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The quark-lepton unification: LHC data and neutrino masses



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ABSTRACT

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1. Introduction

The standard model (SM) [1] of elementary particles is now a completely successful unified theory with an answer to the origin of masses of quarks, leptons and gauge bosons if the recently discovered new boson at Large Hadron Collider (LHC) at a mass around 125 GeV [2] is the standard model Higgs boson. The present data on neutrino masses indicate see-saw scale equal or very close to the grand unification scale. Again, recent result after Daya Bay and RENO experiments [3] $\theta_{13} \approx \theta_C$ (equal up to a factor of $\sqrt{2}$) possibly be a hint of a connection between leptonic mixing and quark mixing. The connection, particularly, the quark-lepton unification has been first enunciated in the grand unified theories (GUTs) with an additional family symmetry in [4] and then worked out in a series of papers [5]. The family symmetry (e.g., S₄, A₄ and SO(3), etc.) dictates the organizing principle for the structure of Yukawa matrix (to generate the observed mass pattern in both lepton and quark sector at weak scale).

The quark masses originate from electroweak symmetry breaking (EWSB), while neutrino masses have different origin around the GUT scale — the see-saw mechanism. All observed quark mixing angles are very small, while in neutrino sector 1–2 and 2–3 mixing angles are large and 1–3 mixing angle is small. The large magnifications of solar and atmospheric mixing angles

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through renormalization group evolution (RGE) from a high scale around the GUT scale to weak scale are possible [6] for quasidegenerate neutrinos. Here, the required quasi-degenerate hierarchical (normal) neutrinos can be generated in GUT models with type II see-saw with an additional family symmetry [5] (discussed later).

The intermediate scale between GUT scale and electroweak (EW) scale is relevant for see-saw mechanisms, but not necessary at all for quark masses and their mixing. The lower limits of the see-saw scales are increased as the neutrino masses becomes smaller and smaller. The improvement on sky survey data are showing more stronger lower limit on the sum of neutrino masses ~ 0.6 eV or more smaller, which leads to see-saw scales equal or very close to the GUT scale. If the see-saw scale is at the GUT scale, the RG evolved neutrino masses at the EW scale are fitted well over the allowed ranges obtained from sky survey data [7] as well as the mass squared differences from neutrino oscillation experiments. Moreover, this range can also be accessible in future double beta decay experiments [8] and in KATRIN experiment [9].

Here, we consider the hypothesis of equal quark and lepton mixings and hierarchical (normal) neutrino masses at GUT scale as in [4]. In this scenario we find a correlation between the upper limit on the sum of the active neutrino masses $(\sum_i m_{\nu_i})$ and lower bound on tan β as the quark-lepton unification fixes the lower limit on tan β (discussed later) for a given upper limit on $\sum_i m_{\nu_i}$. Once lower limit on tan β is fixed, lower bounds on sparticle masses are fixed from EWSB condition. It determines the stable the electroweak symmetry breaking minima and fixes μ^2 :

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$$\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan\beta}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2.$$
 (1)

If $\mu^2 < 0$, Higgsino mass is imaginary and the EWSB minima becomes unstable. The RGE of Higgs mass parameters $m_{H_u}^2$ and $m_{H_u}^2$ strongly depends on the sparticle masses and tan β . This leads to a correlation between lower limit on tan β and lower bounds on sparticle masses at weak scale.

For $\sum_i m_{\nu_i} \leq 1$ eV (constrained from large scale structure of universe), sparticle masses are $\gtrsim 1$ TeV (consistent with LHC bounds) and only $\mu > 0$ is allowed (supported by (g-2) of muon). Again, a small change in lower limit on θ_{13} from zero ($\approx \theta_C / \sqrt{2}$ after Daya Bay and RENO experiments) leads to a large increase in lower limits on sparticles masses ($\gtrsim 2$ TeV) from quark-lepton unification, which is also the case if recently discovered boson at LHC with mass around 125 GeV is the Higgs boson. We present our result for constrained minimal supersymmetric extension of the SM (CMSSM) [10]. In our study the running of neutrino parameters are exact as they are coupled with the running of minimal supersymmetric standard model (MSSM) parameters using the ISASUGRA program of the ISAJET package (V7.81) [11].

