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Axis position dependent dynamics of multi-axis milling machines

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Abstract

Numerous analytical methods are available to predict the stability of milling processes. Most of these methods base on the assumption, that the dynamics of the machine tool are time invariant. This assumption seems to be valid in many cases. However, in case of huge translational or rotatory axes movements or process-induced changes in the work piece's mass and elasticity a time variant dynamic model might be needed. This paper presents a method to model the axis position dependent dynamics of a multi-axis milling machine. According to this method, the modal parameters of the machine tool are predetermined in different discrete axis positions. An interpolation strategy allows calculating the modal parameters in arbitrary resolution along arbitrary tool paths. Here, an exemplary 2.5-dimensional milling process serves as an example. The conventional step-by-step time domain simulation procedure is complemented by the modal interpolation strategy to account for changing machine dynamics. The effect of changing dynamics on the process is determined and a comparison to a cutting test is performed.

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Keywords: Machine tool; Dynamics; Milling

1. Introduction

Vibrations during machining can have different reasons. One of the reasons is the regenerative effect that can cause excessive chatter vibrations [1]. This kind of vibration has already been described by Tlusty, Tobias and Opitz [2, 3, 4]. They have introduced the so called stability chart that separates all possible combinations of revolution speed and depth of cut into stable and instable combinations. In the recent decades research has focused on the development of algorithms to efficiently calculate such stability charts for turning and milling processes. Altintas and Budak [5] have presented the so-called zero order approximation, where the time-periodic unperturbed milling force is averaged out and the stability chart is efficiently approximated in the frequency domain. This method is particularly useful for modelling high immersion milling processes. For low immersion milling with only few teeth in cut Merdol and Altintas [6] presented the multi frequency solution which was later extended by Bachrathy [7] to cope with more complex tool geometries. Besides the methods in frequency domain several time domain approaches are available. In time domain, the vibrational perturbation of the cutting process can be described by a system of delay differential equations (DDE). These DDE can be solved approximately with help of discretization techniques. Bayly and his colleagues have presented the temporal finite element method [8]. For each temporal element they parameterize polynomial functions to approximate the vibration of the tool during the cut. Insperger and Stépán have introduced the semi-discretization method [9]. According to this method the delay term is kept constant for a short time. For this small time the resulting ordinary differential equation (ODE) is solved under the restriction, that its solution is compatible to the solution of the preceding and subsequent ODEs. Besides solving the DDEs approximately, a numerical step-by-step time-domain approach is possible. For each time step the force acting on the tool and work piece is calculated based on the present

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cutter-work piece engagement. The deflection response of the machine structure to this force is determined. This deflection influences the engagement and thus the force changes. The new force is used to repeat the calculations in the next time step. This step-by-step approach has been adopted by several authors, e.g. [10, 11, 12].
The abovementioned research works are announced here to represent the large amount of work carried out on modelling regenerative chatter in cutting processes. Most of the documented models are built for time invariant structural dynamics. This approximation seems to be valid in many cases. However, the dynamic properties of machine tool structures can vary for various reasons. Tool and work piece exchanges alter the dynamic system. Moreover the removal of material from the work piece can have a noticeable effect. As soon as a machine tool undergoes excessive axis movements or synchronous movements of several axes, the changing stiffness and mass distributions lead to changing dynamics which might influence the process vibrations.
According to [13], modal parameters can be interpolated to describe the dynamics of a work piece, whose dynamic properties change remarkably, when material is removed. Law and his colleagues [14] set up a reduced three axis milling machine model that can be moved to different positions. Based on this model Frequency Response Functions (FRFs) are determined for different axes positions and are fed to frequency domain stability simulations. Moreover [15] has developed a method to interpolate FRFs, that can represent crossing eigenmodes.
As far as we now, time domain process simulations that account for changing dynamics due to axes movements have not been studied excessively so far. However, time domain approaches can handle more aspects (e.g. nonlinearities, changing cutter-work piece engagements) than frequency domain approaches. As a consequence, this paper deals with a method that allows incorporating the time variant structural dynamics in the step-by-step time domain simulation of process vibrations. A three axis milling machine serves as an application example and an exemplary 2.5-dimensional milling process is simulated.

