An analytical and computational study of crack initiation under transient creep conditions

C.M. Davies a, N.P. O’Dowd b,*, K.M. Nikbin a, G.A. Webster a

a Department of Mechanical Engineering, Imperial College London, South Kensington Campus, London SW7 2AZ, United Kingdom
b Department of Mechanical and Aeronautical Engineering, University of Limerick, Limerick, Ireland

Received 30 April 2006; received in revised form 28 August 2006
Available online 6 September 2006

Abstract

An analytical method has been developed to predict creep crack initiation (CCI), based on the accumulation of a critical level of damage at a critical distance. The method accounts for the re-distribution of stress from the elastic or elastic–plastic field, experienced on initial loading, to a steady state creep stress distribution, via a transient creep region. The method has been applied to predict CCI times in a fracture specimen of type 316H stainless steel at 550 °C. The failure model has been also been implemented into a finite element (FE) framework. Reasonable and conservative predictions of CCI time can be obtained from the analytical solution relative to FE solutions. Conservative predictions of experimental CCI times are obtained when stress redistribution is taking into account. However, CCI times predicted from a steady state creep model are found to be non-conservative.

Keywords: Creep crack initiation; Fracture mechanics; Transient creep; Finite element analysis; HRR; Stress redistribution

1. Introduction

Failure of components operating at high temperature can occur by crack growth from pre-existing defects due to the accumulation of time dependent inelastic (creep) strains. Creep crack initiation (CCI) in a ductile material may be considered to occur at the time when micro-cracks (formed by the nucleation, growth and coalescence of voids ahead of a pre-existing defect) first link up with the main defect (Holdsworth, 1992). These voids and micro-cracks can generally be referred to as ‘creep damage’. The CCI period may occupy a large fraction of the components lifetime in high temperature plants. Thus the prediction of the CCI time is fundamental in high temperature component life assessments.

At high temperatures, crack tip stresses will re-distribute from the linear elastic $K$ field or elastic–plastic HRR field towards a steady state creep field via a transient creep region. Many initiation prediction methods assume that initiation occurs under steady state conditions, controlled by the $C^*$ parameter, for example the
Riedel and Rice (1980) predictions for widespread creep conditions and the NSW crack growth models (Austin and Webster, 1992).

In this work an analytical method to estimate CCI times is described. The method accounts for stress redistribution from elastic or elastic–plastic conditions on initial loading to small scale creep and eventually to widespread creep conditions. The sensitivity of the CCI predictions to input parameters is considered and the method is assessed by comparing its predictions with those obtained from finite element (FE) solutions and with experimental data.

2. High temperature deformation

2.1. Elastic–plastic deformation

The Ramberg–Osgood material model is widely used to describe the stress–strain behaviour of isotropic strain hardening materials. It may be written in non-dimensional uniaxial form as

\[
\frac{\varepsilon}{\varepsilon_{p0}} = \frac{\sigma}{\sigma_{p0}} + \alpha \left( \frac{\sigma}{\sigma_{p0}} \right)^N,
\]

where \(\sigma\) and \(\varepsilon\) are the stress and strain, respectively, \(\sigma_{p0}\) and \(\varepsilon_{p0}\) are the normalising (plastic) stress and strain, respectively, \(N\) is the power-law plastic hardening exponent and \(\alpha\) is an additional material constant.

2.2. Creep deformation

The average creep strain rate, \(\dot{\varepsilon}_A\), at a given stress is defined by the ratio of the uniaxial creep ductility, \(\varepsilon_f\), to the time to rupture, \(t_r\),

\[
\dot{\varepsilon}_A = \frac{\varepsilon_f}{t_r}.
\]

The average creep strain rate can be used to describe the three regions of the creep curve–primary, secondary and tertiary creep, as shown in Fig. 1. The stress dependency of the average creep strain rate, \(\dot{\varepsilon}_A\), can often be represented as a power-law, such that,

\[
\dot{\varepsilon}_A = \dot{\varepsilon}_0 \left( \frac{\sigma}{\sigma_0} \right)^n,
\]

where \(n\) is the power-law creep stress exponent and \(\sigma_0\) and \(\dot{\varepsilon}_0\) are normalising creep stress and creep strain rate, respectively.

Fig. 1. Definition of the average creep strain rate, \(\dot{\varepsilon}_A\).
3. Multiaxial creep failure strain relations

Failure is to be predicted using a critical creep strain based criterion. Under multiaxial stress conditions a method is required to determine the critical creep strain for failure, denoted $e_{\text{crit}}^*$. This multiaxial failure strain may be related to the experimentally measurable creep failure strain under uniaxial conditions, $e_{\text{crit}}$, through a multiaxial strain factor (MSF).

$$e_{\text{crit}}^* = e_{\text{crit}} \times \text{MSF}$$

The value of the MSF for a particular material may be determined experimentally or estimated using a failure mechanism model, such as the Cocks and Ashby (1980) model for grain boundary void growth, which is employed here. The MSF derived from the Cocks and Ashby model depends on the ratio between the mean (hydrostatic) and equivalent Mises stress, $\sigma_m/\sigma$, often referred to as the triaxiality and here denoted $h$, and also depends on creep stress exponent, $n$. Using this model it can be shown (see e.g. Davies, 2006) that the MSF is given by

$$\text{MSF}(h; n) = \frac{\text{Sinh}^{2(n-1/2)} h^{n+1/2}}{\text{Sinh}^{2(n-1/2)} h^{n+1/2}}.$$  \hspace{1cm} (5)

The dependency of the Cocks and Ashby MSF on triaxiality, $h$, is illustrated in Fig. 2, for $n = 10$, in which it is seen that the MSF is a strongly decreasing function of triaxiality (in other words an increase in triaxiality leads to a significant decrease in the multiaxial failure strain).

