The effect of bonding layer properties on the dynamic behaviour of surface-bonded piezoelectric sensors

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A B S T R A C T

The dynamic behaviour of piezoelectric sensors depends on the bonding condition along the interface between the sensors and the host structure. This paper provides a comprehensive theoretical study of the effect of the bonding layer on the coupled electromechanical characteristics of a piezoelectric sensor bonded to an elastic substrate, which is subjected to a high frequency elastic wave. A sensor model with a viscoelastic bonding layer, which undergoes a shear deformation, is proposed to simulate the two dimensional electromechanical behaviour of the integrated system. Analytical solution of the problem is provided by using Fourier transform and solving the resulting integral equations in terms of the interfacial stress. Numerical simulation is conducted to study the effect of the bonding layer upon the dynamic response of the sensor under different loading frequencies. The results indicate that the modulus and the thickness of the bonding layer have significant effects on sensor response, but the viscosity of the bonding layer is relatively less important.

1. Introduction

Newly developed piezoelectric materials with strong electromechanical coupling, such as piezoceramics, have attracted significant attention from both research and industrial communities because of their potential usage as sensors/actuators in the design of the so-called smart structures (Gandhi and Thompson, 1992; Banks et al., 1996; Chee et al., 1998; Cohen, 2000; Boller, 2000). These new piezoelectric sensors are low in cost, compact in size, highly sensitive over a large range of frequency, and can be easily fabricated into different desired shapes. As a result, they are suitable for real-time health monitoring of structures (Park et al., 2005, 2006; Dalton et al., 2001; Giurgiutiu et al., 2002; Fukunaga et al., 2001). As sensors, piezoelectric patches are usually bonded to structures to measure the strain by transforming mechanical deformation into electric voltage. The existence of sensors may disturb the mechanical field to be measured, especially for the case where the stiffness of the piezoelectric sensors, piezoceramic ones for example, is comparable to that of the host structure. In addition, the bonding condition between the sensor and the host structure will also affect the performance of the sensor. It becomes, therefore, an important issue to study the coupled electromechanical behavior of these sensors with bonding layers to reliably evaluate the relation between the measured signal and the local mechanical deformation.

Due to the presence of material discontinuity between the piezoelectric sensors and the host structure, complicated local electromechanical fields will be generated near the edge of sensors. The local stress distribution will affect the load transfer between the sensor and the host structure, and therefore influence the performance of the sensor. To study the static load transfer between the piezoelectric elements and the host structure, simplified sensor/actuator models have been established.
A beam-like structure with surface-bonded and embedded thin-sheet piezoelectric elements is first analyzed to study the load transfer (Crawley and de Luis, 1987). In this analysis, the axial stress in the piezoelectric elements is assumed to be uniform across their thickness. A Bernoulli–Euler model of a piezoelectric thin-sheet bonded to a beam is further developed by considering the linear stress distribution across the thickness of the piezoelectric element (Crawley and Anderson, 1990). A refined actuator model based on the plane stress condition is studied to investigate the electromechanical behavior of a beam with symmetrically surface-bonded actuator patches (Lin and Rogers, 1993b). Plate and shell models have also been extensively used in modelling the electromechanical behavior of piezoelectric structures (Dimitriadis et al., 1991; Tzou and Tseng, 1991; Mitchell and Reddy, 1995; Banks and Smith, 1995; Han and Lee, 1998). The static local stress field near a thin-sheet piezoelectric element attached to an infinite elastic medium is studied to investigate the stress concentration and the load transfer between the piezoelectric element and the host medium (Wang and Meguid, 2000). Similar analysis is also conducted to determine the static electromechanical field of a piezoelectric layer bonded to an elastic medium with both interfacial and normal stresses being considered (Zhang et al., 2003a,b).

A one dimensional actuator model has recently been developed to study the dynamic load transfer between piezoelectric actuators and elastic host medium when the system is subjected to in-plane mechanical and electrical loads (Wang, 1999). This actuator model is further modified to study problems with varying electric field distribution along the actuators (Wang and Huang, 2006), with the effect of both the longitudinal and the transverse deformations of the actuators being considered. Similar to this actuator model, a one dimensional sensor model has also been established to study the relation between the sensor response and the deformation of the host medium under static and dynamic loads (Wang and Huang, 2006).