2. Time and positon variant dynamics of machine tool

Changes of the machine axes positions can affect the dynamic properties. Here, a method to account for the resulting time variant machine dynamics is presented. The proposed method interpolates the modal parameters (poles and eigenvectors) between a set of spatial sampling points. The interpolation strategy can be used to calculate FRFs or time domain forced responses for complex tool paths accounting for changing dynamics. The following paragraph describes how FRFs have been determined experimentally for a three axis milling machine. The FRFs determined for different axis positions are compared. The subsequent paragraph presents the basic idea of modelling time or axis position dependent dynamics. The issue of changing mode orders is addressed as well as the extension of a time domain simulation approach of forced responses.

2.1. FRFs measured in different axis positions

The dynamic compliance of a three axis milling machine is determined experimentally by measuring frequency response functions in two axis positions. In each of the axes positions the 3 by 3 FRF-matrix

$$\boldsymbol{H} = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix} \quad (1)$$

is determined. This frequency dependent matrix describes the relative dynamic compliance between tool and work piece. Figure 1 shows a picture of the measurement setup that is used to determine G_{xx}. The force excitation is done with a hydraulic exciter that is positioned between a dummy tool and a dummy work piece. The generated force $F = F_{stat} + F_{dyn}$ is measured as well as the accelerations on the work piece and the tool side ($\ddot{x}_{wp}$, $\ddot{x}_{tool}$). Moreover the relative displacement x_{rel} between work piece and tool is directly captured by an inductive sensor.

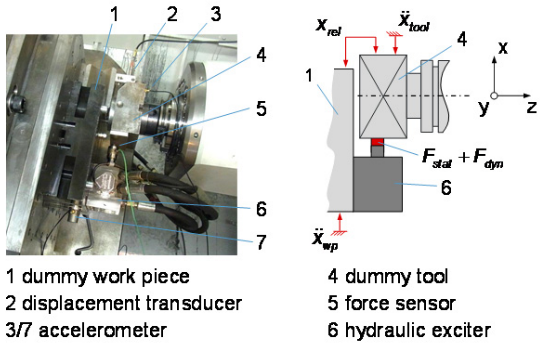

Figure 1: Measurement setup for determination of G_{xx}.

The sketch in Figure 2 shows the two different y-positions that have been considered. The absolute values of the direct FRFs G_{xx}, G_{yy} and G_{zz} are plotted with a linearly scaled ordinate for the two different y-positions. Naturally the dynamic compliance differs between the Cartesian directions, but noticeable differences appear between the tested y-positions as well. The filled areas give a good impression of the differences due to the y-position-variation. The differences in the static compliances are easy to explain by considering the changing leverages. The changes in the dynamic properties cannot be explained just as intuitively. Especially in the frequency range between 60 and 90 Hz the dynamic compliance in x-directions seems to be sensitive to modifications of the y-position. The z-direction seems to show a major sensitivity in the range between 120 and 150 Hz. The dynamic properties between the discrete set of measured positions is unknown. Although the filled areas illustrate the range between the measured compliances, it is theoretically possible, that positions in between show compliances that exceed the filled areas. This issue can be resolved by considering the modal parameters instead of the frequency dependent compliance as is explained in the following paragraph.

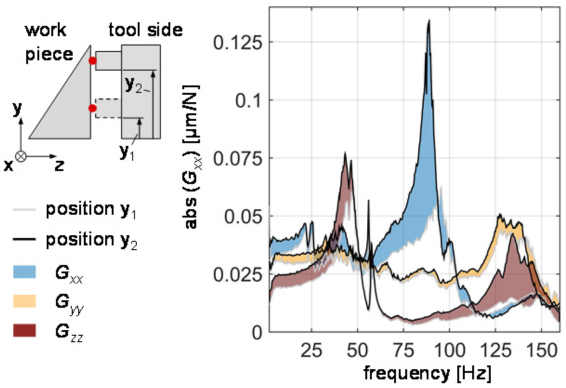

Figure 2: Measurement of direct FRFs in different y-positions.