4. Crack tip parameters and crack tip fields

Fracture mechanics relies on the use of characterising parameters to describe the conditions in the vicinity of a sharp crack tip. Depending on the material and/or loading conditions different parameters may be used to characterise the stress and strain fields near the crack tip. For a linear elastic material, or for elastic–plastic materials below their yield stress, the stress field in the vicinity of a sharp crack, $\sigma_y$, may be written as (Williams, 1957)
\[ \sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(0), \]  

where \( K \) is the linear elastic stress intensity factor, \( r \) is the radial coordinate from the crack tip and \( f_{ij} \) is a non-dimensional function of angle, \( \theta \). Defining \( f_m \) and \( \bar{f} \) as the corresponding mean (hydrostatic) and Mises equivalent stress functions, respectively, which are non-dimensional functions of \( \theta \) and \( \nu \), the triaxiality under linear elastic plane strain conditions, here designated, \( h_k \), is given by,

\[ h_k = \frac{f_m(\theta; \nu)}{f(\theta; \nu)} = \frac{2}{3} (1 + \nu) \left( (1 - 2\nu)^2 + 3\sin^2 \left( \frac{\theta}{2} \right) \right)^{-\frac{1}{2}}. \]

Solutions for the angular functions \( \bar{f} \) and \( f_m \) are widely available, (see e.g. Webster, 1994).

For most materials, plastic deformation occurs near the crack tip where the stress exceeds the yield stress of the material, leading to the formation of a plastic zone close to the crack tip. In a material which deforms according to Eq. (1), the crack tip stress and strain distributions are given by the HRR field (Hutchinson, 1968; Rice and Rosengren, 1968). The amplitude of the stress is given by the path independent \( J \) parameter (Rice, 1968) such that

\[ \sigma_{ij} = \frac{J}{\sigma_0 \sigma_0 \sigma_0 I_N R} \bar{\sigma}_{ij}(0; N), \]

where \( I_N \) is a dimensionless constant depending on \( N \). The functions \( \bar{\sigma}_{ij} \) are non-dimensional function of crack tip angle, \( \theta \) and \( N \). Solutions for \( \bar{\sigma}_{ij} \) are tabulated (Shih, 1983). Defining \( \bar{\sigma}_m \) and \( \bar{\sigma} \) as the corresponding mean (hydrostatic) and Mises equivalent stress functions, respectively, which are also non-dimensional functions of \( \theta \) and \( N \), we can write the triaxiality within the HRR field, designated \( h_{HRR} \) as

\[ h_{HRR} = \frac{\bar{\sigma}_m(N, \theta)}{\bar{\sigma}(N, \theta)}. \]

For a material which exhibits time dependent (creep) deformation, a creep zone will develop close to the crack tip for times, \( t > 0 \). The size of this creep zone will increase with time, ultimately encompassing the whole specimen or component. At short times, the creep zone is small in relation to the crack length (or uncracked ligament width) and conditions are designated as small scale creep (SSC). During transition creep (intermediate times) the creep zone is comparable to the relevant specimen dimensions. Under these transient creep conditions the elastic and/or plastic strain rates in the vicinity of the crack tip may be significant. The crack tip fields have the same form as the HRR field and are analogous to Eq. (8), with the amplitude controlled by a time dependent parameter designated as \( C(t) \) (Bassani and McClintock, 1981) and the hardening exponent \( N \) and the normalising plastic stress and strain replaced by the creep stress exponent \( n \) and the normalising creep stress and strain rate, respectively. Hence, the transient creep fields are described as (Riedel and Rice, 1980)

\[ \sigma_{ij} = \frac{C(t)}{\sigma_0 \sigma_0 I_n R} \bar{\sigma}_{ij}(0; n), \]

where \( I_n \) is a dimensionless constant depending on creep stress exponent, \( n \), and \( \bar{\sigma}_{ij} \) is a non-dimensional function of crack tip angle, \( \theta \) and \( n \). These fields are referred to here as the RR fields. The \( C(t) \) parameter can be defined by a path dependent crack tip integral, the value of which is determined along a path very close to the crack tip. Thus in a numerical analysis a fine mesh is required to resolve the value of \( C(t) \).

At long times, the creep zone extends through the entire uncracked region (widespread creep) and steady state condition are established with stress and strain no longer varying with time. Under widespread creep conditions \( C(t) \) becomes both path and time independent and is designated \( C^* \) (Landes and Begley, 1976; Nikbin et al., 1976). The crack tip stress fields are then given as

\[ \sigma_{ij} = \frac{C^*}{\sigma_0 \sigma_0 I_n R} \bar{\sigma}_{ij}(0; n). \]
Eq. (11) is here denoted the RRss field (the steady state Riedel and Rice field). The triaxiality during steady state and transient creep, denoted \( h_{RRss} \) and \( h_{RR} \), respectively, are equal and given by

\[
h_{RR} = h_{RRss} = \frac{\tilde{\sigma}_m(\theta; n)}{\bar{\sigma}(\theta; n)},
\]

where, in this case \( \tilde{\sigma}_m \) and \( \bar{\sigma} \) are, respectively, non-dimensional functions of \( \theta \) and \( n \), corresponding to the mean (hydrostatic) and Mises equivalent stress functions in the RR and RRss field.

Eqs. (6), (8), (10) and (11) describe the range of stress conditions which may prevail near a sharp crack tip. Different parameters may be relevant at different times and positions for a given specimen or component. For example, at short times in an elastic–plastic material it is expected that close to the crack tip the stress field will be given by the HRR field scaled by \( J \) (Eq. (8)) while further away from the crack tip the appropriate parameter is \( K \). Under SSC the stress fields close to the crack tip will be given by the RR distribution, Eq. (10), with amplitude \( C^* \) while as steady state conditions are reached the stress will be given by the constant RRss distribution with amplitude \( C^* \). For a fixed material point therefore, the stress field will change with time and thus any method to predict crack initiation should take this transient behaviour into account.

### 4.1. Estimation of crack tip parameters

To carry out a failure assessment, methods are required in order to evaluate the appropriate crack tip characterising parameter.