2. Models for degenerate neutrino masses at high scale

The most natural way to understand the smallness of neutrino mass is the see-saw mechanism. Here, the neutrino mass matrix is generated by the effective dimension 5 Weinberg operator [12]. In conventional type I see-saw [13], the SM is extended with additional heavy right handed neutrinos (which are not connected with the left handed fermions) and the mass matrix can be written as

$$M_{\nu} = -M_D (M_N)^{-1} M_D^T;$$
(2)

where, M_N is the mass matrix of right handed neutrinos and M_D is the Dirac neutrino mass matrix. Here, the neutrino masses are expected to be hierarchical in the similar way to the quark masses.

However, in case of type II see-saw [14] the SM model is extended by a charged Higgs triplet Δ and the neutrino mass matrix can be expressed as

$$M_{\nu} = Y_{\Delta} \langle \Delta \rangle. \tag{3}$$

Now, the most general neutrino mass matrix can be written as

$$M_{\nu} = Y_{\Delta} \langle \Delta \rangle - M_D (M_N)^{-1} M_D^T.$$
⁽⁴⁾

The Yukawa coupling matrix Y_{Δ} depends on high scale physics and it is unconstrained by SM data. One can therefore choose it to be unit matrix. If one considers the neutrino mass matrix dominated by the first term, then the neutrino masses are quasi-degenerate in conjunction with the lepton mixing angles close to quark mixing angles.

The realization of above type II see-saw scenario has been shown to be achieved in GUT models with gauge group $SU(2)_L \times$ $SU(2)_R \times SU(4)_C$ and with an additional S_4 global symmetry in [4]. This is not an adhoc assumption, while the gauge group can be a subgroup of number of GUTs like SO(10), E_6 , SO(18), etc.

Here, we have not considered the running of the parameters between two scales for decoupling of heavy fields as the values of neutrino parameters are considered as the input at the lowest scale of decoupling of Heavy fields and this lowest scale is assumed as the GUT scale.

3. Renormalization group evolution

The solution of the coupled RGEs for neutrino parameters along with SUSY parameters are obtained by an iterative cyclic process

(weak-to-GUT and then GUT-to-weak) with GUT boundary conditions following CMSSM [10]. The neutrino parameters are also set at GUT scale. The Higgsino mass μ and the soft Higgs bilinear term *B* are fixed from radiative electroweak symmetry breaking (REWSB). This has been done by the following steps. First, we set the gauge couplings and Yukawa couplings at EW scale and run only these couplings up to GUT scale (where g_1 and g_2 meet) setting other required mass parameters at SUSY breaking scale with approximate values. Now, we set the GUT boundary conditions for neutrino and SUSY parameters; put μ , B = 0 (one can also put arbitrary values), and run down to weak scale. After adding the loop corrections to $m_{H_u}^2$ and $m_{H_d}^2$ we calculate μ and *B* from EWSB condition. Taking these μ and *B* values as well as the RGE evolved SUSY parameters and neutrino parameters, we run up to GUT scale and put the GUT values of μ and B as they come and reset all other parameters at the GUT scale as earlier. We iterate this process until all parameters converge to a certain tolerance. The iteration are needed as the μ and *B* are involved in the running of other parameters.

To understand the results the analytical formula for RGE of neutrino mixing angles and masses [6] are very important:

$$\dot{\theta}_{12} = -\frac{Cy_{\tau}^{2}}{32\pi^{2}}\sin 2\theta_{12}s_{23}^{2}\frac{|m_{1}e^{i\varphi_{1}} + m_{2}e^{i\varphi_{2}}|^{2}}{\Delta m_{21}^{2}} + O(\theta_{13}), \quad (5a)$$

$$\dot{\theta}_{13} = \frac{Cy_{\tau}^{2}}{32\pi^{2}}\sin 2\theta_{12}\sin 2\theta_{23}\frac{m_{3}}{\Delta m_{32}^{2}(1+\zeta)} \times \left[m_{1}\cos(\varphi_{1}-\delta) - (1+\zeta)m_{2}\cos(\varphi_{2}-\delta) - \zeta m_{3}\cos\delta\right] + O(\theta_{13}), \quad (5b)$$

