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Rotating black holes and Coriolis effect

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ABSTRACT

In this work, we consider the fluid/gravity correspondence for general rotating black holes. By using the suitable boundary condition in near horizon limit, we study the correspondence between gravitational perturbation and fluid equation. We find that the dual fluid equation for rotating black holes contains a Coriolis force term, which is closely related to the angular velocity of the black hole horizon. This can be seen as a dual effect for the frame-dragging effect of rotating black hole under the holographic picture.

1. Introduction

It is well-known that gauge/gravity correspondence is a great breakthrough on theoretical physics. If we consider a space–time with a boundary, such conjecture claims that there exists a correspondence between the gravity theory in this space–time and the field theory on its boundary. This conjecture offers us a powerful tool to study properties of strongly coupled systems. An important application of this conjecture is the correspondence between gravity and fluid dynamics. Such correspondence was first observed by Policastro, Son and Starinets [1]. The crucial idea of this fluid/gravity correspondence is quite clear. Because of the gauge/gravity correspondence, a gravity theory in space–time should correspond to a field theory on its boundary. Therefore the gravitational perturbation in the space–time will induce a perturbation in the dual field theory on the boundary. Since the infrared behavior of a field theory is governed by hydrodynamics, there should be a natural relation between the perturbed Einstein equation in the long wavelength limit and the hydrodynamical equation. By considering long wavelength model of perturbation on black brane solutions, Son et al. established such correspondence and calculated the associated shear viscosity of the dual fluid. During the last decade, this topic has attracted great attention of researchers [2–10]. Many interesting fluid phenomenon have been realized holographically, e.g. turbulence [11] and Hall viscosity [12]. In 2011, Strominger et al. develop a new method to relate the perturbed Codazzi equation on boundary with the Navier–Stokes equation [13,14]. Soon later, with the idea of local boost transformation Compére et al. re-realized such correspondence and generalize the fluid/gravity correspondence to higher order perturbation [15]. Since the fluid/gravity duality is a quite natural corollary of gauge/gravity correspondence, it is reasonable to believe that such correspondence should hold for general stationary black holes. Unfortunately, the long wavelength conditions used in the original method in [1,13–15] can not be generalized to deal with non-plane symmetric black hole cases. Especially, the method can not be applied to rotating black holes. In 2011, Strominger and his colleagues proposed a new idea to realize the correspondence [16]. They found that, by imposing a carefully chosen boundary condition, the perturbed Einstein equation exactly reduces to the Navier–Stokes equation in one lower dimension. Mathematically, this method is much simpler than the original one and can be generalized to the cases of more general black holes. Following this idea, the fluid/gravity correspondence for general non-rotating black holes has been established in[17–26]. In this paper, we consider the case of rotating black holes.

On the gravity side, rotating black holes have an important phenomenon, i.e. the frame-dragging effect [27]. Near horizon, the stationary observer will be forced to rotate with the black hole because of the distorted space–time geometry. In fact, such effect exists for any rotating massive objects and has been observed by the GPB experiment [28]. An interesting question is what the dual effect for the frame-dragging on the field theory side is. In this paper, we will study the fluid/gravity duality and focus on the physical effect on the dual fluid caused by the rotation of black holes. We discover that the dual fluid equation is an incompressible Navier–Stokes equation with the Coriolis force. Our result implies that the holographic dual effect of frame-dragging is just the Coriolis force, at least at the hydrodynamical limit level.

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In this paper, we first introduce some basic properties of our black hole background. To describe a general stationary black hole, we use the theory of isolated horizon which is developed by Ashtekar and other authors [29]. Then we study the fluid/gravity correspondence for rotating black holes using Strominger's boundary condition method. Finally, we conclude our results.

