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# The Fermi motion contribution to $J/\psi$ production at the hadron colliders

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## Abstract

We investigate the relativistic Fermi motion effect in the case of  $J/\psi$  production in various hadron colliders. A light-cone wave function is adopted to represent the  $J/\psi$  final state. The change in the confinement parameter which sets a scale for the size of the final state, allows one to see the effect in an explicit manner. While the effect has considerable influence on the fragmentation probabilities and the differential cross sections, the total cross sections essentially are left unchanged. Such a feature is in agreement with the momentum sum rule which the fragmentation functions should satisfy.

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## 1. Introduction

Evaluation of the  $J/\psi$  cross section at Tevatron energies has been one of the interesting problems of QCD in theory [1–3] and in experiment [4]. Predictions using the QCD calculations where the production of  $J/\psi$  is assumed to occur in color singlet form, fails to agree with the experimental results. To bring about the agreement the mechanism of color octet is introduced in which the bound state is produced originally in the color octet state at the production point [5]. Then by emitting a soft gluon the colored object is transformed into a color singlet state [6].

Generally in the formation of quarkonia bound state it is assumed that the constituents are not relativistic and therefore they are let to fly together within the bound state ignoring their respective motion. Here we propose that the constituents within such states specially the charm quark is not that heavy to let one to ignore its relativistic effects particularly in relation to the energies at which such a particle is produced. Therefore the effect should be accounted for in the production process in a more explicit and accurate form. The results of such a study may shed light on the problem of  $J/\psi$  production cross section.

In this work, we introduce the Fermi motion into the  $J/\psi$  production in direct fragmentation process using a light-cone wave function. Since our aim is to show the size of the effect, we have not included other contributions. We demonstrate the enhancement of the fragmentation function due to this effect. With such a significant enhancement, we evaluate the differential cross section times the branching ratio and the total integrated cross section for the process  $\bar{p}p \rightarrow c \rightarrow J/\psi \rightarrow \mu^+\mu^-$  at the Tevatron Run I energies and compare them with the CDF data. We also present similar results for the case of the RHIC, the Tevatron Run II, and the RHIC and the CERN LHC  $pp$  collisions.

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## 2. The light-cone wave function and the bound state formation via fragmentation

There have been different approaches to introduce the Fermi motion into the production and decay processes of various meson states and quarkonia using different forms of wave functions [7]. The light-cone wave functions have been employed in the case of charmonia to study their production in B-decay [8] and in photoproduction [9]. Here motivated by harmonic oscillator model, we have picked up a wave function in the light-cone quantization to represent the  $J/\psi$  bound state. It has the following form [7]

$$\psi_{c \rightarrow J/\psi}(x_1, x_2, q_T) = A_{c \rightarrow J/\psi} \exp\left[-\frac{1}{8\beta^2} \frac{m^2 + q_T^2}{x_1 x_2}\right], \quad (1)$$

where  $A_{c \rightarrow J/\psi}$  is the normalization coefficient,  $m$  is the quark (anti-quark) mass and  $q_T$  is the transverse momentum of the constituents. The  $x$ 's are the energy momentum ratios and finally the parameter  $\beta$  is known as the confinement parameter which controls the width of the wave packet representing the bound state. The normalization condition is

$$\sum_{n, \lambda_i} \int [dx] [d^2 \mathbf{q}_T] |\psi_n(x_i, \mathbf{q}_{T_i}, \lambda_i)|^2 = 1, \quad (2)$$

where

$$[dx] \equiv \prod_{i=1}^n dx_i \delta\left[1 - \sum_{i=1}^n x_i\right], \quad (3)$$

and

$$[d^2 \mathbf{q}_T] \equiv \prod_{i=1}^n d^2 \mathbf{q}_{T_i} 16\pi^3 \delta^2\left[\sum_{i=1}^n \mathbf{q}_{T_i}\right]. \quad (4)$$

The sum is over all Fock states and helicities.