$$\dot{\theta}_{23} = -\frac{Cy_{\tau}^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m_{32}^2} \\ \times \left[c_{12}^2 |m_2 e^{i\varphi_2} + m_3|^2 + s_{12}^2 \frac{|m_1 e^{+i\varphi_1} + m_3|^2}{1 + \zeta} \right] + O(\theta_{13}),$$
(5c)

where $\zeta = \Delta m_{21}^2 / \Delta m_{32}^2$, $\Delta m_{21}^2 = m_2^2 - m_1^2$, $\Delta m_{32}^2 = m_3^2 - m_2^2$; C = 1 in MSSM and -3/2 in SM.

From the RGE it is clear that large value of y_{τ} (which requires large tan β), quasi-degenerate neutrino masses, and normal hierarchical mass pattern are needed to generate large radiative magnification of the 1–2 and 2–3 mixing angles at electroweak scale from quark-lepton unified mixing angles at the GUT scale.

4. Result

The sparticle masses are complicated function of soft SUSY breaking parameters which are obtained at weak scale through running of their coupled RGE from GUT scale. We find the allowed parameter space (and lower limits on masses) by scanning randomly over the following ranges of the parameters; common scalar mass (m_0): 0.05–3 TeV, common gaugino mass ($m_{1/2}$): 0.05–3 TeV, common trilinear coupling (A_0): $-3m_0$ to $+3m_0$, sign(μ): ± 1 , and tan β (ratio of two vacuum expectation values of H_u and H_d): 35–70. We set the ranges for neutrino masses $m_1^0, m_2^0, m_3^0 = 0-0.7$ eV, neutrino mixing angles $\theta_{12}^0 = 0.22(1 \pm x_1), \theta_{13}^0 = 0.0039(1 \pm x_2), \theta_{23}^0 = 0.034(1 \pm x_3)$, CP phase $\delta_{CP}^0 = 60^\circ(1 \pm x_4)$, and Majorana phases $\varphi_1^0, \varphi_2^0 = 0-360^\circ$. We have chosen x_i s (uncertainties in the unification) randomly within the range $0 \le x_i \le 40\%$ (i = 1, 2, ...) as an uncertainty due to approximate evolution [15] of CKM parameters. We have checked varying the upper limits of x_i s from 30% to 50% that the results do not change drastically; the bounds (discussed later) become gradually stronger as the upper limits of x_i s are decreased.



Fig. 1. The lower bound on tan β for a given $\sum_i m_{\nu_i}$, and the allowed parameter space for Majorana phases in $\varphi_1 - \varphi_2$ plane at the weak scale for both sign of μ .

We set the experimental bounds obtained from LEP data: $m_h >$ 114.5 GeV, $m_{\tilde{\chi}^{\pm}} > 103$ GeV [16] as these masses can be dominated by the value of μ . If we withdraw the LEP bounds, the relatively lower sparticle masses are allowed, but the correlation of the bounds (discussed later) with $\sum_{i} m_{\nu_i}$ remains. We consider only the points in the parameter space that can produce neutrino oscillation parameters at weak scale within the 3σ range obtained from global-fit [17]: $\sin^2 \theta_{23} \simeq 0.52^{+0.12}_{-0.13}$; $\sin^2 \theta_{12} \simeq 0.312^{+0.048}_{-0.042}$, and $\sin^2\theta_{13} \lesssim 0.039$ (the case for nonzero bound on θ_{13} has been discussed later). It is expected that the addition of threshold corrections to the mass squared differences [18] may restrict the parameter space as it restricts the choices of m_{ν_i} s for a given $\sum_i m_{\nu_i}$. We have studied the unification with and without threshold corrections varying the ranges of mixing angles and mass squared differences. But, these are not very significant in this scenario with normal slepton mass hierarchy and no significant change in result is observed as we have used large allowed ranges for all parameters in our scanning (both neutrino parameters as well as SUSY parameters), where the parameters are chosen randomly. We present the plots for Δm_{21}^2 : 5-10 × 10⁻⁵ eV² and Δm_{32}^2 : 2-3 × 10⁻³ eV², respectively, considering the threshold corrections. Obviously, more stronger constraints are obtained for narrower ranges of oscillation parameters (discussed later).