2.2. Modelling time-variant dynamics

Being only limited by the resolution of the translational or rotatory encoders a machine tool can move its axes to a vast amount of different axis positions. The determination of the dynamic properties in either all the possible axis positions or at least in all positions along the different tool paths would be expensive regarding the number of necessary calculations or measurements. Thus we propose to determine the modal properties (they can be used to synthesize the FRF-matrix) in several discrete positions and then use an interpolation procedure to predict the dynamic properties between these discrete points. The mentioned procedure was presented in [15, 16] and is used there to predict the position dependent dynamics of a single linear axis. Here, the method is adapted for use with multiple axis. Now the interpolation is performed along an arbitrary multi-dimensional path. Positions on this path can be selected by a single path parameter s. The basic idea is presented in Figure 3. The modal parameters are calculated for the first position (s=0). They are arranged according to the increasing eigenfrequencies. To account for a possible change of the mode order between subsequent tool path increments a MAC correlation (see [17], for example) between the current and the preceding eigenvector matrix is performed. As soon as such mode switches appear, this is indicated by two off diagonal entries in the MAC matrix with values close to one. The modal parameters of the current position are then reordered according to the switch indicated by the MAC correlation. This procedure is repeated for all the discrete sampling positions.

Exemplary trends of eigenfrequencies over position s are plotted in the lower part of Figure 3 for the tracked and for the untracked case. The eigenfrequencies show several discontinuities that are related to the edge length of the finite elements in the direction of the axis movement. As soon as the modal parameters are tracked properly, a smoothing can be performed to minimize this discontinuities. Finally an interpolation can be performed to increase the number of sampling points along the tool path. Thus, in the time domain step-by-step simulation, a new set of modal parameters is available for every time step. The changing dynamics are modeled by means of digital filters. The digital filter approach for the calculation of forced responses is presented in [18] as a more accurate alternative to common finite-difference approaches. According to [18] the filter coefficients $b_{0...p}$ and $a_{1...q}$ of the recursive relationship

$$x_n = b_0 f_n + b_1 f_{n-1} + \ldots + b_p f_{n-p} - a_1 x_{n-1} - a_2 x_{n-2} - \ldots - a_q x_{n-q} \quad (2)$$

can be efficiently calculated from modal parameters. Thus the deflection x_n at the current time step n is calculated based on the deflections at q preceding time steps and the forces at the current and p preceding time steps. The number of preceding time steps involved (q and p) depend on the strategy (e.g. ramp invariant, step invariant or impulse invariant approach) that is used to determine the filter coefficients. As soon as multiple structural modes are considered, the deflections are calculated for each mode separately before they are summed. Moreover if multiple degrees of freedom (DOF) are considered, e.g. if multiple forces act on the structure, cross compliances are taken into account. For this purpose, filter coefficients are calculated for all the considered direct and cross compliances and the total deflection at a selected DOF results from the superposition of all responses at this DOF that result from the different force components.

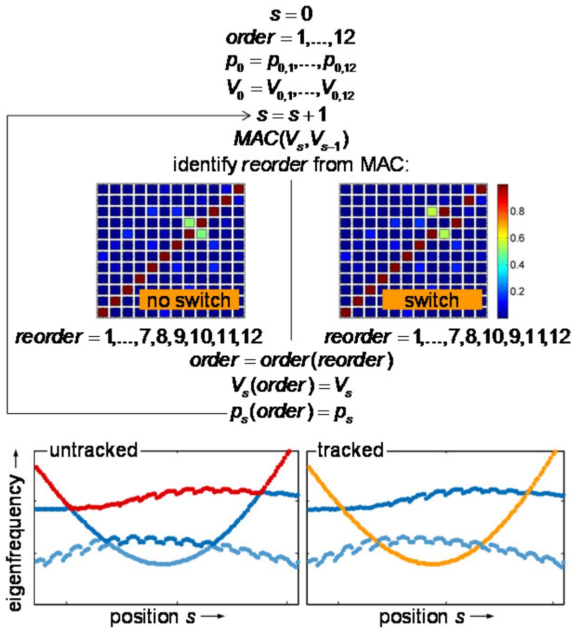

Figure 3: Principle of mode tracking along tool path positions s.

