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## Weak Signal Detection Technology Based on Holmes Duffing Oscillator

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### Abstract

In this paper, the principle and application of weak signal detection based on Holmes Duffing oscillator is introduced. The relationship between the state equation of Duffing oscillator and the control of signal by scanning is analyzed and the signal is detected. The simulation and design the circuit of weak signal detection are researched based on Duffing oscillator. Control experiments are carried out on the circuit with sine signal of different frequencies. We achieve the purpose of detecting weak period signal from noise background by scanning. This study is a useful exploration in Chaos control using engineering signal detection.

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*Keywords:* Duffing oscillator, weak signal control, scanning signal, detection circuit

### 1. Introduction

The use of chaotic dynamics in detecting weak signal under strong noise has become a hot research field. Chaotic oscillator can be used to detect weak signal, because it's based on the sensitive dependence of the chaotic response to initial values and system parameters. Study shows that, chaotic oscillator is very sensitive to the input periodic signal, but it's immune to the noise. This characteristic makes it possible that chaotic oscillator can be used in detecting weak signal under noise<sup>[1]</sup>. Duffing oscillator is one representative signal detection system which can extract weak periodic signal from strong noise. In 1992, Dr. Donald L. Bir x, from Dayton University in America, detected weak sine signal under Gaussian noise with chaotic oscillator for the first time<sup>[2]</sup>. In 1994, Feng Qi studied the influence of noise on the Duffing oscillator. He found that noise can make the system produce complex movement only in a limited period of time, but the system tended to regular movement finally<sup>[3]</sup>. In paper [4], they proposed periodic signal and harmonic signal detection methods by using the synergy among noise, non-linear systems and signals.

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In paper [5], they distinguished the phase of Duffing oscillator by using phase difference, then identified weak signal. Wang Guanyu and others achieved to measure sine signal under white noise by using chaos measurement system, the SNR can be as low as -66dB. They extracted harmonic signal successfully by using complementary space relations of local tangent space of manifold where the attractor was<sup>[67]</sup>. In paper[8], Zhao Xiangyang used it in weak signal frequency detection of silicon micro-resonant sensor. He achieved the flexible measuring of different testing accuracy and different sensitivity. In paper [9], they transformed the scale and designed the chaotic oscillator arrays, detecting a wider range of voice frequency signals. In paper [10][11],they studied the simulation of circuit system according to Duffing oscillator.

Upon simulation of EWB circuit, in this paper, we carry out research on Holmes Duffing oscillator by means of the implementation of hardware circuit and chaos control. We extract weak periodic signal from strong noise successfully by scanning frequency with LabVIEW. The study of chaos makes a useful exploration in the project signal detection by using chaos control circuit.

## 2. The theory of Duffing oscillator used for the weak signal detection

We use Duffing oscillator to detect weak signal. It's essential that we change the amplitude of the sine scan signal, thus the phase trajectory of chaotic system changes from chaotic critical state to large-scale periodic state. Thereby, we achieve the detection of weak periodic signal. In this paper, we choose Holmes Duffing oscillator as chaotic system model<sup>[12]</sup>.

$$\ddot{x} + k\dot{x} + (-x^3 + x^5) = F \cos(\omega_n t) \quad (1)$$

To make the system detect the signal at any frequency, we modify the equation (1):

$$\begin{cases} \dot{x} = y \\ \dot{y} = -ky + x^3 - x^5 + F \cos(\omega_n t) + a_x \cos(\omega t) + n(t) \end{cases} \quad (2)$$

Where  $k$  is damping ratio,  $x^3 - x^5$  is non-linear restoring force,  $F \cos(\omega_n t)$  is control signal,  $a_x \cos(\omega t) + n(t)$  is signal to be detected,  $n(t)$  is Gaussian white noise. According to Melnikov method, we know that the chaos threshold for different control signals is<sup>[1]</sup>

$$\frac{F}{k} = R(\omega_n) = -\frac{\sqrt{2}(3\pi^2 + 16\omega_n^2)^{\frac{3}{2}}}{256\omega_n\pi} \quad (3)$$