Under linear elastic conditions \( J \) is related to the stress intensity factor, \( K \), by

\[
J = \frac{K^2}{E'},
\]

where \( E' \) is the effective elastic modulus (= \( E \) for plane stress and \( E/(1 - \nu^2) \) for plane strain conditions). Eq. (13) applies for a linear elastic material. Under small scale yielding conditions (SSY) when the plastic zone size is small, Eq. (13) can be adjusted for crack tip plasticity by evaluating \( K \) at an effective crack length, \( a_e \). The EPRI (Kumar et al., 1981) estimate for \( J \) covers the full elastic–plastic range and is given by the sum of the SSY and plastic components:

\[
J = \frac{K^2(a_e)}{E'} + \varepsilon \sigma_{p0} \sigma_{p0} ch_1(a/W; N) \left[ \frac{P}{P_{p0}} \right]^{N+1},
\]

where \( h_1 \) is a dimensionless function of normalised crack length and hardening exponent, \( N \), \( P_{p0} \) is a normalising load related to \( \sigma_{p0} \) and \( c \) is the characteristic length scale for the geometry (e.g. crack length). Solutions for \( h_1 \) have been obtained numerically and are tabulated in, e.g. Kumar et al. (1981).

For a power-law creep material, \( C^* \) may be evaluated using the fully plastic component of the EPRI solution for \( J \) (the second term on the RHS of Eq. (14)) by replacing the quantities \( \varepsilon_{p0}, \sigma_{p0}, N, \) and \( P_{p0} \) in Eq. (14) by \( \dot{\varepsilon}_0, \sigma_0, n \) and \( P_0 \), respectively, that is

\[
C^* = \dot{\varepsilon}_0 \sigma_0 ch_1(a/W; n) \left[ \frac{P}{P_0} \right]^{n+1},
\]

where \( P_0 \) is a normalising load related to the normalising stress \( \sigma_0 \).

Eq. (15) may be considered to be an exact solution for \( C^* \) with \( h_1 \) determined from finite element solutions. No equivalent result is available for \( C(t) \) but a number of approximate solutions are available. Ehlers and Riedel (1981) provide an expression to estimate \( C(t) \) for times ranging from SSC to widespread creep conditions in an elastic-power-law creeping material,

\[
C(t) = \left( 1 + \frac{K^2/E'}{C^*(n + 1)t} \right) C^*.
\]

Ainsworth and Budden (1990) have developed an alternative expression for estimating \( C(t) \) under elastic-creep conditions,
\[ C(t) = \left( \frac{(1 + t/\tau)^{n+1}}{(1 + t/\tau)^{n+1} - 1} \right) C^*. \]  
(17)

In Eq. (17) \( \tau \) is the redistribution time, an estimate of the time taken for \( C(t) \) to reach its steady state value, \( C^* \), and is determined from
\[
\tau = \frac{J}{C},
\]  
(18)

where \( J \) takes its value on initial loading (Ainsworth and Budden, 1990). Similar predictions of \( C(t) \) are obtained from Eqs. (16) and (17) (see Ainsworth and Budden, 1990) and at short times the predictions from the two equations are indistinguishable. The expression shown in Eq. (17) for \( C(t) \) has the advantage over Eq. (16) that it may be easily integrated with respect to time, though in general Eq. (16) is preferred as it provides a more accurate solution (compared to FE solutions), particularly for times close to the redistribution time, \( \tau \).

For an elastic–plastic-creeping material, where the creep stress exponent is greater than the plasticity hardening exponent \((n > N)\), at short times \( C(t) \) may be estimated from (Kim et al., 2000; Riedel, 1987)
\[ C(t) = \frac{J}{(n+1)t}, \]  
(19)

where \( J \) takes its elastic–plastic value on initial loading. Following the approach of Ehlers and Riedel (1981) the short time solution of Eq. (19) and the long time solution \((C(t) = C^*)\) can be combined to provide an estimate for \( C(t) \) under elastic–plastic conditions
\[ C(t) = \left( 1 + \frac{\tau}{(n+1)t} \right) C^*, \]  
(20)

where \( \tau \) is evaluated from Eq. (18) using the elastic–plastic value of \( J \) on initial loading. Under elastic-creep conditions or small scale plasticity, Eq. (20) becomes equal to Eq. (16). An easily integrable expression for \( C(t) \), in the form of Eq. (17), for elastic–plastic-creep conditions has yet to be determined.

5. Initiation prediction models

A number of different methods exist to predict CCI which can be classified into two main approaches. Firstly the individual microstructural damage mechanisms can be modelled (see e.g. Onck and van der Giessen, 1999; Michel, 2004; Sun et al., 1997) and fracture examined at the length scale at which the damage mechanisms are operative. Such approaches generally require a large number of material parameters, which may be difficult to obtain. Alternatively a continuum damage based approach can be employed (see e.g. Becker et al., 2002; Bouchard et al., 2004) where the effects of the specific fracture phenomena are accounted for through a single damage variable (Kachanov, 1986). In this work, the latter approach is taken.

In previous work, analytical models have generally assumed that crack initiation and growth occur under steady state conditions, and are thus based on the steady state \( C^* \). Riedel and Rice (1980) introduced a model to describe initiation from a stationary crack tip with initiation deemed to occur at the attainment of a critical equivalent strain at a characteristic distance. The model accounts for the angular variation in stress and strain, but the critical equivalent strain is assumed to be a material property (independent of stress triaxiality). More recently, a model known as the NSW-MOD model was developed by Yatomi et al. (2006) to predict steady state creep crack growth. The model has been applied to predict creep crack initiation (Davies et al., 2006) taking the initiation time to be the time for a measurable amount of crack growth to occur (under steady state conditions). In the NSW-MOD model, crack growth is deemed to occur when a critical amount of damage has accumulated ahead of a growing crack tip. The model accounts for the angular variation in the crack tip stress and strain fields and the associated variation in the crack tip triaxiality and allows for crack initiation and growth along an arbitrary angle.