3. Application to machine tool

3.1. Component models

In this paper the method of modal interpolation is extended to cope with multi-axis machines. Although the modal parameters are only calculated in several discrete axis positions, a model reduction on component level is performed to speed up the calculations. The Dual Craig Bampton [19] method is adopted to reduce the number of component DOF. The original component DOF $\boldsymbol{u}$ are approximated by a limited number of free vibration modes $\boldsymbol{\Phi}$ and residual flexibility attachment modes $\boldsymbol{\Psi}_r$. The former are weighted by modal amplitudes $\boldsymbol{\eta}$, the latter are weighted by boundary forces $\boldsymbol{g}_b$ that act on the boundary DOF. Constraints can be put on the boundary forces to connect different components with each other or to prevent a component from floating, for example. As a formula the reduction writes

$$\begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{g}_b \end{bmatrix} \approx \begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{\Psi}_r \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{g}_b \end{bmatrix} = \boldsymbol{R}_{DCB} \begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{g}_b \end{bmatrix}. \quad (3)$$

The huge number of displacement DOFs are replaced by a few modal amplitudes. This way, the system size is drastically reduced. The boundary forces (only a small number) remain accessible after the reduction. The reduction basis of the quite new Dual Craig Bampton Method [19] is physically more intuitive than the basis of the common Craig Bampton method [20]. The latter includes the constraint flexibility modes which are experimentally difficult to identify. Moreover, using the Dual Craig Bampton Method, modal damping can be introduced on component level to account for the inner damping of the component. For a welded steel component 0.1 % modal damping led to a good correlation with experiments as documented in [21]. Although several parts of the machine tool considered are made of cast iron, the same damping ratio is chosen here, for now. After the reduction process mass, stiffness and damping matrices are available for each structural component.

Figure 4 shows the finite element model of the machine column. All nodes that are meant to be connected to neighboring components are defined as boundary nodes. Consequently their DOF remain accessible even after the component reduction. The most expensive part of the component reduction is the calculation of the residual flexibility attachment modes, since an inversion of the full stiffness matrix is necessary.

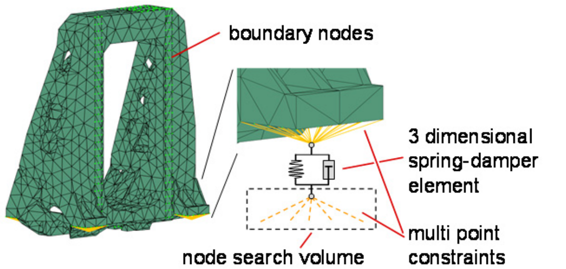

Figure 4: Modelling structural components and joints between components.

3.2. Realization and updating of multi point constraints

The different components are connected by multi-point constraints (MPCs) and three dimensional linear spring-damper elements. This component connection is presented in Figure 4 for the case of a linear guide rail-guide shoe connection. A three dimensional spring-damper element connects two nodes, that are positioned on top of each other in the center of the rail-shoe connection (in the figure, a gap is present to show the spring-damper element). The motions of the nodes of the spring-damper element are determined as a weighted average of the movement of a set of slave nodes, which are subsets of the boundary nodes of the connected structural components. Here, the multi-point interpolation constraint formulation presented in [22] is implemented. The DOFs of the single node that is connected to the spring-damper element are the slave DOFs ($\boldsymbol{u}_S$). Their motion is related to the motion of the master DOFs ($\boldsymbol{u}_M$) via

$$\begin{aligned} &\boldsymbol{R}_M \boldsymbol{u}_M + \boldsymbol{R}_S \boldsymbol{u}_S = \boldsymbol{0}\,, \\ &\boldsymbol{u}_S = \boldsymbol{B}_S \boldsymbol{u}\,,\ \ \boldsymbol{u}_M = \boldsymbol{B}_M \boldsymbol{u}\,. \end{aligned} \quad (4)$$