From equation (3), we know that there are different chaos thresholds for different frequency. To detect weak signal, we must find the amplitude of control signal, which is corresponding to the chaos threshold at different frequencies. For example  $\omega_1$  corresponds to  $F_1 = kR(\omega_1)$ ,  $\omega_2$  corresponds to  $F_2 = kR(\omega_2)$ ,  $\omega_n$  corresponds to  $F_n = kR(\omega_n)$ . The specific detection process is: First, we should estimate the frequency range of the signal to be detected and make the frequency range of control signal in this estimate, making the phase trajectory of the system is chaotic critical state. Then, we input signal to be detected and adjust the control signal, making its frequency is  $\omega_n$  and amplitude is  $F_n$ . When the frequency  $\omega_n$  of control signal equals to the frequency  $\omega$  of the signal to be detected, the system trajectory will go into the large-scale periodic state. That indicates that the signal to be detected contains weak sine signal. Simultaneously,  $\omega_n$  is the frequency  $\omega$  of the signal to be detected. Last, we decrease the amplitude of control signal until the phase trajectory of the system changes from large-scale periodic state

to chaotic critical state, this moment amplitude of control signal is  $F'_n$ . Thus, amplitude of the signal to be detected is:

$$a_x = |F'_n - F_n| \tag{4}$$

If the system is still in chaotic state, it illustrates that the signal to be detected does not contain sine signal at frequency  $\omega_n$ .

### 3. Detection circuit design

We change Duffing oscillator system shown as equation (2) to the following form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -0.5 \end{bmatrix} \times \begin{bmatrix} x^3 - x^5 + F \cos(\omega_n t) + a_x \cos(\omega t) + n(t) \\ y \end{bmatrix} \tag{5}$$

Where  $k = 0.5$ ,  $F \cos(\omega_n \tau) + a_x \cos(\omega \tau) + n(\tau)$  is the input.

Therefore, based on equation (5), we design simulation circuit of chaotic system with EWB. The devices we use are sine voltage source, analog operational amplifier, analog multipliers, resistors and capacitors. The circuit is shown as Fig.1.

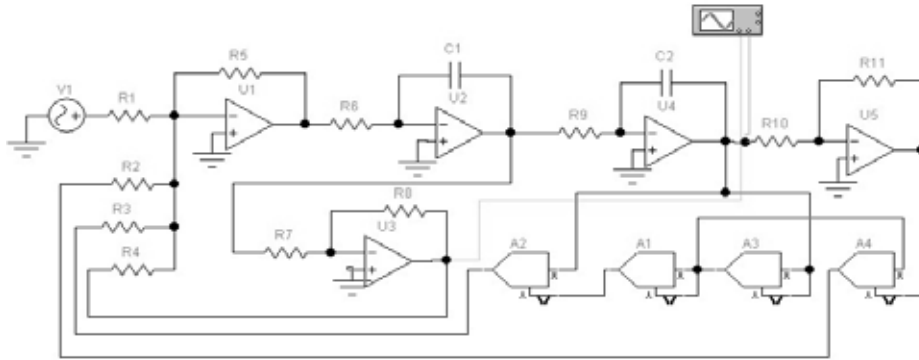


Fig. 1 The circuit of Duffing oscillator

We can get the equation of the Duffing oscillator system circuit according to Fig. 1

$$U_1 = -\left(\frac{R_5}{R_1} V_1 + \frac{R_5}{R_2} U_4^2 U_5 + \frac{R_5}{R_3} U_4^5 + \frac{R_5}{R_4} U_3\right)$$

$$U_2 = -\frac{1}{R_6 C_1} U_1, U_3 = -\frac{R_8}{R_7} U_2$$

$$U_4 = -\frac{1}{R_9 C_2} U_2, U_5 = -\frac{R_{11}}{R_{10}} U_4 \tag{6}$$

The sine voltage source is used to generate the control signal  $V_1$ . We can obtain  $U_2$  ( $U_2$  corresponds to  $-y$  from Duffing equation) from  $U_1$  via the inverting integrator and obtain  $U_4$  ( $x$ ) from  $U_2$  via the inverting integrator.  $A_4$  ( $-x$ ) and  $A_2$  ( $x^5$ ) can be obtained via the following inverting amplifier circuit and multiplication circuit.  $U_3$  ( $x$ ) can be obtained from  $U_2$ . We can obtain  $U_1$  ( $y$ ) from  $V_1, A_4, A_2$  and