With the choice of the wave function (1), we fix all degrees of freedom (transverse and longitudinal) of the constituents within the  $J/\psi$  bound state by matching them with those in the matrix elements relevant to the fragmentation function and vary the parameter  $\beta$  to change the size of the wave packet representing the bound state. Benefiting such a method, in the leading order perturbative regime, the fragmentation functions for  $J/\psi$  production without and with the Fermi motion are obtained as follows [10].

### 2.1. Fermi motion off

In this case the transverse momenta of the constituents are set equal to zero and that the longitudinal components are chosen to be equal. The confinement parameter is  $\beta = 0$  in this case. The fragmentation function is obtained as

$$D_{c \rightarrow J/\psi}(z, \mu_o, \beta = 0) = \frac{\alpha_s^2 C_F^2 \langle k_T^2 \rangle^{1/2}}{16mF(z)} \{z(1-z)^2 [\xi^2 z^4 + 2\xi z^2(4-4z+5z^2) + (16-32z+24z^2-8z^3+9z^4)]\}, \quad (5)$$

where  $\alpha_s$  is the strong interaction coupling constant and  $C_F$  is the color factor. The quantity  $\langle k_T^2 \rangle$  is the average transverse momentum squared of the initial state heavy quark, the parameter  $\xi$  is defined as  $\xi = \langle k_T^2 \rangle / m^2$  and finally  $F(x)$  is given by

$$F(z) = [\xi^2 z^4 - (z-2)^2(3z-4) + \xi z^2(8-7z+z^2)]^2. \quad (6)$$

### 2.2. Fermi motion on

Here the different momentum components of the constituents, i.e. the quantities  $q = |\mathbf{q}_T|$  and  $x = x_1 = 1 - x_2$  in the matrix elements and the wave function (1) are integrated over. The only remaining parameter is the confinement parameter  $\beta$ . The fragmentation function in this case is obtained as

$$D_{c \rightarrow J/\psi}(z, \mu_o, \beta) = \frac{\pi^2 \alpha_s^2 C_F^2 \langle k_T^2 \rangle^{1/2}}{2m} \int \frac{dq dx |\psi_M|^2 x^2 (1-z)^2 z q}{G(z)} \times \{1 - 4(1-x)z + 2(4-10x+7x^2)z^2 + 4(-1+x^3-5x^2+4x)z^3 + (1-4x+8x^2-4x^3+x^4)z^4 + \eta \xi z^2 [1-2x+z^2+x^2(2-2z+z^2)] + \eta [2+(-6+4x)z + (9-8x+2x^2)z^2 - 2(2-x+x^2)z^3 + (1+x^2)z^4] + \xi z^2 [1+2x^3(2-3z)z + z^2 + 2x^4 z^2 + 2x(-1+z-2z^2) + x^2(2-8z+9z^2)] + \eta^2(1-z)^2 + \xi^2(1-x)^2 x^2 z^4\}. \quad (7)$$

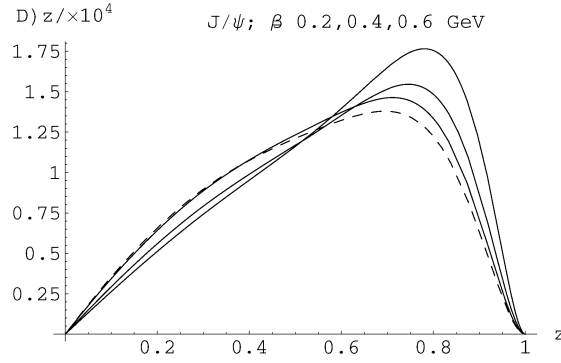


Fig. 1. Fragmentation function for  $J/\psi$  production. While the dashed curve represents the function when the Fermi motion is off, the solid ones show the behavior of this function when the Fermi motion is on. The  $\beta$  values are indicated. The function picks up as  $\beta$  increases. This gives raise to increase in the fragmentation probability.