The relation between the DOFs is put in the matrices $\boldsymbol{R}_M$ and $\boldsymbol{R}_S$, respectively. The DOFs affected by the MPC are selected from the total DOFs ($\boldsymbol{u}$) with help of the Boolean matrices $\boldsymbol{B}_S$ and $\boldsymbol{B}_M$, respectively. Further details on the realization of constraints for finite element models can be found in [23], [24] or [25], for example.

Regarding the machine tool assembly, the connection to the component to which the shoe is mounted remains unchanged, when the machine axis moves. The connection to the rail, by contrast, is updated, when the corresponding machine axis moves. For each time step or position increment, respectively, the boundary nodes in the node search volume are identified, and the MPC is updated. Consequently the subsets of boundary nodes that serve as master nodes in the MPC change depending on the machine axis positions. As the boundary nodes are all the nodes that will become MPC master nodes during the machine axis movements, the component reduction process has to be done only once for each structural component.

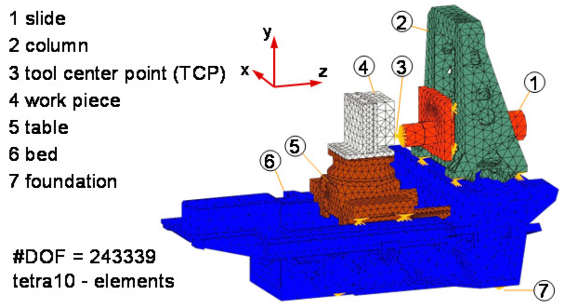

Figure 5: Machine assembly.

3.3. Machine assembly

The studies presented her aim at the introduction of the method of modal interpolation for multiple machine axes. As a subject of study, a three axis milling machine as depicted in Figure 5 is chosen. This machine consists of different structural components, whose finite element models are reduced as explained above. Here, only the guide-shoe connections and foundation elements are taken into account. They are modeled as spring-damper elements. Ball-screw nuts and spindles, motors and drives are neglected, so far. All in all, the machine model is kept simple for the assessment of the simulation of the time-variant machine dynamics. For the assembly, the reduced component models and the linear spring-damper models are arranged in block diagonal form. The assembly is realized by relating the unassembled not unique DOFs to a unique set of DOFs:

$$\boldsymbol{u}_d = \boldsymbol{L}\boldsymbol{u}_{da} \quad (5)$$

The DOFs $\boldsymbol{u}_d$ and $\boldsymbol{u}_{da}$ contain modal amplitudes, displacements and boundary forces. More details on the formulation of these vectors and the assembly matrix L can be found in [16] or [25].

The machine model considered here has not yet undergone a model updating process. The spring-damper elements of guides and foundation elements are parameterized with stiffness and damping coefficients that are known for other machines of similar dimensions. Consequently a perfect match between simulation and experiment cannot be expected at this point of research. The modal parameters are calculated as the solution of an eigenvalue problem. The eigenvalue problem is formulated in state-space form with the reduced mass, stiffness and damping matrices. Thus, complex eigenvectors and poles are the results of the eigenvalue problem.