$U_3$  via inverting adder. We connected  $U_3$  and  $U_4$  separately to ends of oscilloscope, then the phase trajectory of the system will be displayed on the oscilloscope. According to Fig.1, we make the circuit board with kinds of components .We do control and detecting test of the circuit with signal generator and oscilloscope. During the experiment, we may adjust the resistances (R2, R4) or the integral capacitance to make the circuit adapt to control and detection of sine signal at different frequency. In this paper, we choose to change the integral capacitance.

**4. System control and signal detection**

*4.1 Control the system with 10Hz sine signal*

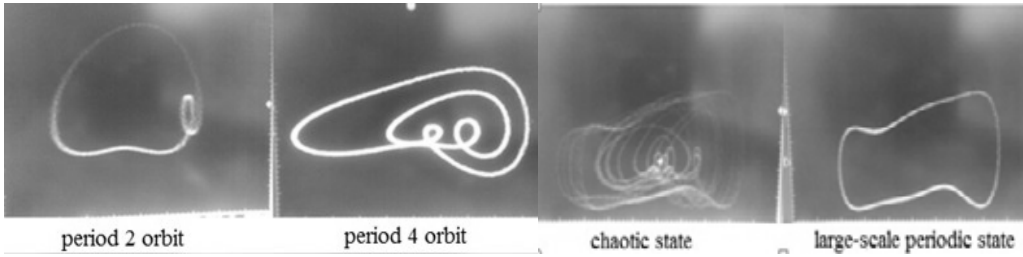


Fig.2 The control circuit of 10Hz sine signal

We input sine signal at 10Hz into the system. With the amplitude  $F$  of control signal increasing, the trajectory of the system will be homoclinic orbit, period-doubling bifurcation, chaotic state, large-scale periodic state orderly. When  $F < 0.38V$ , the trajectory is period 1 orbit. The trajectory will be period 2 orbit since  $F > 0.72V$ . It will be period 4 orbit since  $F > 1.35V$ . We continue to increase the amplitude, the trajectory will be chaotic state. When  $F = 2.89V$ , the trajectory is critical state, which is from chaotic state to large-scale periodic state. The system will be large-scale periodic state since  $F > 2.90V$ . The trajectory is shown by Fig.2. According to equation (4), we know that the amplitude of signal to be detected is  $0.01V$ .

*4.2 The detection of 70Hz sine signal under noise background*

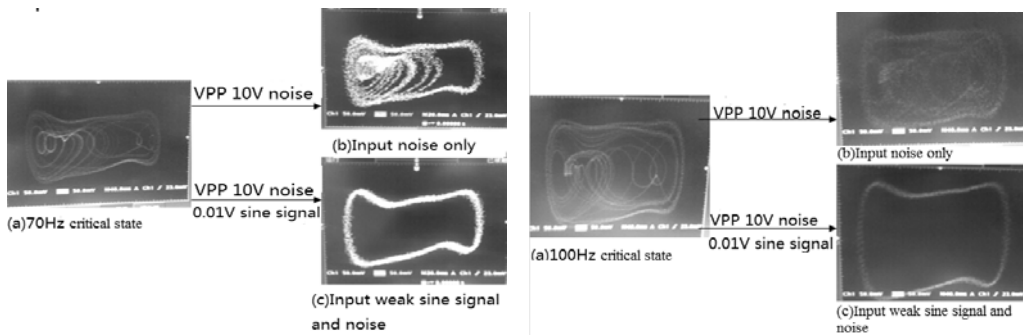


Fig.3 The detection of 70Hz sine signal under noise background; Fig.4 The detection of 100Hz sine signal under noise background