In these studies, however, the sensors/actuators are assumed to be perfectly bonded to the host structures. Typically, piezoelectric sensors are bonded to the host structure by epoxy or conductive epoxy. As a result, a bonding layer will be generated. Since the modulus of the bonding layer is usually lower than that of the sensors and the host structure, it may significantly affect the local stress distribution. Recently, several studies have been conducted to investigate the effect of bonding layers for piezoelectric structures under low frequency vibration. The result indicates that the controllability of piezoelectric structures can suffer when the mechanical properties of the bonding layer are ignored in their dynamic analysis (Akella et al., 1994; Nakra, 2005). In the study of the vibration properties of composite beams with attached sensors and actuators (Park et al., 2000), it is found that, by selecting the proper adhesives, the controllability of cantilever beams can be optimized. The influence of the bonding layer parameters on the active damping response of a controlled simply-supported beam is analyzed using attached piezoelectric sensors under low frequency harmonic loads (Pietrzakowski, 2001). A three dimensional sensor model, in which the bonding medium is considered as an elastic layer with a finite stiffness in the thickness direction is also proposed to study the effect of the shear stress in the bonding layer on the sensing effectiveness of piezoelectric patches (de Faria, 2003). These work contributed significantly to the study of the effect of the bonding layer in these integrated structures under low frequency loads. For the cases where high frequency loads are applied, the inertia effect must be considered and the bonding layer may play a more important role because of the fact that a wave with a shorter wavelength will be more sensitive to interfacial layers.

It is, therefore, the objective of the current paper to provide a comprehensive theoretical study of the effect of the bonding condition on the dynamic behavior of surface-bonded piezoelectric sensors. An integrated model containing a piezoelectric thin-sheet, a viscoelastic bonding layer and an elastic medium (host) is proposed to evaluate its dynamic property under different loading frequencies. Numerical simulation is conducted to simulate the effect of the geometrical and material property of the system, especially that of the bonding layer upon the coupled response of the sensors.
2. Formulation of the problem

The system considered is a homogeneous and isotropic elastic medium with a surface-bonded piezoelectric sensor. The sensor is attached to the host medium through a thin bonding layer, as illustrated in Fig. 1. Plane strain deformation is assumed to simulate the case where the width of the sensor is large in comparison with its length. The dimension of the sensor is assumed significantly smaller than that of the host structure. Therefore, the host structure is modeled as a semi-infinite medium. The lengths of the sensor and the bonding layer are denoted as 2a, and the thickness of the sensor and the bonding layer are denoted as h and h₀, respectively. It is assumed that the poling direction of the piezoelectric sensor is along its thickness, parallel to the z-axis shown in Fig. 1. The system is subjected to a harmonic incident wave with a frequency \( \omega \) and an incident angle \( \theta_0 \). For the steady state response of the system, the time factor \( \exp(-i\omega t) \), which applies to all the field variables, will be suppressed in the following discussions.

This study will focus on a thin-sheet sensor, with relatively small thickness in comparison with its length. Therefore, the axial stress and strain can be assumed to be uniform across the thickness of the sensor. Since the thickness of the bonding layer is usually smaller than that of the sensor, the same assumption is also used for the bonding layer. The interfacial shear stress distributed between different layers, and the longitudinal displacements on the upper and lower surface of the bonding layer are shown in Fig. 2.

2.1. The bonding layer

The bonding layer is the medium between the sensor and the host structure. Its shear modulus, thickness, and coefficient of viscosity, will govern the property of the layer.

The real shear stress \( \tau \) distributed in the layer is determined by the following constitutive relation,

\[
-\tau = \mu_b \dot{\varepsilon}_y + c_b \ddot{\varepsilon}_y
\]

where \( \dot{\varepsilon}_y \) is the real longitudinal strain of the bonding layer, and \( \mu_b \) and \( c_b \) are the shear modulus and the coefficient of viscosity of the bonding layer, respectively, with the subscript ‘b’ representing the bonding layer. In Eq. (1),

\[
\begin{align*}
\tau &= \tau \exp(-i\omega t) \\
\dot{\varepsilon}_y &= \frac{u^+ - u^-}{h'} \exp(-i\omega t) \\
\ddot{\varepsilon}_y &= \frac{\delta}{\delta t} \left[ \left( \frac{u^+ - u^-}{h'} \right) \exp(-i\omega t) \right]
\end{align*}
\]

where \( \tau \) is the magnitude of the shear stress distributed in the layer, \( u^+ \) and \( u^- \) are the magnitudes of the longitudinal displacements of the upper and lower bonding layer surfaces, respectively. Since \( u^+ \) and \( u^- \) are not functions of time, Eq. (1) can be rewritten as:

\[
-\tau = \left( \frac{\mu_b}{h'} - \frac{i\omega c_b}{h'} \right) (u^+ - u^-)
\]

It should be mentioned that when \( h' \) approaches zero singularity may occur. In the numerical simulation, both sides of the equation is multiplied by \( h' \) to remove this singularity. Because of the continuity of the displacements, \( u^+ \) and \( u^- \) also represent the longitudinal displacements of the lower surface of the sensor layer and the upper surface of the host medium,
respectively. Therefore, their expression can be obtained by analyzing the dynamic behaviour of the sensor and the host medium.