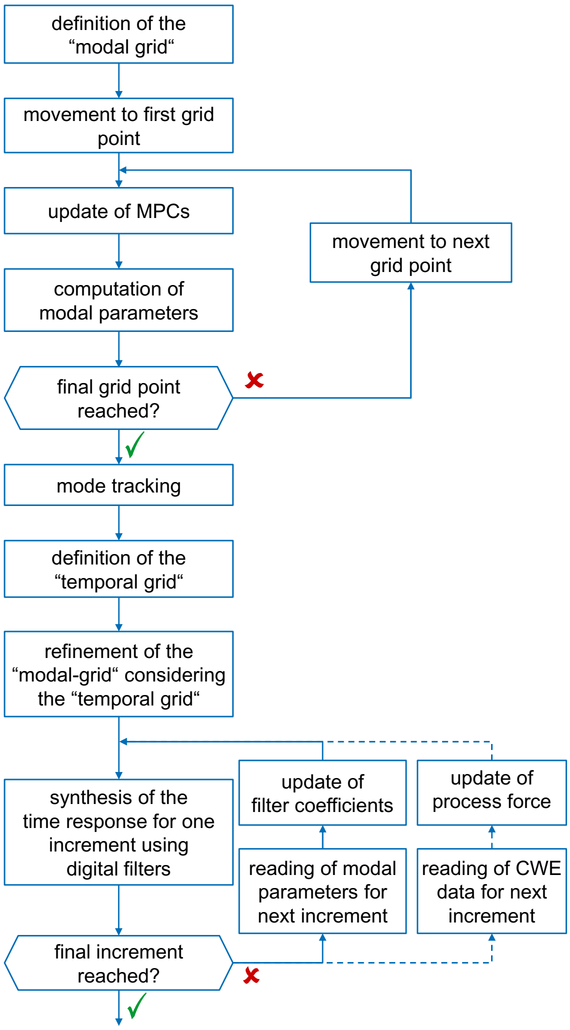

Figure 6: Steps for the time domain simulation of machine tool with time-variant dynamics.

3.4. Mode tracking along tool path

The method of modal interpolation can easily be extended to multiple machine axis. The necessary steps are presented in Figure 6. First, a "modal grid" along the tool path is defined. Then, the machine model is moved to the first grid point and the MPC are formulated for the master nodes within the search volumes. The modal parameters for the current position are determined as the solution of an eigenvalue problem. If the last point of the modal grid is not reached, the machine moves to the next grid point and again, the modal parameters are computed. If the last grid point is handled a database with modal parameters for all the modal grid points along the tool path is available. Using this database, mode tracking is performed according to Figure 3. Next, a "temporal grid" is defined. The resolution of this grid depends on the feed rate of the considered process and the selected sampling frequency. If the feed rate is constant, the distance between the nodes of the temporal grid are equidistant along the tool path. The modal grid is refined according to the temporal grid. This refinement is done via a simple one dimensional (linear) interpolation. For every time step the digital filter coefficients are determined from the current set of modal parameters and the current deflections at the tool center point are determined according to Equation (2).

4. Extension of process machine simulation and application to a reference process

Most of the studies related to the simulation of regenerative chatter and machine tool vibrations neglect the changing machine dynamics. Figure 6 illustrates with a dashed line how the process machine simulation can be done by taking into account the changing machine dynamics. Based on the current cutter-work piece engagement, the current process forces are calculated and the current modal parameters are read from the database. The digital filter coefficients and the acting forces are updated and the deflections for the next time increment are simulated.

4.1. Dynamics along exemplary tool path

Here, a 2.5-dimensional reference process is defined to evaluate the method of modal interpolation for multiple axis movements. The work piece and the tool path are depicted in Figure 8. The movements of the y- and x-axis of the machine both cover approximately 280 mm. The generation of the depicted tool path is realized by a synchronous movement of slide and column. In the top of Figure 7 the absolute value of the residues R_{xx} are plotted over the eigenfrequency and the tool path position s. From this graph it can be seen, that major differences in the magnitudes of the residues occur for different positions on the tool path. The eigenfrequencies do not change as much, but still a few mode switches occur. According to the relationships