The function  $G(z)$  reads

$$G(z) = \{[\eta(1-z)^2 + \xi x^2 z^2 + (1 - (1-x)z)^2][\eta(-1+z) + \xi(-1+x)xz^2 - 1 + (1-x+x^2)z]\}^2. \quad (8)$$

Here we have defined  $\eta = q^2/m^2$ . In our study we have set the charm quark mass equal to 1.25 GeV.

The fragmentation functions (5) and (7) provide a comparison of the cases of the Fermi motion off and on. The two functions coincide at sufficiently low  $\beta$ . As  $\beta$  increases, (7) raises considerably at the peak region which leads to the respective increase in fragmentation probability. To choose the  $\beta$  value, we note that different values within the range of  $\beta = 0.250\text{--}0.750$  GeV have been employed for different meson states [7]. Even the case of  $\beta = m_q$  is adopted by Tao Huang et al. in [7]. On the other hand in [10], we have argued that the size of the wave packet representing the bound state is important here. Therefore we have raised the value of  $\beta$  up to 0.6 GeV which we believe is quite safe. The behavior of the fragmentation function (7) is shown in Fig. 1 for  $\beta = 0, 0.2, 0.4$  and 0.6 GeV. We take the maximum value of  $\beta = 0.6$  GeV and use it for our further considerations.

### 3. Inclusive production cross section

We have employed the idea of factorization to evaluate the  $J/\psi$  production cross section at hadron colliders. For  $\bar{p}p$  collisions we may write

$$\begin{aligned} \frac{d\sigma}{dp_T}(\bar{p}p \rightarrow c \rightarrow J/\psi(p_T)X) \\ = \sum_{i,j} \int dx_1 dx_2 dz f_{i/\bar{p}}(x_1, \mu) f_{j/p}(x_2, \mu) [\hat{\sigma}(ij \rightarrow c(p_T/z)X, \mu) D_{c \rightarrow J/\psi}(z, \mu, \beta)]. \end{aligned} \quad (9)$$

Where  $f_{i,j}$  are parton distribution functions with momentum fractions of  $x_1$  and  $x_2$  (different from  $x_1$  and  $x_2$  which appear in (1)),  $\hat{\sigma}$  is the charm quark production cross section and  $D(z, \mu, \beta)$  represents the fragmentation of the produced heavy quark into  $\bar{c}c$  state with confinement parameter  $\beta$  at the scale  $\mu$ . We have used the parameterization due to Martin–Roberts–Stirling (MRS) [11] for parton distribution functions and have included the heavy quark production cross section up to the order of  $\alpha_s^3$  [12]. The dependence on  $\mu$  is estimated by choosing the transverse mass of the heavy quark as our central choice of scale defined by

$$\mu_R = \sqrt{p_T^2(\text{parton}) + m_c^2}, \quad (10)$$

and vary it appropriate to the fragmentation scale of the particle state to be considered. This choice of scale, which is of the order of  $p_T$  (parton), avoids the large logarithms in the process of the form  $\ln(m_Q/\mu)$  or  $\ln(p_T/\mu)$ . However, we have to sum up the logarithms of order of  $\mu_R/m_Q$  in the fragmentation functions. But this can be implemented by evolving them by the Altarelli–Parisi equation [13]. The following form of this equation is used here

$$\mu \frac{\partial}{\partial \mu} D_{c \rightarrow J/\psi}(z, \mu, \beta) = \int_z^1 \frac{dy}{y} P_{Q \rightarrow Q}(z/y, \mu) D_{c \rightarrow J/\psi}(y, \mu, \beta). \quad (11)$$

Here  $P_{Q \rightarrow Q}(x = z/y, \mu)$  is the Altarelli–Parisi splitting function. The boundary condition on the evolution equation (11) is the initial fragmentation function  $D_{c \rightarrow J/\psi}(z, \mu, \beta)$  at some scale  $\mu = \mu_0$ . In principle this function may be calculated perturbatively as a series in  $\alpha_s$  at this scale.