2.2. Modelling of the sensor

The current study focuses on the case where the system is subjected to a high frequency elastic wave, which has a wave length comparable to the length of the sensor. In this case, the inertia effect of the sensor must be considered (Wang and Huang, 2006), and the equation of motion of the sensor can be expressed as,

\[
\frac{d\sigma_y}{dy} + \tau(y)/h + \rho_o \omega^2 u^s_y = 0
\]

(6)

where the superscript ‘s’ represents the sensor, \(\sigma^s_y\), \(\rho_s\), and \(u^s_y\) are the axial stress along the sensor, the mass density, and the axial displacement along the sensor, respectively.

The electromechanical behavior of the piezoelectric sensor can be described by

\[
\sigma^s_y = E_s \frac{\partial u^s_y}{\partial y} - e_s E_z
\]

(7)

\[
D_z = e_s \frac{\partial u^s_y}{\partial y} + \lambda_s E_z
\]

(8)

where \(E_s\), \(e_s\), and \(\lambda_s\) are effective material constants given in Appendix A, \(D_z\) and \(E_z\) represent the electric displacement and the electric field intensity, respectively.

Since there is no external force applied at the two ends of the sensor, traction free boundary condition can be applied as follows,

\[
\sigma^s_y = 0, \quad |y| = a.
\]

(9)

In addition, the sensor will be assumed to operate in an open-loop mode with no external charge supplied to it (Lee and Moon, 1989). Therefore, the electric displacement across the sensor will be zero,

\[
D_z = 0.
\]

By using Eqs. (7)–(10), Eq. (6) can be solved, and the longitudinal displacement of the sensor can be obtained as,

\[
u^s_y(y) = - \frac{\cos{k_s(a + y)}}{k_s E_s \sin 2k_s a} \int_a^y \cos{k_s(\xi - a)}|p(\xi)| d\xi - \frac{1}{k_s E_s} \int_a^y \sin{k_s(y - \xi)}|p(\xi)| d\xi \quad |y| < a
\]

(11)

where

\[
p(y) = \frac{\tau(y)}{E_s}, \quad k_s = \frac{\omega}{c_s}, \quad c_s = \sqrt{E_s/\rho_s}, \quad E_s = E_s + e_s^2/\lambda_s
\]

(12)

with \(k_s\) and \(c_s\) being the wave number and the axial wave speed of the sensor, respectively.

2.3. Dynamic behaviour of the host medium

The dynamic plane strain displacement field in a homogeneous isotropic elastic medium is governed by Achenbach (1973),

\[
u_y = \frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial z}, \quad u_z = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y}
\]

(13)

where \(\Phi\) and \(\Psi\) are two displacement potentials which satisfy

\[
(\nabla^2 + k^2)\Phi = 0, \quad (\nabla^2 + k^2)\Psi = 0
\]

(14)

in which the Laplace operator \(\nabla^2\) stands for \(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\) and \(K\) and \(k\) are two wave numbers defined as

\[
K = \omega/c_L, \quad k = \omega/c_T
\]

(15)

with \(c_L\) and \(c_T\) being the longitudinal and transverse shear wave speed of the elastic medium, respectively.

The stress field generated inside the host medium can be divided into two parts, as shown in Fig. 3. The first is caused by the incident wave in the host medium with a traction free boundary and the second is caused by surface shear stress \(\tau\) resulted from the sensor. For the first subproblem, once the incident wave reaches the free surface of the host medium, it will be reflected and the displacement along the surface of the host medium \(u^s\) can be easily obtained using the theory of wave propagation and the boundary condition, as given in Appendix B. For the second subproblem, the displacement induced by \(\tau\) can be determined by solving Eq. (14), as described in the following discussion.

Using the following Fourier transform
2.4. Dynamic load transfer

the general solution of Eq. (14) for the second subproblem can be expressed as

\[ \Phi^*(s,y) = A(s) e^{i\omega z}, \quad \Psi^*(s,y) = B(s) e^{i\omega z} \] (17)

from which the displacement components can be obtained as

\[ u_i^* = -iA(s) e^{i\omega z} + \beta B(s) e^{i\omega z} \] (18)
\[ u_z^* = A(s) e^{i\omega z} + iB(s) e^{i\omega z} \] (19)

where (*) represents Fourier transform. \(A(s)\) and \(B(s)\) are two unknown functions of \(s\), and \(\alpha\) and \(\beta\) are given by

\[ \alpha = \left\{ \begin{array}{ll} \sqrt{s^2 - K^2} & |s| > K \\ -i\sqrt{K^2 - s^2} & |s| < K \end{array} \right. \]
\[ \beta = \left\{ \begin{array}{ll} \sqrt{s^2 - k^2} & |s| > k \\ -i\sqrt{k^2 - s^2} & |s| < k \end{array} \right. \] (20)

which ensure that the induced stress field satisfies the radiation condition of the problem.