An enhancement to these models was presented in Davies et al. (2006). In this model CCI is predicted from a stationary sharp crack tip. Failure is phrased in terms of the accumulation of creep damage and crack initiation occurs when damage reaches a critical condition at the critical distance, \( d \). The critical damage may first
be attained at any crack tip angle, \( \theta \), (see Fig. 3) and therefore the angular dependency of the critical equivalent strain and equivalent stress must be accounted for. The distance, \( d \), may be related to a characteristic microstructural length, e.g. grain size. The stress/strain at \( d \) may also be considered as a representative measure of the stress/strain within a volume of the creep process zone. In this work, a ductility exhaustion model is assumed so that the rate of creep damage is given by the ratio of the equivalent creep strain rate, \( \dot{\varepsilon}_c \), to the critical creep strain under multiaxial conditions, \( \varepsilon_{crit} \). For a power-law creeping material the rate of creep damage, denoted \( \dot{\omega} \), is written as

\[
\dot{\omega} = \frac{\dot{\varepsilon}_c}{\varepsilon_{crit}} = \frac{\dot{\varepsilon}_0}{\varepsilon_{crit} \sigma_0} \left( \frac{\sigma(t)}{\sigma_0} \right)^n,
\]

where the equivalent stress, \( \sigma \), and the triaxiality, \( h \), take their appropriate values, depending on whether the stress at the initiation distance, \( d \), for a given time follows the \( K \), HRR, RR or RRss field solution. Note that when employing the Cocks and Ashby relation the MSF is also an explicit function of creep exponent, \( n \). The critical strain, \( \varepsilon_{crit} \), is assumed to be constant for a given material and temperature. The total creep damage accumulated in a time \( t \) is then evaluated from the integral

\[
\omega(t) = \int_0^t \dot{\omega} \, dt = \frac{\dot{\varepsilon}_0}{\varepsilon_{crit} \sigma_0} \int_0^t \frac{\sigma^n(t)}{MSF(h)} \, dt.
\]

At \( t = 0 \), the material is assumed to be undamaged and \( \omega = 0 \). Failure is conceded when the damage parameter, \( \omega \), attains the value of unity. No additional complication is introduced if it assumed that the material has experienced some creep damage prior to the current loading. In this case Eq. (22) can be replaced by

\[
\omega(t) = \omega_0 + \int_0^t \dot{\omega} \, dt = \omega_0 + \frac{\dot{\varepsilon}_0}{\varepsilon_{crit} \sigma_0} \int_0^t \frac{\sigma^n(t)}{MSF(h)} \, dt,
\]

where \( \omega_0 \) is the initial damage and may vary from point to point within the material.

5.1. Initiation under steady state creep

Assuming that creep strain accumulation occurs predominantly during steady state creep, which may be a reasonable assumption for materials with a very high creep failure strain and/or under widespread plasticity when \( n = N \) (Kim, 2001; Kim et al., 2001), then the crack tip stress distribution remains constant with time and is given by the RRss field, Eq. (11). The creep damage accumulated under RRss stress field control at a time \( t \) is then

\[
\omega_{RRss} = \frac{\dot{\varepsilon}_0}{\varepsilon_{crit} \sigma_0} \int_0^t \frac{\sigma^n_{RRss}(t)}{MSF_{RRss}} \, dt,
\]

where \( \sigma_{RRss} \) denotes the equivalent stress distribution in the steady state RRss field, Eq. (11), and MSF_{RRss} the multiaxial strain factor associated with the triaxiality value during steady state creep, \( h_{RRss} \). The time to failure at a given crack tip angle, \( \theta \), and radial distance, \( d \), from Eqs. (4), (11) and (24), is obtained directly by integrating the term on the RHS of Eq. (24) to give,
Initiation is defined here as the time for first failure at the characteristic distance, \(d\), at any angle, \(\theta\), and occurs at the angle, denoted \(\theta_c\), where the function \(MSF_{R\kappa_{ss}}/\bar{\sigma}_{R\kappa}\) is a minimum, and thus

\[
t_{R\kappa}^{\text{crit}} = \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_{\text{crit}} (\sigma_0/\bar{\sigma}_0)} \left( \frac{\varepsilon_0 \sigma_0 d}{C} \right)^{n - 1} \frac{MSF_{R\kappa_{ss}}}{\bar{\sigma}_{R\kappa}}. \tag{26}
\]

Eq. (26) may be presented in a fully normalised fashion, depending only upon the creep exponent, \(n\), by plotting \(t_i/\dot{\varepsilon}_{\text{crit}}\) normalised by \(\dot{\varepsilon}_0/\sigma_0 d\).

### 5.2. Initiation under \(K\) field control

If it is assumed that on initial loading a point at the initiation distance \(d\) from the crack tip is within the \(K\)-dominant region, then the stress is given by the \(K\)-field, Eq. (6), and the creep damage parameter at time \(t\), denoted \(\omega_K\), is evaluated using Eq. (22) as

\[
\omega_K = \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_{\text{crit}} \sigma_0} \left( \frac{K^*}{\sigma_0 \sqrt{2\pi d}} \right)^n t, \tag{27}
\]

where \(\sigma_K\) is the equivalent (von Mises) stress in the linear elastic \(K\) field and \(MSF_K\) is evaluated using the \(K\) field triaxiality value, \(h_K\), Eq. (7). The term on the RHS of Eq. (27) may be integrated to give

\[
\omega_K = \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_{\text{crit}} MSF_K} \left( \frac{K^*}{\sigma_0 \sqrt{2\pi d}} \right)^n t, \tag{28}
\]

where the dimensionless function \(f\) is a function of crack tip angle \(\theta\) and Poisson’s ratio, \(v\). For this situation the initiation time, \(t_i\), is given in normalised form as

\[
\frac{t_i^{K} \dot{\varepsilon}_0}{\dot{\varepsilon}_{\text{crit}}} = \left( \frac{\sigma_0 \sqrt{2\pi d}}{K^*} \right)^n MSF_K \left( \frac{K^*}{\sigma_0 \sqrt{2\pi d}} \right)^n \frac{\sigma_K}{f^n} \bigg|_{\text{min}}. \tag{29}
\]

Eq. (29) may be rewritten in terms of \(C^*\), noting that \(\tau = J/C^*\), using Eqs. (13) and (18), to give

\[
\frac{t_i^{K} \dot{\varepsilon}_0}{\dot{\varepsilon}_{\text{crit}}} = \left( 2\pi(1 - v^2) \frac{\sigma_0 \varepsilon_0 \sigma_0 d}{\sigma_0 \varepsilon_0 \sigma_0} \frac{C^*}{C^*} \frac{J}{J} \right) MSF_K \frac{\sigma_K}{f^n} \bigg|_{\text{min}}, \tag{30}
\]

under plane strain conditions. Note that while in Eq. (29), the normalised initiation time, \(t_i^{K} \dot{\varepsilon}_0/\dot{\varepsilon}_{\text{crit}}\), is unique for a given normalised load, \(K/\sigma_0 \sqrt{d}\), and \(n\), the normalised initiation time in Eq. (30) is dependent upon groups of dimensionless parameters in addition to the normalised load parameter, \(C^*/\dot{\varepsilon}_0 \sigma_0 d\), and \(n\).