$$G_{xx} = \sum_{k=1}^{n} \frac{R_{xx,k}}{j\omega - p_k} + \frac{\overline{R}_{xx,k}}{j\omega - \overline{p}_k}, \quad (6)$$

$$R_{xx,k} = R_{xx,k}^{tool} - R_{xx,k}^{wp},$$

$$R_{xx,k}^{tool} = v_{x,k}^{tool} v_{x,k}^{tool}, \quad R_{xx,k}^{wp} = v_{x,k}^{wp} v_{x,k}^{wp},$$

the relative FRF in x-direction between tool and work piece can be determined as a sum over n modes. The contribution of each mode depends on the residue $R_{xx,k}$ and the pole p_k. The residue results from summing the residues of tool and work piece side. Each of these residues is calculated as a product of eigenvector entries. Here the DOF at the TCP in x-direction are selected. It is assumed that the eigenvectors have been scaled to unity modal A, as described in [26], for example. Figure 7b shows the magnitude of G_{xx} along the tool path.

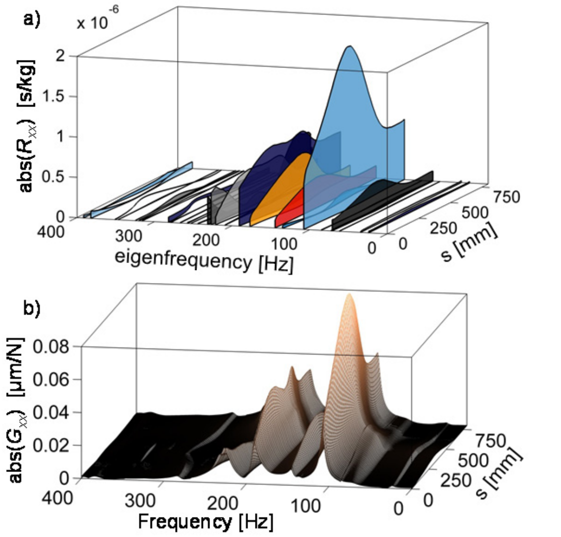

Figure 7: a) Magnitude of residues R_{xx} and eigenfrequencies along tool path. b) Magnitude of G_{xx} along tool path.

4.2. Simulation and measurement of in-process forces and accelerations

The work piece features three holes, which lead to changes of the cutter work piece engagement. The entry and exit angles of the milling cutter with respect to the tool coordinate system are depicted on the right hand side of Figure 8. The theoretical cutting force on the work piece in x-direction is plotted in the bottom of Figure 8. This simulated force signal is determined based on the cutter-work piece engagement and a linear force model. Changes of chip thickness due to vibrations are not considered. It can be seen, that the force depends on the tool path positon s. If the cutter (four inserts, $z = 4$) is fully immersed, the simulation shows no periodic influence from the inserts entering and leaving the material. This periodic influence can clearly be seen if there is no full immersion, e.g. if the tool enters the material or it comes to intermittent cuts at holes in the material.

A time domain milling process simulation is performed for the example process. Figure 9 presents the simulated deflections of the tool in x-direction. On the one hand the dynamic properties of the machine are kept constant. In this case the modal parameters are determined only once in the initial tool path position ($s = 0$). On the other hand the changing dynamics are considered. The machine vibrates in a similar fashion, but differences can clearly be seen. First of all, the static compliance changes along the tool path which leads to low frequency changes of the deflections. Moreover, the forced vibrations resulting from the intermittent cuts differ between the two considered cases.

The differences in the simulated deflections motivate to consider the changing dynamics, when searching for stability limits as a small difference in the dynamic compliance might change the process stability state.