By making use of the general solution of \(u_i^*\) and \(u_z^*\) and the following boundary conditions

\[ \begin{cases} \sigma_{\alpha\alpha}(y,0) = -\tau, & |y| < a, \\ \sigma_z(y,0) = 0 \end{cases} \] (21)

the dynamic displacement of the second subproblem along the interface between the bonding layer and the host medium can be determined. The total displacement can then be obtained by superimposing the solutions of both parts, which can be expressed as

\[ u_b^*(y,0) = \frac{1}{\pi \mu_h} \int_0^\infty \left[ \frac{\beta k^2}{(\omega^2 - k^2)^2 - 4s^2 x' \beta} + \frac{\lambda_0}{2 \omega} \right] \int_{-a}^a \tau(\xi) \cos(s(y - \xi)) \, d\xi \, ds + \frac{\lambda_0}{\pi \mu_h} \int_{-a}^a \int_{-a}^a \tau(\xi) \frac{1}{y - \xi} \, d\xi \, dy + u_p^*, \quad |y| < a \] (22)

where the superscript and subscript ‘h’ represents the host medium, \(\mu_h\) is the shear modulus of the matrix, \(\lambda_0 = 2(1 - \nu)\) with \(\nu\) being the Poisson’s ratio.

2.4. Dynamic load transfer

By substituting Eqs. (11) and (22) into Eq. (5), the governing equation for the sensor system with a bonding layer can be rewritten as

\[ \frac{\cos[k_a(y + a)]}{k_a E_s \sin 2k_a a} \int_{-a}^a \cos[k_a(y - \xi)] p(\xi) \, d\xi - \frac{1}{k_a E_s \sin[ka(y - \xi)] p(\xi) \, d\xi} - \frac{1}{\pi \mu_h} \int_{-a}^a \int_{-a}^a \tau(\xi) \cos[s(y - \xi)] \, d\xi \, ds \]
\[ + \frac{\lambda_0}{2 \pi \mu_h} \int_{-a}^a \int_{-a}^a \tau(\xi) \frac{1}{y - \xi} \, d\xi \, dy + \frac{h'}{\mu_h - \nu \mu_a} \tau(y) = u_b^*(y), \quad |y| < a \] (23)

Eq. (23) provides an integral equation in terms of the shear stress \(\tau(y)\). To solve this equation, \(\tau(y)\) will be expanded in terms of the first kind of Chebyshev polynomials, \(T_j\), as

\[ \tau(y) = \sum_{j=0}^\infty c_j T_j(y/a) / \sqrt{1 - y^2/a^2} \] (24)

where

\[ T_j(y/a) = \cos(j\theta), \quad \cos \theta = y/a \] (25)
From Eq. (5), it can be concluded that, unlike the cases with perfectly bonded sensors (Wang and Huang, 2006), the shear stress $\tau$ will not be singular, since displacements $u'$ and $u''$ will always be bounded. The singularity involved in Eq. (24) is for mathematical reasons, i.e. simplifying the integral Eq. (23) and satisfying the orthogonality of Chebyshev polynomials.

If $\tau$ is expanded into $N$ terms, and Eq. (23) is satisfied at the following collocation points along the sensor

$$y = a \cos \left( \frac{l}{N+1} \pi \right), \quad l = 1, 2, \ldots, N, \quad |y| < a$$

Eq. (23) becomes $N$ linear algebraic equations in terms of $\{c\} = \{c_0, c_1, \ldots, c_{N-1}\}^T$, the coefficient of Chebyshev expansion. These $N$ equations can be written in the form of:

$$[Q]\{c\} = \{F\}$$

where $[Q]$ is a resulting known matrix, and $\{F\}$ represents the applied load matrix, which are given in Appendix C. By solving Eq. (27), $\{c\}$ can be obtained. It can then be used to determine the axial displacement $u_y^0$ and the axial strain $\varepsilon_y^0$ of the sensor.

### 3. Results and discussion

In this section, the electromechanical behavior of the sensor system subjected to an incident longitudinal wave is considered. The attention will be focused on the strain distribution along the sensor, which represents the load transfer between the sensor and the host medium. The effect of material properties of the bonding layer, the loading frequency, the material combination, and the sensor geometry upon the load transfer will be studied. Different material combinations of the sensor and the host structure are represented by the material mismatch factor $q$, which is defined as

$$q = \frac{\pi E_h}{2(1 - v_h^2)E_b}$$

The sensor geometry parameter, denoted as $v$, is defined as the ratio of the sensor length to its thickness:

$$v = \frac{a}{h}$$

The material constants of a typical piezoceramic (PZT) sensor are shown in Table 1 (Park, 1990), and the material parameters of the host medium and the bonding layer are shown in Table 2 (Wang and Huang, 2006). The elastic constants of the bonding layer are assumed based on existing material constants (Park et al., 2000). The mass densities of the sensor and the host medium, $\rho_s$ and $\rho_h$, are assumed to be 2700 Kg/m$^3$ (Wang and Huang, 2006). The geometry of the sensor is assumed to be $a = 1.0$ cm, $h = 0.5$ mm. The length of the bonding layer is the same as the sensor, and the range of its thickness $h/a$ is from 0 to 0.032.