The generation of neutrino mixing angles at EW scale in the ranges allowed by global-fit of neutrino oscillation data needs large radiative magnifications and demands very high value of y_{τ} . As the ranges of the mixing angles at present are very narrow, it almost fixes y_{τ} and consequently determines the lower bound on $\tan \beta$ for a given $\sum m_{\nu_i}$ at the EW scale. As $\sum_i m_{\nu_i}$ is lowered, higher value of y_{τ} is required and it demands more larger value of $\tan \beta$. This is shown in the first plot of Fig. 1.

The solar and atmospheric mass squared differences are different by two order of magnitude as well as the magnification for solar angle is ~ 3 and for atmospheric angle is ~ 20 . To accommodate all parameters in the experimentally allowed ranges for a given neutrino mass scale the Majorana phases are constrained in very narrow regions (see Fig. 1). This can be understood from Eq. (5a) and from Eq. (5c).

The upper bound on tan β is either fixed from REWSB or from the LEP bounds on $m_{\tilde{\chi}^{\pm}}$ or m_h (which are lowered for smaller μ values). As tan β increases $m_{H_d}^2$ decreases through RG evolution; and it can even be negative. This leads to smaller μ at larger tan β . At more higher tan β , μ^2 becomes negative as the minima of Higgs potential become unstable and then REWSB becomes impossible. In case of $\mu < 0$, the loop correction to $m_{H_d}^2$ leads to a more lower value compared to $\mu > 0$ and we find an upper limit on tan $\beta \lesssim 55$ from REWSB. This restricts the increase in y_{τ} and consequently leads to a lower bound on $\sum_i m_{\nu_i} \approx 1$ eV, which is strongly disfavored by the present cosmological data [7]. On the other hand, for $\mu > 0$ one can increase tan β up to 65 leading to a decrease in $\sum_{i} m_{\nu_i} \approx 0.6$ eV, which is very highly favored by sky survey data [7].

The REWSB and the value μ^2 depends on the GUT scale values of $m_{H_u}^2$ and $m_{H_d}^2$ as well as on other soft mass parameters (which change the values of $m_{H_u}^2$ and $m_{H_d}^2$ through RGE and loop corrections). Again, for a given $\tan \beta$, y_{τ} depends on the value of μ , gaugino mass parameters and scalar mass parameters through radiative corrections at EW scale. This leads to strong lower bounds on sparticle masses correlated with the upper limit of $\sum_i m_{\nu_i}$. If $\mu > 0$ and $\sum_i m_{\nu_i} \leq 1$ eV, then we find $m_{\tilde{g}} \gtrsim 1$ TeV, $m_{\tilde{t}_1} \gtrsim 0.7$ TeV, and $m_{\tilde{e}_L} \gtrsim 0.5$ TeV. The lower bounds on sparticle masses depend only on upper limit on $\sum_i m_{\nu_i}$ as other parameters are scanned over their whole ranges. All these bounds follow the recent LHC results [2]; and again, the neutrino mass limit $\sum_i m_{\nu_i} \lesssim 1$ eV is strongly favored by the sky survey data.

In Fig. 2 we present the allowed parameter space in the planes of $m_{\tilde{e}_L} - m_{\tilde{\tau}_1}$ and $m_{\tilde{g}} - m_{\tilde{t}_1}$, respectively. For each plot the allowed points are separated for three $\sum_i m_{\nu_i}$ ranges: < 0.7 eV, 0.7–1.0 eV and > 1 eV, respectively, to show the dependence of lower bounds of sparticle masses on the upper bound of $\sum_i m_{\nu_i}$. As an example, in case of $\mu > 0$, we find $m_0 \gtrsim 1.5$ TeV only when $m_{1/2} \gtrsim 0.4$ TeV for whole range of $\sum_i m_{\nu_i}$ (0.6–2 eV); but, if $\sum_i m_{\nu_i} \lesssim 1$ eV, then $m_{1/2} \gtrsim 0.5$ TeV over whole range of m_0 (0–3 TeV). We have randomly chosen all the parameters and there is no correlation among them. So, these bounds depends only on upper bound of $\sum_i m_{\nu_i}$. From these plots one can easily find the values of individual sparticle masses and the differences $m_{\tilde{e}_L} - m_{\tilde{\tau}_1}$ or $m_{\tilde{g}} - m_{\tilde{t}_1}$, etc., which have definite pattern and one can predict the interesting possible collider signatures at LHC.