2. Asymptotic behavior of metric near horizon

In order to study the fluid/gravity correspondence in general cases, one needs to consider the properties of general stationary black hole configuration. To do this, Ashtekar's isolated horizon theory is a suitable tool [29]. This theory was developed by Ashtekar and other authors for four dimension at the end of last country. Later, Lewandowski and his colleagues generalized it to general dimensions [30]. Roughly speaking, an isolated horizon is a null hyper-surface with constant area. It is easy to check that stationary horizon is a special case of isolated horizon since the whole space–time is time-independent. In fact, isolated horizon is a natural generalization of the stationary black hole horizon. It does not require the space–time to be stationary, but only requires the geometry inside the horizon is time-independent. Physically, isolated horizon describes the case that all dynamical processes in the neighborhood of black hole has been fixed, but the exterior space–time region may still be dynamical.

In the rest part of this paper, we will focus on the gravitational perturbation near the isolated horizon. As has been discussed in [22], near the horizon, one can establishes a special coordinates $\{t, r, x^i\}$ $(i = 1, 2, \cdots, p)$, the Bondi-like coordinates, with the horizon at $r = 0$. In terms of such coordinates, there is a set of null tetrad $\{l, n, E_I\}$ $(I = 1, 2, \cdots, p)$, which can be expressed as

$$\begin{aligned} n &= \partial_r, \\ l &= \partial_t + U\partial_r + X^i\partial_i, \\ E_I &= W_I\partial_r + e_I^i\partial_i, \qquad I, i = 1, 2, \cdots, p, \end{aligned} \tag{1}$$

where (U, X^i, W_I, e_I^i) are functions of (t, r, x^i) whose near horizon behavior can be obtained from Cartan structure equations as

$$\begin{aligned} U &= \hat{\epsilon}r + \frac{1}{2}(\hat{R}_{nlnl} + 2|\hat{\pi}|^2)r^2 + O(r^3), \\ W_I &= -\hat{\pi}_I r + \frac{1}{2}(\hat{R}_{nInl} + 2\hat{\theta}'_{IJ}\hat{\pi}_J)r^2 + O(r^3), \\ X^i &= -\hat{\pi}^i r + \frac{1}{2}(\hat{R}_{nInl}\hat{e}_I^i + 2\hat{\theta}'_{IJ}\hat{\pi}_I\hat{e}_J^i)r^2 + O(r^3), \\ e_I^i &= \hat{e}_I^i - \hat{\theta}'_{IJ}\hat{e}_J^i r + \frac{1}{2}(\hat{R}_{nInJ}\hat{e}_J^i + 2\hat{\theta}'_{IJ}\hat{\theta}'_{JK}\hat{e}_K^i)r^2 + O(r^3), \end{aligned} \tag{2}$$

where $\hat{e}_I^i$ is the tetrad on the section of horizon and $\theta'_{IJ} := \langle E_I, \nabla_J n\rangle$, $\epsilon := \langle n, \nabla_l l\rangle$ with $\hat{\epsilon}$ being the surface gravity of horizon. The hatted quantities represent the initial data of the horizon. What need to be emphasized is the quantity $\pi_I := \langle E_I, \nabla_l n\rangle$. Based on the discussion in ref. [29], it is just the rotational 1-form potential. $\pi_I \neq 0$ implies that the black hole is rotating. In previous works, all black holes considered are non-rotating The main concern of this paper is to study the effect induced by the non-zero π_I.

Finally, using the null tetrad (1), the most general form of the metric in the neighborhood of a stationary horizon can be written as

$$(g^{\mu\nu}) = \begin{pmatrix} 0 & 1 & \vec{0} \\ 1 & 2U + W_I W_I & X^i + W_I e_I^i \\ \vec{0} & X^j + W_I e_I^j & e_I^i e_I^j \end{pmatrix}. \tag{3}$$