We adjust the control signal frequency, making it about 70Hz. At the same time we adjust the amplitude  $F_n = kR(\omega_n)$  corresponding to the threshold of chaos. Thus, the system is chaotic critical

state. Then we input the system signal to be detected. The signal is composed of noise whose VPP is  $10V$  and sine signal whose frequency is  $70Hz$  and amplitude is  $0.01V$ . If  $\omega_n \neq 70Hz$ , the system will be still in chaotic state. We adjust  $\omega_n$  and  $F_n = kR(\omega_n)$  constantly, when  $\omega_n = 70Hz$ , the trajectory turns into large-scale periodic state, shown as Fig.3(c). In this time, the amplitude of control signal is  $1.53V$ . After that, we decrease the amplitude of the control signal gradually until the trajectory turns into chaotic state, shown as Fig.3 (b) and meanwhile, the amplitude of the control signal is  $1.52V$ . According to equation (4), we get that the amplitude of the weak sine signal is  $0.01V$ .

#### 4.3 The detection of 100Hz sine signal under noise background

The detection process of 100Hz sine signal is the same as that of 70Hz sine signal. We adjust the control signal frequency, making it about 100Hz. At the same time we adjust the amplitude  $F_n = kR(\omega_n)$  corresponding to the threshold of chaos. Thus, the system is chaotic critical state. Then we input the system signal to be detected. The signal is composed of noise whose VPP is  $10V$  and sine signal whose frequency is 100Hz and amplitude is  $0.01V$ . If  $\omega_n \neq 100Hz$ , the system will be still chaotic state. We adjust  $\omega_n$  and  $F_n = kR(\omega_n)$  constantly, when  $\omega_n = 100Hz$ , the trajectory turns into large-scale periodic state, shown as Fig.4(c). In this time, the amplitude of control signal is  $2.19V$ . After that, we decrease the amplitude of the control signal gradually until the trajectory turns into chaotic state, shown as Fig.4 (b) and meanwhile, the amplitude of the control signal is  $2.18V$ . According to equation (4), we get that the amplitude of the weak sine signal is  $0.01V$ .

Both Fig.3 and Fig.4 show: we input noise to the system, the trajectory of the system become rough, but still in chaotic state. That verifies the system's immunity to noise. When the frequency of the control signal does not equal to that of weak sine signal, the system is always in chaotic state. Instead, when their values are the same, the system will turn into large-scale periodic state. That verifies the system's sensitivity to weak signal. Therefore, we can detect weak signal under noise by scanning frequency.

#### 4.4 Detecting any signal by scanning frequency with LabVIEW

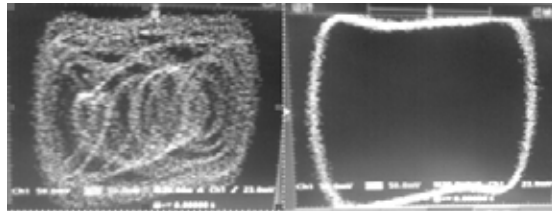


Fig.5 The trajectory of the system when sweeping frequency (a) Chaotic critical state (b) Large-scale periodic state

In order to achieve the detection of weak signals at any frequency, we should control output of the signal generator with LabVIEW, the output is the control signal, detecting unknown signal by scanning frequency. The detection process is: We put all amplitude corresponding to each frequency of some frequency band to the command list of LabVIEW. We set procedure to make signal generator output control signal periodically, making the system be chaotic critical state, shown as Fig. 5(a). When there is weak periodic signal at certain frequency in the input, the phase trajectory of the system will turn into large-scale periodic state, shown as Fig. 5(b). According to the frequency of scanning signal and the

difference of amplitude between chaotic critical state and large-scale periodic state, we can get the frequency and amplitude of the unknown signal.

In the scanning process, the time of output signal should not too short, or it would too short to let the system react, causing the omission of useful signal. It also should not too long, or it would lower the detection efficiency of the system. In order to ensure the efficiency and accuracy, we should estimate the range of signal to be detected before detecting.

## 5. Conclusion

After a brief introduction about the weak signal detection theory based on Duffing oscillator and simulation circuits based on EWB. We make circuit of chaotic system. We do control and detecting experiments with multiple frequency sine signal respectively. We control the output signal with LabVIEW and achieve the detection of unknown signal. Our work has practical significance in using Duffing oscillator to detect low-frequency signal under strong noise common in mechanical engineering.

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