### 5.3. Initiation under HRR field control

If on initial loading a point at a radial distance \(d\) from the crack tip is in the plastic zone, the stress is expected to be given by the HRR field solution, Eq. (8), and the value of creep damage at time \(t\), denoted \(\omega_{HRR}\), is

\[
\omega_{HRR} = \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_{\text{crit}}} \left( \frac{\sigma_{p0}}{\sigma_0} \right)^n \left( \frac{J}{2\dot{\varepsilon}_0 \sigma_{p0} \varepsilon_0} \right)^{n - 1} \frac{\bar{\sigma}_{HRR}^n}{\bar{\sigma}_{HRR}} t, \tag{31}
\]

where \(\bar{\sigma}_{HRR}\) is the equivalent stress in the HRR field and \(MSF_{HRR}\) is evaluated using the HRR field triaxiality value, \(h_{HRR}\), Eq. (9). The initiation time, \(t_i^{HRR}\), is then

\[
\frac{t_i^{HRR} \dot{\varepsilon}_0}{\dot{\varepsilon}_{\text{crit}}} = \left( \frac{\sigma_0}{\sigma_{p0}} \right)^n \left( \frac{2\dot{\varepsilon}_0 \sigma_{p0} \varepsilon_0}{J} \right)^{n - 1} MSF_{HRR} \frac{\bar{\sigma}_{HRR}^n}{\bar{\sigma}_{HRR}} \bigg|_{\text{min}}. \tag{32}
\]
It may be seen from Eq. (32) that for a given material (fixed \( N, n \) and \( r_0/r_p \)) normalised initiation time, \( t^{HRR}_{i_e_0}/\varepsilon_{crit} \), depends only on the normalised load parameter, \( J/\dot{\varepsilon}_p\sigma_p\dot{\sigma}_0 d \). Eq. (32) may be re-written in terms of \( C^* \) as

\[
\frac{t^{HRR}_{i_e_0}}{\varepsilon_{crit}} = \left( \frac{\sigma_0}{\sigma_p} \right)^n \left( \frac{\dot{\varepsilon}_p\sigma_pI_N}{\sigma_0\dot{\sigma}_0 d} \right)^{\frac{1}{n}} \frac{\pi^{\alpha}}{\sigma_{HRR}^\alpha} \left| \frac{\text{MSF}_{HRR}}{\sigma_{HRR}^\alpha} \right|_{\text{min}}
\]

and \( t^{HRR}_{i_e_0}/\varepsilon_{crit} \) may be written in terms of \( C^*/\sigma_0\dot{\varepsilon}_0 d \). Note that the relationship is now dependent upon \( \dot{\varepsilon}_p\sigma_p/\tau\dot{\varepsilon}_0\sigma_0 \) in addition to \( N, n \) and \( \sigma_0/\sigma_p \).

### 5.4. Initiation under transient creep stress conditions

The previous initiation time relations were based on the assumption that during the period up to initiation the stress and strain fields are given either by \( K \), \( J \) or \( C^* \). More generally, at a given point in a creeping body, the parameter characterising the stress/strain distribution will change with time. This is illustrated schematically in Fig. 4. If the point is initially (at time zero) situated in the elastic zone, Fig. 4(a), the stress is initially controlled by \( K \), and denoted \( \sigma_K \). After a period of time, here designated the changeover time and denoted \( t_{K-RR} \), the material point will enter the creep zone controlled by \( C(t) \) and the stress will follow the transient RR field distribution, \( \sigma_{RR}(t) \). At long times, \( C(t) \) tends to \( C^* \), and the stress is given by the RRss distribution.

Similarly, if the point is initially within the plastic zone controlled by \( J \), as shown in Fig. 4(b), then initially the stress at that point is expected to follow the HRR field. After the changeover time, which in this case is

![Fig. 4. Evolution of the stress field at a given point initially in (a) the \( K \) dominant zone and (b) the \( J \) dominant zone, on initial loading.](image-url)
denoted $t_{KRR-RR}$, the material point will enter the creep zone controlled by $C(t)$ and the stress will follow the transient RR field distribution, $\sigma_{RR}(t)$. Again as $C(t)$ approaches $C^*$, $\sigma_{RR}(t)$ approaches $\sigma_{RR,ss}$.

### 5.4.1. Elastic-creep conditions

The changeover time, $t_{K,RR}$, for the stress field to change from $K$ field control to the transient RR stress field, defined by the $C(t)$ parameter, is estimated by equating the von Mises equivalent stress given by Eqs. (6) and (10)

$$\frac{K}{\sqrt{2\pi d}} \tilde{f} = \sigma_0 \left( \frac{C(t)}{\sigma_0 \varepsilon_0 d} \right)^{ \frac{1}{n} } \tilde{\sigma}_{RR}. \tag{34}$$

Rearranging Eq. (34) and dividing by $C^*$ we obtain

$$\frac{C(t)}{C^*} = \left( \frac{K}{\sigma_0 \sqrt{2\pi d} \tilde{\sigma}_{RR}} \right)^{ \frac{1}{n} } \left( \frac{\sigma_0 \varepsilon_0 d}{C^*} \right) \frac{\tilde{f}}{n}. \tag{35}$$

Substituting Eq. (16) into Eq. (35), the time at which the $K$ field and RR stress solutions equate, at a radial distance $d$, is given by

$$t_{K,RR} = \left[ \left( \frac{K}{\sigma_0 \sqrt{2\pi d} \tilde{\sigma}_{RR}} \right)^{ \frac{1}{n} } \left( \frac{\sigma_0 \varepsilon_0 d}{C^*} \right) \frac{\tilde{f}}{n} - 1 \right]^{-1} \frac{\tau}{n + 1}. \tag{36}$$

Note that the changeover time is a function of crack tip angle, $\theta$, (through the functions $\tilde{f}$ and $\tilde{\sigma}_{RR}$) i.e. different points in the body will have a different changeover time.