Detection of final state requires kinematical cuts of the transverse momentum,  $p_T$ , and the rapidity,  $y$ . We have imposed the required  $p_T^{\text{cut}}$  and  $y^{\text{cut}}$  in our simulations for different colliders as required and have used the following definition of rapidity

$$y = \frac{1}{2} \log \left[ \frac{E - p_L}{E + p_L} \right]. \tag{12}$$

#### 4. Results and discussion

We have used a light-cone wave function to introduce the Fermi motion in  $J/\psi$  production in direct fragmentation channel and obtained its fragmentation function in leading order perturbative regime. In this function the confinement parameter switches the effect of Fermi motion and within its physically acceptable values, the function demonstrates the effect of Fermi motion in an interesting manner. The motivation of introducing this effect is to see its role in improving the QCD versus experimental results for the  $J/\psi$  cross section at the Tevatron energies.

In the case of the  $J/\psi$  state such a fragmentation function, with a reasonable choice of the confinement parameter, gives raise to a significant increase in the fragmentation probability due to the Fermi motion. Such a behavior is illustrated in Fig. 1.

To see the effect in the production rates, we have used the usual procedure of factorization for  $J/\psi$  production in hadron colliders. We present the  $p_T$  distribution of  $\text{BR}(J/\psi \rightarrow \mu^+\mu^-)d\sigma/dp_T$  for the cases of the Fermi motion off and on along with the CDF data at Run I in the Fig. 2. The branching ratio  $\text{BR}(J/\psi \rightarrow \mu^+\mu^-) = 0.0597$  is taken from [14]. The poor agreement with data is due to the fact that here we have only considered the contribution of  $\bar{p}p \rightarrow c \rightarrow J/\psi \rightarrow \mu^+\mu^-$ . Similar behavior at Tevatron Run II energies is shown in Fig. 3. We have also extended our study to the cases of the RHIC and the CERN LHC  $pp$

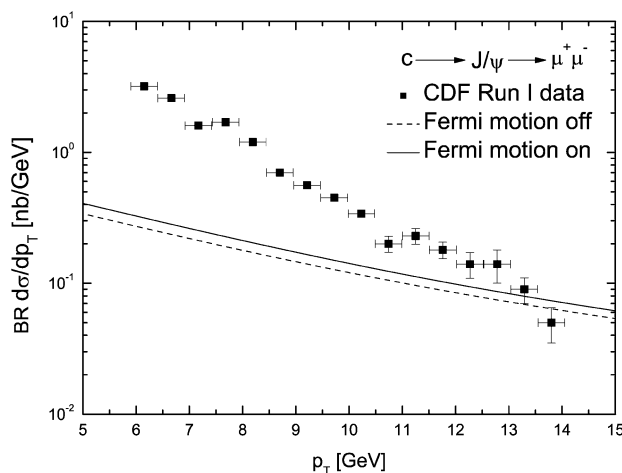


Fig. 2. The differential cross section for direct fragmentation production of  $J/\psi$  and its subsequent decay  $J/\psi \rightarrow \mu^+\mu^-$  at the Tevatron Run I energies. While the dashed curve is obtained using (5) or equally (7) with  $\beta = 0$ , the solid one is due to (7) with  $\beta = 0.6$  GeV. The result is compared with the CDF Run I data. Other contributions are not included. The scale is chosen to be  $2\mu_R$ .

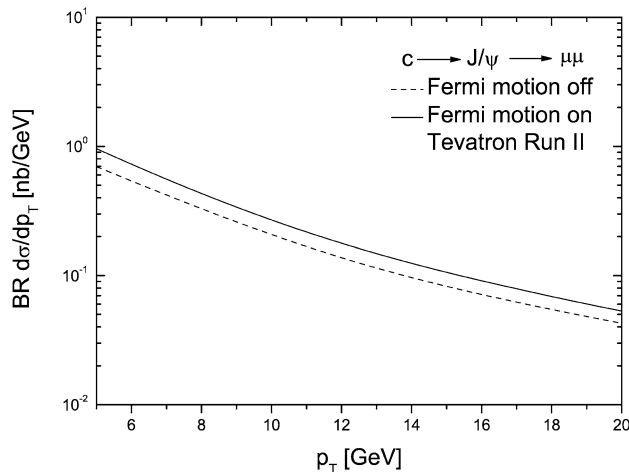


Fig. 3. The differential cross section for direct fragmentation production of  $J/\psi$  at Tevatron Run II. The two curves are obtained using (7) with  $\beta = 0$  and  $0.6$  GeV respectively. The scale has been set to  $2\mu_R$ .