3. Brown–York tensor of the boundary near horizon

Based on the basic gauge/gravity correspondence, Brown–York tensor corresponds to the energy-momentum tensor of the dual field theory and can be obtained by

$$t_b^a = K h_b^a - K_b^a, \tag{4}$$

where K_b^a is the extrinsic curvature and h_b^a is the induced metric of the boundary. It is well-known that the dynamical equation of fluid comes from the conservation law of the energy-momentum tensor. To study the fluid/gravity correspondence by using Strominger's boundary condition method [16], it is crucial to have the asymptotic behavior of the Brown–York tensor near horizon. Such behavior can be obtained by the direct calculations based on the asymptotic results of metric in last section. Another important technique of Strominger's method is to take the near horizon limit.[1] Based on the gauge/gravity dictionary, the radius of the boundary is related to the energy scale of the dual field theory on the boundary. So the boundary approaching to the horizon implies the low energy limit in the dual field theory, i.e. the hydrodynamic limit. We summarize our approach as follows: introduce a rescaling parameter λ and consider the boundary at $r = r_c$ in the neighborhood of the horizon, then define rescaled coordinates as $\tau = 2\hat{\epsilon}\lambda^2 t$ and $r_c = 2\hat{\epsilon}\lambda^2$. Taking $\lambda \to 0$ limit corresponds to take the near horizon and non-relativistic limit at the same time. We denote the Brown–York tensor of the background as $t_b^{a(B)}$ and expand the components of the non-perturbed Brown–York tensor in terms of λ,

$$\begin{aligned} t_\tau^{\tau(B)} &= \xi\lambda + O(\lambda^3), \\ t_i^{\tau(B)} &= \hat{\pi}_i\,\lambda + O(\lambda^3), \\ t_\tau^{i(B)} &= O(\lambda^3), \\ t_j^{i(B)} &= \frac{1}{2\lambda}\delta_j^i + \left[(\beta + \xi)\,\delta_j^i - \xi_j^i\right]\lambda + O(\lambda^3), \\ t^{(B)} &= \frac{p}{2\lambda} + [p\,(\beta + \xi)]\,\lambda + O(\lambda^3). \end{aligned} \tag{5}$$

where β, ξ_j^i and $\xi = \xi_i^i$ are constants depending on the initial data of the horizon,

$$\begin{aligned} \beta &= \frac{1}{4}(3\hat{R}_{nlnl} + |\hat{\pi}|^2), \\ \xi_j^i &= -2\hat{g}^{ik}\tilde{\nabla}_{(j}\hat{\pi}_{k)} + 2\hat{\pi}^i\hat{\pi}_j + 2\hat{\epsilon}\,\hat{g}_{jm}\hat{\theta}'_{IJ}\hat{e}_I^i\hat{e}_J^m, \end{aligned} \tag{6}$$

and $\tilde{\nabla}$ is the induced derivative on the section of the horizon.

4. Petrov-like boundary condition and gravitational perturbation

Following fluid/gravity correspondence, one needs to consider the gravitational perturbation in the space–time. A basic requirement for the perturbation is to satisfy the regular condition at horizon. In early works, people solved the perturbed equation concretely to ensure the regularity [13,14]. For general stationary horizons, this method fails to work. One needs other method to ensure the regularity of the perturbation. Thanks to the results on initial-boundary value problem [31], one can ensure such regularity by imposing suitable boundary condition. One of the possible choices is Strominger's Petrov-like boundary condition [16]. The Petrov-like

[1] Such limit also has been used to consider other topics about black hole which is related with AdS/CFT correspondence, such as Kerr/CFT correspondence [32]. Detail analysis will show such limit is equivalent to the "large mean curvature limit" in Strominger's original work [16].

boundary condition requires the perturbed Weyl curvature satisfy the following condition on the boundary,

$$C_{lilj} = 0, \tag{7}$$

where l^a is the out-pointed null normal of the time-like boundary. Physically, this boundary condition implies there is no outgoing perturbation at boundary. As the original paper by Strominger et al., the perturbation is introduced in terms of Brown–York tensor:

$$t^a_b = t^{a(B)}_b + \sum_k t^{a(k)}_b \lambda^k, \tag{8}$$

where $t^{a(k)}_b$ are gravitational perturbations and $\lambda \ll 1$ is the perturbation parameter. Under such rescaling, the perturbed Petrov-like boundary condition implies that

$$t^{i(1)}_j = -2\hat{g}^{ik}\tilde{\nabla}_{(j}(t^{\tau(1)} + \hat{\pi})_{k)} + 2t^{\tau i(1)}\left(t^{\tau(1)}_j + \hat{\pi}_j\right) + \frac{t^{(1)}}{p}\delta^i_j + \xi^i_j - \tilde{R}^i_j + \frac{2(p^2-3)}{p^2(p+1)}\Lambda\delta^i_j. \tag{9}$$

It is easy to check that this equation reduces to the non-rotating result in [22] as $\hat{\pi}$ vanishes.