It should be mentioned that the solution of the current problem is complex in general cases. The resulting physical strain should be the real part of the obtained strain after multiplying $\exp(-i\omega t)$. In the following discussion, only the magnitude of the physical strain is considered, which is the most important parameter of the problem. The convergence of the solution using Chebyshev polynomials, as shown in Eq. (26), has been carefully evaluated. The number of terms of Chebyshev polynomials is selected to be 64, with which the convergence of the results for all the cases considered is ensured.

For the case where the loading frequency is so low that the typical wave length of the incident wave is much longer than the length of the sensor, the problem could be treated as a static one. In the current analysis it is assumed that a constant static strain $\varepsilon_s^0$ is applied along the y-direction at infinity and the bonding between the sensor and the host medium is perfect.

Fig. 4 shows the effect of $v = a/h$ upon the amplitude of the strain ratio $A = |\varepsilon / \varepsilon_s^0| = |\varepsilon_y^0| / \varepsilon_s^0$ along the sensor, with the material constants of the sensor and the host being given by Tables 1 and 2. Significant effect of $v$ upon the strain ratio is observed, with higher value of $v$ corresponding to relatively higher strain ratio. A comparison with the result of the Finite Element analysis is also given in Fig. 4. Excellent agreement is observed for $v = a/h = 20$.

#### 3.1. Wave propagation inside the host medium

In general cases, an incident P-wave in the host medium, as shown in Fig. 5, can be expressed as (Achenbach, 1973)

$$U = A_0 \bar{d} \exp(ik_{y-s} |y| - ik_{z-s} |z|)$$

### Table 1

<table>
<thead>
<tr>
<th>Material properties of the PZT sensor</th>
<th>$c_{11}$</th>
<th>$c_{12}$</th>
<th>$c_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic stiffness parameters (Pa)</td>
<td>$13.9 \times 10^{10}$</td>
<td>$6.78 \times 10^{10}$</td>
<td>$7.43 \times 10^{10}$</td>
</tr>
<tr>
<td>Piezoelectric constants (C/m²)</td>
<td>$-5.2$</td>
<td>$15.1$</td>
<td>$12.7$</td>
</tr>
<tr>
<td>Dielectric constants (C/Vm)</td>
<td>$6.45 \times 10^{-9}$</td>
<td>$6.45 \times 10^{-9}$</td>
<td>$5.62 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
where \( \vec{d} \) represents the direction of the particle motion during the wave propagation and is given in Appendix B, \( A_0 \) is the magnitude of the wave, \( \theta_0 \) is the incident angle, and \( k \) represents the wave number of the incident wave, with \( ka = 1.0 \) corresponding to a loading frequency of 0.12 MHz. The response of the host medium without sensors can be determined by considering the traction free boundary condition at \( z = 0 \). The resulting displacement and strain on the free surface of the host

**Table 2**

Material properties of the host medium and the bonding layer

<table>
<thead>
<tr>
<th></th>
<th>Host medium</th>
<th>Bonding layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus ( E_h ) (Pa)</td>
<td>2.74 ( \times 10^{10} )</td>
<td>1.0 ( \times 10^{9} )</td>
</tr>
<tr>
<td>Poisson’s ratio ( v_h )</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Shear modulus ( \mu_b ) (Pa)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 4.** The static strain ratio.

**Fig. 5.** Reflection of a P-wave.
medium are denoted as $u'$ and $e_I$. When a sensor is attached to the host medium, the strain distribution along the sensor will be different from $e_I$, which can be obtained using the result given in Section 2,

$$
e_I(y) = \frac{\partial u'}{\partial y} = \frac{\sin[k_0(a + y)]}{E} \sin 2k_0 a \int_{-a}^{a} \cos[k_0(z - a)]p(z)\,dz = \frac{1}{E} \int_{-a}^{a} \cos[k_0(y - z)]p(z)\,dz \quad |y| < a. \quad (31)$$

3.2. Strain distribution along the sensor

The relation between the sensor strain $e_I$ and the strain to be measured $e_I$ can be evaluated using the amplitude of a dynamic strain ratio, $\kappa(y)$, defined as,

$$\kappa(y) = \frac{e_I(y)}{e_I(y)}, \quad |y/a| < 1 \quad (32)$$

which represents the percentage of deformation transferred from the host medium to the sensor.