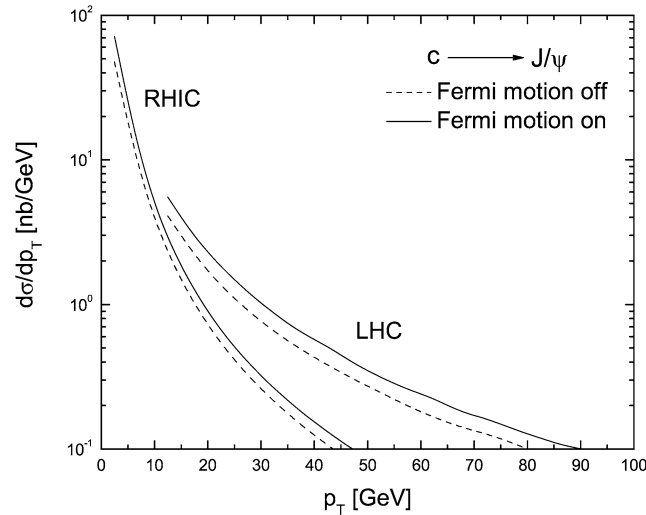


Fig. 4. The differential cross section for direct fragmentation production of  $J/\psi$  at the RHIC and the CERN LHC. The shift due to the Fermi motion is significantly increased in the case of LHC but shows to be less important at the RHIC. In both cases the fragmentation function (7) is employed with  $\beta = 0$  and  $0.6$  GeV, respectively.

colliders. Here we provide the  $p_T$  distributions of the differential cross sections for  $c \rightarrow J/\psi$  and compare the two cases of the Fermi motion off and on in the Fig. 4. In all cases we have used  $\beta = 0.6$  GeV for the confinement parameter in the fragmentation functions. Naturally, the results for  $\beta$  in the range of  $0-0.6$  GeV fall between the above results.

We have also calculated the total integrated cross sections for each case. We found that the total cross sections for with and without Fermi motion essentially remain unchanged within the uncertainties of Monte Carlo simulations. The reason is first due to the momentum sum rule which the fragmentation functions should satisfy. In other words although the modification of fragmentation functions by the Fermi motion redistributes the final states, the integrated cross sections are left unchanged. Alternatively although the Fermi motion increases the fragmentation probability for the state, i.e., introduces a state with overall higher mass, the cross section is lowered by just the same amount when we introduce the effect in calculation of the total integrated cross section.

First of all, we note that the effect of Fermi motion is indeed significant and that the implementation of such a study in all production channels of  $J/\psi$  may give rise to considerable enhancement of the color singlet differential cross section.

It is evident from Figs. 2, 3 and 4 that the effect increases with increasing  $\sqrt{s}$ . The kinematical cuts play an important role apart from  $\sqrt{s}$ . The large cross section at the RHIC compared with the LHC in the Fig. 4 is an example.

Finally, we consider the uncertainties involved in our study. There are two main sources of uncertainties. The first is about the simulation of  $J/\psi$  production at hadron colliders such as the uncertainties along with the fragmentation functions and parton distribution functions. These kind of uncertainties are well discussed in the literature. The second source of uncertainty is due to the choice of the confinement parameter in the fragmentation function. Relying on our discussion in Section 2, our choice of  $\beta = 0.6$  GeV seems to be justified. Future determination of this parameter will shed more light on this situation. It is worth mentioning that our choice of charm quark mass, i.e.,  $1.25$  GeV, have put our results in their upper side and that the change of the charm quark mass in its acceptable range does not have significant impact on the Fermi motion effect in  $J/\psi$  production.

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