5. The dual Navier–Stokes equation

With preparation in previous sections, we are able to study the holographic dual of gravitational perturbations. The basic AdS/CFT dictionary tells us that the Brown–York tensor corresponds to the energy-momentum tensor of dual field theory. On the field theory side, the hydrodynamic limit of the conservation law of energy-momentum tensor should give the fluid equation. On the gravity side, the conservation equation of Brown–York tensor is just the Codazzi equation on the boundary of space–time. Thus the hydrodynamic limit indeed corresponds to the near horizon limit. So what one has to consider the near horizon limit of the Codazzi equation, $\bar{D}_a t^a_b = 0$, where $\bar{D}$ is the induced derivative on space–time's boundary. Since the inner geometry on boundary is fixed, the perturbed Codazzi equation are obtained from Eq. (8) as

$$0 = \bar{D}_a t^{a(B)}_b + \bar{D}_a t^{a(1)}_b \lambda + O(\lambda^2). \tag{10}$$

Considering the τ component of Codazzi equation, under the near horizon limit, the first non-trivial equation is in the $O(\lambda^{-1})$,

$$\tilde{\nabla}_i(\hat{g}^{ij}t^{\tau(1)}_j) = 0. \tag{11}$$

For i components of Codazzi equation, the first non-trivial equation is in the $O(\lambda)$,

$$0 = \partial_\tau t^{\tau(1)}_i - 2t^{\tau j(1)}\tilde{\nabla}_{(i}\hat{\pi}_{j)} - 2t^{\tau j(1)}\tilde{\nabla}_{[i}\hat{\pi}_{j]} - \tilde{\nabla}_j(\xi^j_i - t^{j(1)}_i) + \tilde{\nabla}_i\left[\beta + \xi - \frac{1}{4}(\hat{R}_{nlnl} - |\hat{\pi}|^2)\right]. \tag{12}$$

Combined with Eq. (9) and Eq. (11) and used the concrete expression of β and ξ in Eq. (6), this equation becomes

$$0 = \partial_\tau t^{\tau(1)}_i + \tilde{\nabla}_i\frac{t^{(1)}}{p} + 2t^{\tau j(1)}\tilde{\nabla}_j t^{\tau(1)}_i - 4t^{\tau j(1)}\tilde{\nabla}_{[i}\hat{\pi}_{j]} - \tilde{\nabla}^2(t^{\tau(1)}_i + \hat{\pi}_i) - \tilde{R}^j_i(t^{\tau(1)}_j + \hat{\pi}_j) - \tilde{\nabla}_j\tilde{R}^j_i + \frac{1}{2}\tilde{\nabla}_i\left(\hat{R}_{nlnl} + 9|\hat{\pi}|^2 - 8\tilde{\nabla}_j\hat{\pi}^j\right), \tag{13}$$

where $\tilde{R}^j_i$ is the Ricci curvature of the section metric $\hat{g}_{ij}$.

Now let's identify the geometric quantities with hydrodynamic quantities based on gauge/gravity dictionary. Since t^a_b corresponds to the energy-momentum tensor in the dual field theory, it should be identified with the fluid energy-momentum tensor under the hydrodynamic limit. By comparing the perturbed Brown–York tensor with the energy-momentum tensor (8), we establish the standard identification [16],

$$t^{\tau(1)}_i \to \frac{1}{2}v_i, \qquad \frac{t^{(1)}}{p} \to \frac{P}{2}, \tag{14}$$

where P is the pressure and v_i is the velocity in the dual fluid. The above identification (14) works for curved background geometries [17,20–22,24–26]. With this identification, Eq. (11) reminds us that the dual fluid is incompressible, i.e. $\tilde{\nabla}_i v^i = 0$, and the fluid equation can be finally written as,