3.2.1. The effect of the thickness of the bonding layer

From Eq. (5) it is observed that the effect of the shear modulus $\mu_b$ and the thickness of the bonding layer $h'$ is governed by one parameter, the stiffness of the bonding layer $\mu_b/h'$. For the convenience of discussion, $\mu_b$ value given in Table 2 will be used and $h'$ is varied in the following discussion to achieve different bonding layer stiffness. An incident angle $\theta_0 = 30^\circ$ will be used in the following discussion.

Fig. 6 shows the amplitude of the strain ratio along the sensor for the case where $ka = 1.0$ and for different bonding layer thickness $h' = 0, 10, 40, 80, 160$ and $320 \mu$m. The effect of viscosity is ignored. In this case, a significant deduction of the strain ratio occurs with increasing bonding layer thickness. At $y/a = -0.75$ for example, a deduction of 20% of the strain ratio is observed when $h' = 80 \mu$m, in comparison with the perfectly bonded case ($h' = 0$).

Fig. 7 shows the corresponding results of the strain ratio for a higher loading frequency $ka = 5.0$. Different from the low frequency case shown in Fig. 6, increasing bonding layer thickness results in an increase in the strain ratio in the central part of the sensor, $-0.5 < y/a < 0.35$ for example. This figure clearly shows the coupled effect of loading frequency and layer thickness upon the strain ratio distribution.

When $ka = 8.0$ as shown in Fig. 8, the increase of the strain ratio with increasing $h'$ can be observed for $0.3 < y/a < 1$, except for the case where the layer is very thick ($h' = 320 \mu$m). But under this high frequency, the distribution of the strain becomes very complicated.

3.2.2. The effect of the coefficient of viscosity of the bonding layer

From Eq. (5), it can be observed that, as another parameter effecting the stiffness of the bonding layer, the coefficient of viscosity also contributes to the load transfer between the sensor and the host medium. Different values of $c_b$, from 10 Pa s to 10,000 Pa s, are considered in the simulation to study its effect on the load transfer, with $c_b = 10$ Pa s representing very weak
viscosity, and $c_b = 10,000 \text{ Pa s}$ representing strong viscosity. The loading frequency is chosen to be $ka = 1.0$, and the thickness of the bonding layer is assumed to be $h' = 100 \mu m$. The results of the distribution of the amplitude of the dynamic strain ratio along the sensor are shown in Fig. 9.

It is interesting to mention that although changes in the amplitude of the strain distribution along the sensor can be observed with increasing $c_b$, the change is not significant. When $c_b = 10,000 \text{ Pa s}$, a stiff bonding layer is formed and the result is similar to that from a perfectly bonded sensor as shown by the first curve in Fig. 6.

3.2.3. The effect of material combination

The mismatch between the sensor and the host medium is represented by $q$, defined by Eq. (28). In the current study, the material constants of the sensor are fixed, while the Young’s Modulus of the host medium is adjusted to achieve different $q$ values. In previous examples, the selection of sensor and host medium results in $q = 0.3928$. For the convenience of calculation, $q = 0.1, 0.5, 1.0, 2.0, 5.0$ are selected in the following examples. Two interfacial conditions, $h' = 0$ and $100 \mu m$ are considered, and the loading frequency $ka = 1.0$ are selected.

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**Fig. 7.** Normalized strain distribution along the sensor ($ka = 4.0, q = 0.3928, v = 20.0$).

**Fig. 8.** Normalized strain distribution along the sensor ($ka = 8.0, q = 0.3928, v = 20.0$).
Fig. 10 shows the strain distribution along the sensor for different $q$ values, when the system is under perfect bonding condition ($h^\prime = 0$). With the decrease of the stiffness of the sensor (increasing $q$), the amplitude of the normalized strain increases. When $q$ reaches 5.0, the strain distribution curve becomes very flat, approaching 1.0 in most part of the sensor. This is because the disturbance of a soft sensor is relatively insignificant.

The corresponding result for the case where the bonding condition is not perfect, $h^\prime = 100$ l, is shown in Fig. 11. In this case, the thickness is selected to be 100 l, and the loading frequency remains the same, $ka = 1.0$. Again, significant effect of the material mismatch $q$ is observed. In comparison with the results shown in Fig. 10, the amplitude changes between two consecutive curves are more obvious. With increasing $q$ value, the amplitude of the normalized strain can be higher than one. For example, the maximum value is close to 1.25 when $q = 5.0$. Considering the fact that higher $q$ value corresponding to softer sensor, the current result indicates that, even when the host structure is much stiffer than the sensor, the effect of the bonding layer on the load transfer will still be very important.