For times less than the changeover time ($t < t_{K,RR}$) the equivalent stress is given by the $K$ field (Eq. (6)). For times greater than $t_{K,RR}$, the equivalent stress is given by the RR stress field, controlled by $C(t)$, Eq. (10). If $\omega_K$ at a point (of crack tip distance, $d$, and angle $\theta$) attains the value of unity prior to the changeover time, $t_{K,RR}$, then that point fails under $K$ field control and the failure time is given by

$$\frac{t_K^c \varepsilon_0}{\varepsilon_{crit}} = \left( \frac{\sigma_0 \sqrt{2\pi d}}{K} \right)^{ \frac{n}{n-1} } MSF_K \frac{\tilde{f}}{n}. \tag{37}$$

If $\omega_K$ is less than unity at time $t_{K,RR}$, then the fraction of the creep damage parameter accumulated for $t > t_{K,RR}$, denoted $\omega_{K,RR}^{K,RR}$, is given by

$$\omega_{K,RR}^{K,RR} = \int_{t_{K,RR}}^{t_{K,RR}} C(t) \frac{\tilde{\varphi}}{\varepsilon_{crit}} \left( \frac{\tilde{\sigma}_{RR}}{\sigma_0} \right)^{n} dt = \frac{\varepsilon_0 \tilde{\varphi}_{RR}}{\varepsilon_{crit} MSF_{RR}} \left( \frac{1}{\varepsilon_0 \sigma_0 d} \right)^{ \frac{n}{n+1} } \int_{t_{K,RR}}^{t} C(t)^{ \frac{1}{n+1} } dt. \tag{38}$$

The subscript RR associated with $\omega_{K,RR}^{K,RR}$ signifies that damage is accumulating under RR field control and the superscript $K$-RR indicates that a changeover from the $K$ field to the RR field has taken place.

Using the integrable approximation for $C(t)$ given by Eq. (17), the integral on the RHS of Eq. (38) may be written as

$$\int_{t_{K,RR}}^{t} C(t)^{ \frac{1}{n+1} } dt = C^{ \frac{1}{n+1} } \int_{t_{K,RR}}^{t} \left( \frac{(1 + t/\tau)^{n+1}}{(1 + t/\tau)^{n+1} - 1} \right)^{ \frac{n}{n+1} } dt, \tag{39}$$

which can be integrated and substituted into Eq. (38) to give

$$\omega_{K,RR}^{K,RR} = \frac{\varepsilon_0 \tau}{\varepsilon_{crit} MSF_{RR}} \left( \frac{C^*}{\varepsilon_0 \sigma_0 d} \right)^{ \frac{n}{n+1} } \tilde{\varphi}_{RR} \left[ \left( \frac{1}{n+1} \right)^{ \frac{n}{n+1} } - \left( \frac{1 + t_{K,RR}}{\tau} \right)^{n+1} - \frac{1}{\tau} \right]. \tag{40}$$

The total value of the creep damage parameter accumulated at times $t > t_{K,RR}$, is

$$\omega(t, \theta) = \omega_K(t_{K,RR}) + \omega_{K,RR}^{K,RR}(t). \tag{41}$$
At failure $\omega = 1$, and therefore the creep damage fraction to be accumulated under RR field control for failure to occur at a point after changeover from $K$ field control, denoted $\omega^E_{KRi}$, is given by

$$
\omega^E_{KRi} = 1 - \omega_K(t_{K,RR}).
$$

Substituting the value of $\omega^E_{KRi}$ calculated from Eq. (42) (using Eqs. (28) and (36)) into Eq. (40) the failure time for a material point initially in the $K$-dominant zone can be obtained. The failure time, designated $t_i^{E,RR}$ is then given by

$$
t_i^{E,RR} = \frac{t^E_{K,RR}}{\tau} = \left[ \left( \frac{\sigma_0 \varepsilon_0}{\sigma_0} \right) \left[ \frac{\sigma_0 \varepsilon_0 I_n d}{C^i} \right] \frac{-1}{\sigma^E_{KRi} \text{MSF}_{Ri}} \left[ \left( \frac{t^E_{K,RR}}{\tau} \right)^{n+1} - 1 \right]^{\frac{1}{n+1}} + 1 \right]^{\frac{1}{n+1}} - 1.
$$

Here, $t_i^{E,RR}$ is normalised by $\tau$, rather than $\dot{\varepsilon}_0/\varepsilon_{crit}$. The initiation time is defined as the time for first failure at a given distance $d$. Hence, the initiation time under transient elastic-creep conditions, $t_i^{E,RR}$, is predicted to be the minimum failure time calculated for all angles at $d$, where the failure time for a given point (of crack tip distance, $d$, and angle, $\theta$) is predicted using Eq. (37) or (43), depending on whether failure occurs under $K$ or RR field control (i.e. before or after the changeover time, $t_{K,RR}$).

### 5.4.2. Elastic–plastic-creep conditions

If the material point of interest is initially within the $J$ dominant zone a similar procedure to that adopted in the previous section is applied. However, an additional complication arises in this case. The linear elastic stress is expected to be greater than the steady state creep stress, $\sigma_{RRss}$, at crack tip distances relevant to initiation. Thus, it may be safely assumed that the stress magnitude at the initiation distance, $d$, reduces with time as illustrated in Fig. 4(b). However, under certain conditions the equivalent elastic–plastic stress at $d$, $\sigma_{HRR}$, may be greater than $\sigma_{RRss}$ and thus the stress increases with time (for example for $n > N$, $\sigma_{HRR} > \sigma_{RRss}$ at distances very close to the crack tip, while the opposite is true for $n < N$). Both situations are described in this section.