$$0 = \partial_\tau v_i + \tilde{\nabla}_i P + v^j\tilde{\nabla}_j v_i - \tilde{\nabla}^2 v_i - \tilde{R}^j_i v_j - f_i - 4v^j\tilde{\nabla}_{[i}\hat{\pi}_{j]}, \tag{15}$$

where

$$f_i = 2\tilde{\nabla}^2\hat{\pi}_i + 2\tilde{\nabla}_j\tilde{R}^j_i + 2\tilde{R}^j_i\hat{\pi}_j - \tilde{\nabla}_i\left(\hat{R}_{nlnl} + 9|\hat{\pi}|^2 - 8\tilde{\nabla}_j\hat{\pi}^j\right). \tag{16}$$

Eq. (15) can be realized as the forced incompressible Navier–Stokes equations. The first line in Eq. (15) are standard terms of Navier–Stokes equation in the curved space–time with a total divergence term f_i which is only dependent on background geometry and can be realized as external forces. In addition, there is an unusual term $-4v^j\tilde{\nabla}_{[i}\hat{\pi}_{j]}$ appears in Eq. (15). An interesting recognizing is that this term take the exact form of Coriolis force. According to Eq. (5) and gauge/gravity dictionary, the vector $\hat{\pi}_i$ is the velocity of the reference frame, and $\tilde{\nabla}_i\hat{\pi}_j - \tilde{\nabla}_j\hat{\pi}_i$ is just the angular velocity. In order to see this, we consider the Gauss equation. Under near horizon limit, the perturbed Gauss equation gives

$$t^{\tau(1)}_\tau = -2\hat{g}^{ij}t^{\tau(1)}_i t^{\tau(1)}_j - 2\hat{\pi}^i t^{\tau(1)}_i - \xi + \tilde{R} = -\frac{1}{2}|v + \hat{\pi}|^2 + \left(-\xi + \tilde{R} + \frac{1}{2}|\hat{\pi}|^2\right). \tag{17}$$

Based on the AdS/CFT dictionary, t^τ_τ corresponds to the energy density of the dual fluid. Obviously, the first term can be recognized as the non-relativistic kinematic energy. This agrees with that the Navier–Stokes equation describes the non-relativistic dynamics of fluid. In Eq. (17), $v_i + \hat{\pi}_i$ plays the role of total velocity in the background of an inertial reference frame. Since we identified v_i as the velocity in the dual fluid relative to the background, it is natural to see that $\hat{\pi}_i$ is just the velocity of the background consisting with the second equation in Eq. (5).

In eq. (15), we have obtained that the Codazzi equation with Petrov-like boundary condition takes the form of Navier–Stokes equation in the near horizon limit. Comparing to the standard Navier–Stokes equation, there are two extra terms in eq. (15). One term corresponds to the Coriolis force which has been discussed in previous paragraph. The other term corresponds to the external forces which only depend on background geometry of the horizon. The term of external forces again contains two parts. The first part is the gradient of the Weyl curvature component $\hat{C}_{lnln}$, which can be seen as a induced gravitational potential caused by the curved higher dimensional space–time. The second part depends on $\hat{\pi}_i$. Since we have identified $\hat{\pi}_i$ as the velocity in the non-inertial frame, the second part can be realized as certain non-inertial effect caused by the non-inertial frame. This external force vanishes if the horizon is planar symmetric without rotating. This agrees with the results of [13–16].