At this large value of $\tan \beta$, $\tilde{\tau}_1$ can be the lightest supersymmetric particle (LSP) in some cases depending on the choices of other parameters, mainly A_0 and $\operatorname{sign}(\mu)$. Another consequences of such high values of $\tan \beta$ with positive sign of μ are successful explanation of g - 2 of muon [19]. Again, for positive μ , $t - b - \tau$ unification is also possible [20].

The present global-fit of neutrino data [21] after Daya Bay and RENO experiments [3] gives $\sin^2 \theta_{13} > 0.017$ and $\theta_{13} \approx \theta_C / \sqrt{2}$. We have found that a small change in its lower limit leads to a large increase in lower limits on sparticles masses ($\gtrsim 2$ TeV) from quarklepton unification, which is the case if recently discovered boson at LHC with mass around 125 GeV is the Higgs boson (see Fig. 2). However, since the RGE considered for CKM parameters are approximate, we have not represented the bounds for different lower bounds on θ_{13} . This would be more precise and reliable when one considers exact running of CKM parameters.



Fig. 2. The allowed region in the plane of $m_{\tilde{e}_L} - m_{\tilde{t}_1}$, and $m_{\tilde{g}} - m_{\tilde{t}_1}$, respectively. The allowed points for $\sum_i m_{\nu_i} < 0.7$ eV, 0.7–1 eV and > 1 eV are represented by solid box, circle and triangle, respectively. The bigger point-size denotes the points with further constraints on Higgs mass within the range 125 ± 1 GeV. The shapes of the points signify the same as described earlier.

As the allowed range of $\tan \beta$ (which is determined mainly to fix y_{τ} within an interval) is very narrow there appears a definite pattern in the differences between two sparticle masses (see Fig. 2). In case of other supersymmetry breaking scenarios one can also expect similarly strong bounds on sparticle masses from the quark-lepton unification as one always needs large $\tan \beta$ within a narrow range. But, the differences in the sparticle masses will then have different definite pattern due to different GUT boundary conditions. The difference between the patterns becomes more prominent when the allowed range of $\tan \beta$ is very narrow. This may make the possibility to distinguish different models.

5. Conclusion

The quark-lepton unification not only satisfies and/or predicts all experimental results available till now, but also shows that both results from neutrino experiments and LHC are complementary. The quark-lepton unification leads to a strong constraint on the parameter space along with very strong correlations between the upper limit on $\sum_{i} m_{\nu_i}$ and the lower limits on sparticle masses. This arises due to the fact that there exists a lower limit on $\tan \beta$ for a given $\sum_i m_{v_i}$ when one demands quasi-degenerate neutrino masses at the GUT scale (which can be generated in GUT models with type II see-saw scenario with an additional family symmetry). As $\sum_{i} m_{\nu_i}$ decreases lower limit on tan β increases. For a given high value of $\tan \beta$ there appears very strong lower bounds of sparticle masses form EWSB. As $\tan \beta$ increases lower bounds of sparticle masses increase significantly. We find that $\tan \beta \gtrsim 55$ is not allowed for $\mu < 0$ (as μ^2 becomes negative and EWSB minima is unstable) and it constrains $\sum_i m_{\nu_i} \gtrsim 1$ eV.

For $\sum_{i} m_{\nu_i} \lesssim 1$ eV (constraint from large scale structure of universe) only $\mu > 0$ (which is favored by (g-2) of muon) is allowed and there exists strong lower bounds on sparticle masses \gtrsim TeV (which are also the bounds from LHC). A small change in lower limit of θ_{13} from zero ($\theta_{13} \approx \theta_C / \sqrt{2}$ after Daya Bay and RENO results) leads to a large increase in lower limits on sparticles masses ($\gtrsim 2$ TeV) from quark-lepton unification, which is also the case if recently discovered boson at LHC with mass around 125 GeV is the Higgs boson.

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