., [40], the origin of error system (6) is asymptotically stable, implying that the two systems (1) and (4) are synchronized.

Corollary 1. *For these drive-response neural networks (1) and (4) which satisfy boundary condition (2) and initial condition (3) and assumption* (H_1)*, if the controller gain matrix* M *in (7) is real symmetric and positive definite, and satisfies*

$$2a_i + 2\sum_{j=1}^n |M_{ij}| + 2\sum_{j=1}^n |M_{ji}| > \sum_{j=1}^n L_j|w_{ij}|$$
$$+ L_i \sum_{j=1}^n |w_{ji}| + \sum_{j=1}^n L_j|h_{ij}| + L_i\frac{1}{1-\sigma}\sum_{j=1}^n |h_{ji}|, \qquad (13)$$

for $i,j = 1,2,\ldots,n$*, then the asymptotic synchronization of systems (1) and (4) is obtained.*

REMARK 1. The assumption of reaction-diffusion terms in this paper is almost the same in [7–10,22]. We can see its applied meaning in [7–10,22].

REMARK 2. The sufficient conditions for asymptotic synchronization of systems (1) and (4) are dependent of the delay parameter and rely on the inequality of the system's parameters and the controller gain.

4. ILLUSTRATIVE EXAMPLE

The sufficient condition for asymptotic synchronization of a class of coupled delayed neural networks presented in this paper is demonstrated by a numerical example.

EXAMPLE 1. For simplicity, let $n = 2$ and consider system (1) with $D_{ik}(t,x,u) = 0$, and the system parameters are as follows:

$$A = \operatorname{diag}(a_i)_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad W = (w_{ij})_{2\times 2} = \begin{bmatrix} 2 & -1 \\ -4 & 3.5 \end{bmatrix},$$
$$H = (h_{ij})_{2\times 2} = \begin{bmatrix} -1.5 & -0.1 \\ -3 & 2 \end{bmatrix}, \qquad \sigma = 0.3, \qquad g_i(x_i) = \tanh(x_i),$$

Clearly, $g_i(x_i)$ satisfies condition (H_1) above, with $L_1 = L_2 = 1$. The system parameters of the response chaotic neural network (4) with time-varying delays are designed the same as those in the drive system. Then if the controller gain matrix in (7) is chosen as

$$M = (M_{ij})_{2\times 2} = \begin{bmatrix} 8 & -3 \\ 3 & 9 \end{bmatrix},$$

the following inequalities:

$$\begin{aligned} 46 &= 2a_1 + 2\sum_{j=1}^{n}|M_{1j}| + 2\sum_{j=1}^{n}|M_{j1}| \\ &> \sum_{j=1}^{n} L_j|w_{1j}| + L_1\sum_{j=1}^{n}|w_{j1}| + \sum_{j=1}^{n} L_j|h_{1j}| + L_1\frac{1}{1-\sigma}\sum_{j=1}^{n}|h_{j1}| \\ &= 26.5, \end{aligned} \tag{14}$$

$$\begin{aligned} 50 &= 2a_2 + 2\sum_{j=1}^{n}|M_{2j}| + 2\sum_{j=1}^{n}|M_{j2}| \\ &> \sum_{j=1}^{n} L_j|w_{2j}| + L_2\sum_{j=1}^{n}|w_{j2}| + \sum_{j=1}^{n} L_j|h_{2j}| + L_2\frac{1}{1-\sigma}\sum_{j=1}^{n}|h_{j2}| \\ &= 24, \end{aligned} \tag{15}$$

are satisfied. It follows from Corollary 1 that systems (1) and (4) have been synchronized.

5. CONCLUSIONS

Using a suitable Lyapunov functional, a sufficient asymptotic synchronization condition for a class of neural networks with time-varying delays and reaction-diffusion terms is obtained. From Theorem 1, we conclude if reaction-diffusion terms satisfy weaker conditions, the main effect for the asymptotic synchronization of systems (1) and (4) just comes from the network parameter. The given algebraic criteria are easy to verify, and it will bring some convenience for those who design and verify these chaotic neural networks.

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