If we consider a 5-dimensional space–time with its 3-dimensional horizon section metric $\hat{g}_{ij}$ being flat (based on characteristic initial value problem [33], such solution exists, at least locally), we can write the angular velocity of the background with respect to the inertial reference frame as

$$\mathbf{\Omega} = \tilde{\nabla} \times \hat{\boldsymbol{\pi}}. \tag{18}$$

Then Eq. (15) reduces to the standard 4-dimensional incompressible Navier–Stokes equation in a rotating frame of reference,

$$\partial_\tau \mathbf{v} + \mathbf{v} \cdot \tilde{\nabla} \mathbf{v} + \tilde{\nabla} P - \tilde{\nabla}^2 \mathbf{v} + 2\mathbf{\Omega} \times \mathbf{v} + \mathbf{f} = 0, \tag{19}$$

with $2\mathbf{\Omega} \times \mathbf{v}$ is the Coriolis force induced by the non-inertial reference frame and

$$\mathrm{f}_i = -2\tilde{\nabla}^2 \hat{\pi}_i + \tilde{\nabla}_i \left(\hat{C}_{nlnl} + 9|\hat{\pi}|^2 - 8\tilde{\nabla}_j \hat{\pi}^j \right), \tag{20}$$

being the external force since $\mathbf{f}$ only depends on the back ground geometry.

Finally we try to give a more intuitive way to explain why the rotation of a black hole would induce the Coriolis effect in the dual fluid theory. In ref. [34], Eling et al. gave a quite nice qualitative picture about the fluid/gravity correspondence. A stationary black hole background corresponds to a trivial solution of fluid equation, so the four velocity of the fluid along the direction of Killing vector ∂_t. If one perturbs this black hole, the generator of the perturbed horizon will have small fluctuations on x^i which can be identified as the velocity of the fluid respecting to the static background. If the black hole is rotating, this picture should be modified. Because of the rotation of the black hole, the generator of horizon does not coincide with ∂_t, but becomes $\partial_t + \Omega_H \partial_\phi$ which is just the famous frame-dragging effect. According to Eling's picture, this means that the dual fluid has an additional velocity seen by the inertial observer at infinity due to the frame-dragging effect. Thus the dual fluid on the horizon is in a *non-inertial* frame and the angular velocity of the frame causes the Coriolis effect!

6. Conclusion

In this paper we studied the fluid/gravity correspondence for a general rotating black hole. We considered a rotating black hole with an isolated horizon, which is more general than an usual stationary horizon since only the geometry inside the horizon is required to be stationary in this case. Further calculation has shown that the fluid/gravity correspondence will be fail if one give up the isolated condition. We showed that the fluid/gravity correspondence can be established for general rotating black holes. Further more, the most interesting result is that the dual fluid equation on rotating horizon contains a Coriolis force term, which means the dual fluid is in a non-inertial reference frame. As in Eq. (18), the associated angular velocity $\mathbf{\Omega}$ of the reference frame is determined by $\hat{\boldsymbol{\pi}}$ which is closely related to the angular velocity of horizon. However, it is well-known that the horizon angular velocity characterizes the frame-dragging effect for a rotating black hole. Combine all these facts, we thus proposed that the Coriolis effect should be the holographic dual of the frame-dragging effect in a rotating black hole.

One may ask that whether or not we could get the fluid equation (15) by just replacing v_i by $v_i + \hat{\pi}_i$ in the ordinary N–S equation? The answer is no because of the frame-dragging effect. Due to the frame-dragging effect, any physically reasonable observer near horizon has to be co-rotating with the black hole, and there is no proper inertial frame near the horizon. This is also the reason why one must find a Coriolis term in the final fluid equation. Since the frame obtained by the near horizon limit is non-inertial, one cannot get the correct result by just putting $v_i + \hat{\pi}_i$ into the ordinary fluid equation. Technically, this is because the space and time derivatives appeared in the fluid equation are in the non-inertial frame, but $v_i + \hat{\pi}_i$ is the total velocity of the fluid in the (improper) inertial frame. In summary, the Coriolis force term can not be eliminated by any simple coordinate transformation since it is caused by the frame-dragging effect, which is a geometric effect.

When we finish this paper, there appears another paper [35] discussing Kerr/fluid Duality. In that paper, the authors considered the fluid/gravity correspondence for extreme Kerr metric, a special case of our general rotating black hole. In the dual fluid equation, they found a "surprising" term proportion to $\epsilon_{ij} v^j$ which is resemble to the term $4v^j \tilde{\nabla}_{[i} \hat{\pi}_{j]}$ in our result. We believe that this term should be related to Coriolis force which we found in this paper.

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