3.3. Output voltage of the sensor

According to the relationship between voltage and electric field intensity, the voltage distribution along the sensor can be determined as

$$E(y) = \frac{V}{a} \left(1 - \frac{y}{a}\right)$$
When the upper and lower surfaces of the sensor form two electrodes, the total resulting voltage across the sensor can be obtained by averaging the voltage across the sensor obtained before, i.e.

$$V_{\text{out}} = \frac{1}{2a} \int_{-a}^{a} V(y) \, dy = \frac{e_s h}{2a} \int_{-a}^{a} \frac{\partial u_y}{\partial y} \, dy$$

(33)

Different thicknesses of the bonding layer are selected, and loading conditions with $ka = 0.0, 1.0, 2.0, 4.0, 8.0$ and $12.0$ are chosen to show the variation of the output voltage with the increase of thickness of the bonding layer under different loading frequencies. The static result with $ka = 0.0$ is determined by letting the frequency (or $ka$) approaching zero. In the examples considered in this subsection, the amplitude of the incident strain field is kept to be the same and the output voltage is normalized by the corresponding static result for the perfectly bonded sensor ($h' = 0, ka = 0.0$).

Fig. 12 shows the variation of the normalized output voltage of the sensor with the ratio between the thickness of the bonding layer and the thickness of the sensor ($h'/h$) under different frequencies. For relative low frequencies, $ka = 0.0, 1.0$ and $2.0$, the
effect of the layer is insignificant until it becomes relatively thick ($h' / h > 0.16$). In these cases, the output voltage can be approximately represented by the static result. With the increase of the loading frequency, significant reduction in the output voltage can be obtained, which is caused by the variation of the electric potential along the sensor under high loading frequency.

4. Concluding remarks

The focus of this paper is on the study of the effect of the material and geometric properties of the sensor and the bonding layer on the load transfer from the host medium to the sensor, when the system is under high frequency mechanical loads.

Numerical simulation is conducted for special cases. The simulation results indicate that, for relatively low frequency cases, with the increase of the bonding layer stiffness, the amplitude of the strain distribution along the sensor increases. For very high frequency cases, however, the strain distribution along the sensor becomes very complicated and unpredictable with the decrease of the bonding layer stiffness. As a result, the loading frequency should not be too high in order to ensure accuracy in sensor output. Besides, the material combination of the sensor and the host structure needs to be carefully selected in order to improve sensor efficiency.

Appendix A. Effective material constants

The electromechanical behavior of piezoelectric materials can be described by

$$\begin{align*}
\{ \sigma \} &= \{ c \} \{ \varepsilon \} - \{ e \} \{ E \}, \\
\{ D \} &= \{ e \} \{ \varepsilon \} + \{ \lambda \} \{ E \}
\end{align*}$$

where $\{ \sigma \}$, $\{ \varepsilon \}$, $\{ D \}$ and $\{ E \}$ represent the stresses, the strains, the electric displacement and the electric field intensity, respectively, $\{ c \}$ is a matrix containing the elastic stiffness parameters for a constant electric potential, $\{ e \}$ represents the piezoelectric constants and $\{ \lambda \}$ represents the dielectric constants for zero strains.

Plane strain deformation in $\gamma$–$z$ plane will be considered in the current study, which suggests that $\varepsilon_\phi^z = 0$. For a surface-bonded piezoelectric thin sheet with the direction of its length is defined as $\gamma$-axis, the traction-free condition along its surface suggests that $\sigma_\gamma^z = 0$. Therefore, following condition should be satisfied,

$$\varepsilon_\gamma^z = 0, \quad \sigma_\gamma^z = 0$$

For piezoelectric materials with their poling direction along the $z$-axis, the stress component $\sigma_y$ and electric displacement $D_z$ can be obtained by substituting the above equations into the constitutive equation, and the results are as follow:

$$\begin{align*}
\sigma_y &= \left( c_{11} - \frac{c_{12}}{c_{33}} \right) \varepsilon_y - \left( e_{13} - \frac{c_{13}}{c_{33}} \right) \frac{c_{13}}{c_{33}} E_z \\
D_z &= \left( e_{13} - \frac{c_{13}}{c_{33}} \right) \varepsilon_y + \left( \lambda_{33} + \frac{c_{13}}{c_{33}} \right) E_z
\end{align*}$$

Therefore, the effective material constants of the sensor can be expressed as

$$\begin{align*}
E_x &= c_{11} - \frac{c_{11}}{c_{33}} \quad \text{plane strain} \\
e_x &= e_{13} - \frac{c_{13}}{c_{33}} \quad \text{plane strain} \\
\lambda_x &= \lambda_{33} + \frac{c_{13}}{c_{33}} \quad \text{plane strain}
\end{align*}$$

Appendix B. Plane elastic waves in elastic half-spaces

A convenient expression of a plane longitudinal ($P$) wave is given by

$$\bar{U} = \bar{A} \exp[ik(\bar{x} \cdot \bar{p} - ct)]$$

where $\bar{A}$ and $\bar{p}$ are unit vectors defining the directions of motion and propagation, respectively; $\bar{x}$ is the position vector, $A$ denotes the amplitude of the wave, and $k$ and $c$ are wave number and wave speed, respectively.