#### 5.4.2.1. Case 1: $\sigma_{HRR} > \sigma_{RRss}$

Assuming that $\sigma_{HRR}$ evaluated at distance $d$ is greater than $\sigma_{RRss}$ then, following the method described in Section 5.4.1, the changeover time from the HRR to the RR stress field, $t_{HRR-RR}$, is

$$
t_{HRR-RR} = \frac{t_{HRR} \sqrt{\sigma_0}}{\varepsilon_{crit}} = \frac{t_{HRR} \sqrt{\sigma_0}}{\varepsilon_{crit}} \left( \frac{\sigma_0 \varepsilon_0 I_n d}{C^i} \right) \frac{-1}{\sigma_{HRR} \text{MSF}_{HRR}}.
$$

If failure has not occurred prior to the time $t_{HRR-RR}$, then the total creep damage fraction to be accumulated under RR field control for failure to occur, after changeover from HRR field control, is denoted $\omega^{HRR-RR}_{Ri}$ and is given by

$$
\omega^{HRR-RR}_{Ri} = 1 - \omega_{HRR}(t_{HRR-RR}).
$$

Following the steps between Eqs. (38)–(43), the failure time for a point where $\sigma_{HRR}$ is initially greater than $\sigma_{RRss}$ at a time $t > t_{HRR-RR}$, denoted $t_i^{HRR-RR}$, is predicted to be
\[ \frac{t_{HRR-RR}^f}{\tau} = \left[ \left( \frac{e_{\text{crit}}}{i_0 \tau} \left[ \frac{\sigma_0 \dot{i}_0 I_a d}{C^*} \right] \frac{\dot{e}_{\text{crit}}}{\dot{\sigma}_{\text{RRss}}} \frac{\sigma_{\text{RRss}}}{\sigma_{\text{RR}}} \right] \right]^{\frac{1}{n+1}} + \left[ 1 + \left( \frac{t_{HRR-RR}^f}{\tau} \right)^{n+1} - 1 \right]^{\frac{1}{n+1}} + 1 \right]^{\frac{1}{n+1}} - 1. \] (48)

Note that the expression obtained, elastic-creep conditions for Eq. (17), has been used as an approximation to estimate \( C(t) \) in the derivation of Eq. (48), since an integrable estimate for \( C(t) \) under elastic–plastic-creep conditions is currently unavailable.

5.4.2.2. Case 2: \( \bar{\sigma}_{\text{HRR}} < \bar{\sigma}_{\text{RRss}} \). If the equivalent stress at \( d \) under HRR field control is less than the steady state creep stress, the stress at \( d \) does not relax but increases to the steady state creep value. Where this situation arises a changeover time from a HRR field stress to a transient creep stress, controlled by \( C^* \), cannot be easily defined. It has therefore been assumed that, in this case, the HRR stress field prevails until the time \( \tau \), after which the stress is given by the RRss field, controlled by \( C^* \). This approach, while not exact, leads to reasonable, and conservative, estimates of stress in the transition region (compared to finite element solutions).

If \( \omega_{\text{HRR}} \), calculated using Eq. (31), attains the value of unity before the time \( \tau \) then failure occurs under HRR stress control, and the failure time of that point is predicted from Eq. (45). If \( \omega_{\text{HRR}} \) is less than unity at time \( \tau \) then the total value of the creep damage parameter for a point at time \( t > \tau \), is given by

\[ \omega(t, \theta) = \omega_{\text{HRR}}(\tau) + \omega_{\text{RRss}}(t). \] (49)

Using Eqs. (11) and (22), \( \omega_{\text{RRss}}(t) \) can be expressed as

\[ \omega_{\text{RRss}}^f(t) = \int_0^t \frac{\dot{e}_0}{e_{\text{crit}}} \frac{\bar{\sigma}_{\text{RRss}}}{\bar{\sigma}_{\text{RR}}} \text{d}t = \frac{i_0}{e_{\text{crit}}} \left[ \frac{C^*}{\dot{e}_0 \sigma_0 I_a d} \right]^{\frac{1}{n+1}} \frac{\bar{\sigma}_{\text{RR}}}{\sigma_{\text{RRss}}} (t - \tau). \] (50)

The creep damage fraction to be accumulated under RRss control for failure to occur at a point, after changeover from HRR field control, denoted \( \omega_{\text{RRss}}^f \), is given by

\[ \omega_{\text{RRss}}^f = 1 - \omega_{\text{HRR}}(\tau). \] (51)

The failure time for a point at time \( t > \tau \) is then

\[ \frac{t_{HRR-RR}^f}{\tau} = \frac{\omega_{\text{RRss}}^f}{\dot{e}_0 \tau} \left[ \frac{\dot{e}_0 \sigma_0 I_a d}{C^*} \right]^{\frac{1}{n+1}} \frac{\bar{\sigma}_{\text{RRss}}}{\bar{\sigma}_{\text{RR}}} + 1. \] (52)

5.4.2.3. Initiation time prediction. The initiation time under transient elastic–plastic-creep conditions, \( t_i^{HRR-RR} \), is predicted to be the minimum failure time calculated for all angles at \( d \), where the failure time for a given point is predicted using Eqs. (45), (48) or (52), depending on the conditions described above.

6. Application of procedure

Section 5 has detailed the procedures to predict the CCI time under a range of conditions. To enable initiation time predictions during transient creep conditions where the field characterising the stress changes during the initiation period, knowledge of the ratio \( \tau = J/C^* \) is required. This ratio is, in general, load and geometry dependent, and therefore a geometry must be defined in order to obtain a prediction of the initiation time. An example of the method is presented here for a laboratory specimen for which solutions for \( J \) and \( C^* \) (and therefore \( \tau \)) are readily available.