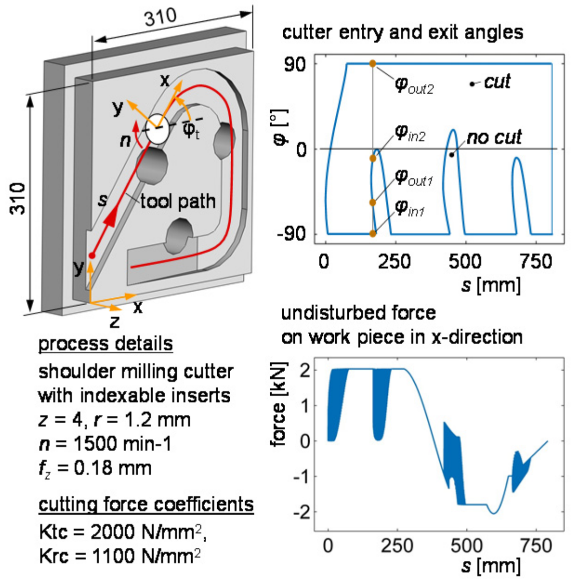

Figure 8: Reference process and undisturbed cutting forces.

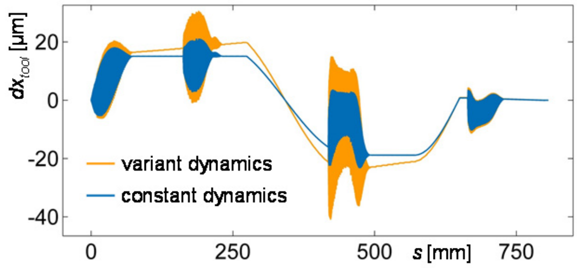

Figure 9: Simulated tool x-deflection with constant and variant machine dynamics.

In order to assess, how big the gap between simulation and experiment is, the reference process is conducted experimentally and the acceleration of the tool is measured in x-direction. This acceleration signal is high-pass filtered using a FIR filter with a cutoff frequency of 5 Hz and integrated twice using a IIR filter, according to [27]. The simulated x-deflections from Figure 9 are also high-pass filtered. The three signals are compared in Figure 10. The overall amplitudes of the signals are comparable, but remarkable differences between measurement and simulation are obvious. In practice the four inserts seem to produce a remarkable varying cutting force, when there is full immersion, although they should not according to theory. At present it is not clear if these vibrations are due to geometric tolerances or a feedback of the vibrations on the cutter work piece engagement, for example.

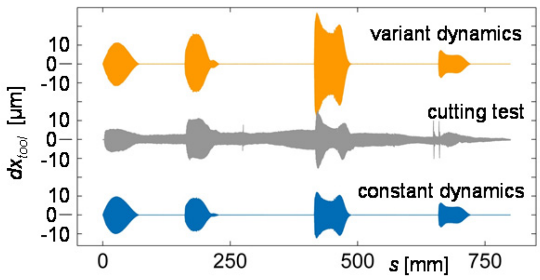

Figure 10: Comparison of simulated and measured deflections along the tool path.

5. Summary and outlook

Much research has already been carried out to optimize the prediction of regenerative chatter vibrations and the construction of the well-known stability charts. In many cases the machine dynamics is represented in terms of modal parameters that are assumed to remain constant during the simulations. Theoretically this assumption is rarely true, as movements of machine axes mean changes of the dynamic properties. This paper presents the method of modal interpolation that allows accounting for the changing machine dynamics during time domain process simulations. The modal parameters are calculated for discrete sample points along the considered tool path. An interpolation refines the grid of modal parameters to the resolution that is needed for a step-by-step time domain simulation. The extended simulation method is used to model a 2.5-dimensional milling process. The machine model used for the studies presented here has not been updated by comparison to measurements. Consequently the correlation between simulated and measured accelerations is low. Moreover, the sample process treated in Figure 10, does not show better correlation with measurement for the case where changing dynamics have been taken into account. Nevertheless, the presented method seems to be appropriate to model the changing dynamics and motivates further investigations.

The machine model will be supplemented by models for the ball-screw spindles and drives. A model updating process will increase the quality the machine model's quality. Several cutting tests will be performed to verify the new simulation model. Guidelines should be worked out giving advice whether to take into account changing dynamics or not. Moreover, the efficiency of the algorithms needs to be improved.

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