By using the notation introduced here, the incident P-wave as well as the reflected P- and S-wave (shear) can be denoted as

$$\bar{U}^{(\text{inc})} = A_n \bar{d}^{(\text{inc})} \exp[ik_n(x_1 p_1^{(\text{inc})} + x_2 p_2^{(\text{inc})} - c_n t)]$$

where the index $n$ serves to label the various types of waves, as shown in Fig. 5.

The index $n$ is assigned the value $n = 0$ for the incident P-wave, and we have

$$\begin{align*}
d_1^{(0)} &= \sin \theta_0, \quad d_2^{(0)} = \cos \theta_0 \\
p_1^{(0)} &= \sin \theta_0, \quad p_2^{(0)} = \cos \theta_0 \\
c_0 &= c_l
\end{align*}$$
The reflected P-wave is labeled by \( n = 1 \), and we have
\[
\begin{align*}
    d_1^{(1)} &= \sin \theta_1, & d_2^{(1)} &= -\cos \theta_1, \\
    p_1^{(1)} &= \sin \theta_1, & p_2^{(1)} &= -\cos \theta_1 \\
    c_1 &= c_L
\end{align*}
\]

The reflected S-wave is labeled \( n = 2 \), and we have
\[
\begin{align*}
    d_1^{(2)} &= \cos \theta_2, & d_2^{(2)} &= \sin \theta_2, \\
    p_1^{(2)} &= \sin \theta_2, & p_2^{(2)} &= -\cos \theta_2 \\
    c_1 &= c_T
\end{align*}
\]

By using the traction free boundary condition, we have
\[
\tau_{22} = \tau_{21} = 0, \quad x_2 = 0
\]

The parameters of the reflected P- and S-wave can be determined
\[
\begin{align*}
    k_1 &= k_0 \\
    k_2/k_0 &= c_L/c_T = \kappa \\
    \theta_1 &= \theta_0 \\
    \sin \theta_2 &= \kappa^{-1} \sin \theta_0 \\
    A_0^1 &= \frac{\sin 2 \theta_0 \sin 2 \theta_2 - \kappa \cos^2 2 \theta_2}{\sin 2 \theta_0 \sin 2 \theta_2 + \kappa \cos^2 2 \theta_2} \\
    A_0^2 &= \frac{2 \kappa \sin 2 \theta_0 \cos 2 \theta_2}{\sin 2 \theta_0 \sin 2 \theta_2 + \kappa \cos^2 2 \theta_2}
\end{align*}
\]

Therefore, once the incident waveform is given, the total displacement along the interface can be obtained as
\[
\bar{U} = \bar{U}^{(0)} + \bar{U}^{(1)} + \bar{U}^{(2)}
\]

### Appendix C. Sensor Solution

The matrix \([Q]\) used in Eq. (27) is given by
\[
Q^2 = -\frac{\pi \cos[k_s(1 + \eta^j)]}{k_s E_s h \sin(2k_s \theta_0)} p_j^2 + \frac{1}{k_s E_s h} \int_{\cos^{-1} \eta^j}^{\pi} \sin[k_s(\cos \theta - \eta^j)] \cos(j \theta) \, d\theta \\
- \frac{1}{\mu_h} \int_{0}^{\pi} p_j \left( \frac{\beta k^2}{(2s^2 - \beta k^2) - 4s^2 \beta} + \frac{\beta k^2}{2s^2} \right) \, ds + \frac{\beta k^2}{2\mu_h} p_j^2 + \frac{\beta k^2}{(\mu_h - i \omega \rho_c \beta)} \cdot T_j(\eta^j)
\]

where
\[
\eta^j = \eta^j/a, \quad k = ka, \quad k_s = k/a, \quad s = sa
\]

and
\[
\begin{align*}
    P_j^1 &= J_j(\tilde{s}) \begin{cases} (-1)^n \sin(3\eta^j) & j = 2n + 1 \\ (-1)^n \cos(3\eta^j) & j = 2n \end{cases} \\
    P_j^2 &= J_j(\tilde{k}) \begin{cases} (-1)^n \sin(\tilde{k}_s) & j = 2n + 1 \\ (-1)^n \cos(\tilde{k}_s) & j = 2n \end{cases} \\
    P_j^3 &= \begin{cases} \cos(n \cos^{-1} \eta^j) (-1)^n & j \neq 0 \\ 0 & j = 0 \end{cases}
\end{align*}
\]

with \( J_j(j = 1, 2, \ldots) \) and \( T_j \) being the Bessel functions of the first kind and the Chebyshev polynomials, respectively. \( \tilde{s}, \tilde{k} \) can be obtained from \( s, k \) directly, with \( s \) and \( k \) being replaced by \( \tilde{s} \) and \( \tilde{k} \), respectively.

The loading matrix \([F]\) used in Eq. (27) is given by
\[
\{F\} = u_j^j(\eta^j, 0)
\]
References