6.1. Material model

A Ramberg–Osgood fit to data from tensile tests on austenitic type 316H stainless steel at 550 °C has been made. This enables \( J \) and \( C^* \) and therefore \( \tau \) to be estimated from the EPRI solutions, Eqs. (14) and (15). The material properties are \( E = 140 \text{ GPa}, N = 3 \text{ and } \sigma_{p0} = 170 \text{ MPa}, \dot{e}_{p0} = \sigma_{p0}/E \text{ and } \alpha = 5.8 \) (Davies, 2006).
The creep properties have been chosen to represent the average creep strain rate determined from a number of uniaxial creep tests on austenitic type 316H stainless steel at 550 °C (Bettinson, 2002). The creep stress exponent in Eq. (1) is \( n = 10 \) and taking \( \dot{e}_0 = 1 \) h\(^{-1} \) then \( \sigma_0 = 900 \) MPa (Davies, 2006). When the average creep strain rate is used to describe the creep strain rate in conjunction with the Cocks and Ashby void growth model the critical failure strain \( \varepsilon_{\text{crit}} \) may be taken to be the uniaxial creep ductility, \( \varepsilon_f \), which is approximately 7% for this material and temperature (Bettinson, 2002).

6.2. Example procedure

An example of the use of the method is presented here for a compact tension, \( C(T) \), specimen with crack length to specimen width ratio \( a/W = 0.45 \). Results are presented for a normalised crack tip distance \( d/a = 2.2 \times 10^{-3} \) (corresponding to an initiation distance \( d = 0.05 \) mm on a standard \( C(T) \) specimen of width \( W = 50 \) mm).

For the purposes of comparison, results are presented in terms of initiation time multiplied by \( \dot{e}_0/\varepsilon_{\text{crit}} \) plotted against \( C^*/\dot{e}_0\sigma_0d \). When plotted in this manner, the prediction lines may have certain dependencies on geometry or combination of material properties. The RRss line, Eq. (26), is however independent of all parameters (for a given \( n \)) and therefore is included on all figures as a reference line. Scales may be exaggerated in places for illustration purposes. The region of practical relevance for this material is considered to be \( 1 \times 10^{-8} < C^* \) (MPa m/h) \( < 1 \); \( 0.05 \leq d \) (mm) \( \leq 0.2 \) and \( 1 \leq t_i \) (h) \( \leq 1 \times 10^5 \) corresponding approximately to \( 1 \times 10^{-12} \leq C^*/\dot{e}_0\sigma_0d \leq 1 \times 10^{-3} \) and to \( 14 \leq t_i\dot{e}_0/\varepsilon_{\text{crit}} \leq 1.4 \times 10^6 \), as indicated in some of the figures presented.

6.2.1. Predictions under elastic-creep conditions

The \( K\)-RR initiation prediction is shown in Fig. 5 together with the \( K \) and RRss predictions. The former two curves are relevant if it is assumed that the initiation distance, \( d \), lies in the \( K \) dominant region on initial loading (\( t = 0 \)). The latter curve assumes that all damage takes place in the steady state creep regime. The \( K\)-RR line is expected to provide the most accurate estimate of initiation times under elastic-creep conditions, as it takes full account of stress redistribution during creep.

At low values of \( C^*/\dot{e}_0\sigma_0d \) (low loads for a given \( d \), or a high \( d \) for a given load) the \( K\)-RR line tends to the RRss line. As \( C^*/\dot{e}_0\sigma_0d \) increases, creep damage accumulation occurs increasingly under \( K \) field stress control and the \( K\)-RR line tends to the \( K \) line. The initiation angle from the RRss and \( K \) predictions are constant and

![Fig. 5. Normalised initiation time predictions at a distance with a linear elastic \( K \)-field stress distribution, \( K \)-field followed by transient creep stress distribution and steady state creep stress distribution.](image-url)
equal to $\theta_i^{\text{RRss}} = 60^\circ$ and $\theta_i^K = 80^\circ$ for $n = 10$. The initiation angle from the $K$-RR prediction is equal to $\theta_i^{\text{RRss}}$ at low values of $C^*/\dot{\varepsilon}_0\sigma_0 d$, and increases towards $\theta_i^K$ for $C^*/\dot{\varepsilon}_0\sigma_0 d > 1 \times 10^{-12}$.

6.2.2. Predictions under elastic–plastic-creep conditions

The initiation times from the HRR-RR prediction are presented in Fig. 6 together with the HRR and RRss predictions. The scales are again extended in order to show the transition from initiation under plasticity dominated conditions (high values of $C^*/\dot{\varepsilon}_0\sigma_0 d$) to creep dominated conditions (low values of $C^*/\dot{\varepsilon}_0\sigma_0 d$). The HRR line is relevant if all damage is accumulated under HRR field control. The HRR-RR line, which has previously been presented in Davies et al. (2006), is expected to provide the most accurate estimate of initiation times under elastic–plastic-creep conditions, as it takes full account of stress redistribution during creep. To describe the behaviour of the HRR-RR prediction, five regions have been identified (see Fig. 6). At low values of normalised $C^*$ (Region 1) the HRR-RR prediction approaches the RRss line. In this region the majority of the damage is accumulated under steady state creep, ($\omega_{\text{HRR}} \approx 0$) and the HRR-RR and RRss predictions are indistinguishable. Conversely, at high values of normalised $C^*$ (Region 5) the HRR-RR and HRR line are identical as damage is accumulated solely under HRR control, ($\omega_{\text{RR}} = 0$). Regions 2–4 describe the transient region between HRR control and steady state creep conditions. It may be seen that for this combination of material properties the region of practical interest is confined to regions 4 and 5 and, in fact, the simple HRR prediction provides a good estimate of initiation time (relative to the more complex HRR-RR relation) over the region of practical relevance.

Note that the predicted angle of crack initiation is a function of $C^*/\dot{\varepsilon}_0\sigma_0 d$ and takes different values in the five regions identified in Fig. 6. In Region 4, $\theta_i$ gradually changes from that predicted by the HRR distributions ($\theta_i = 80^\circ$ for $N = 3$) to $\theta_i = 0^\circ$ and $\theta_i$ remains at $0^\circ$ in Region 3. It may be noted that in this region the HRR-RR line is parallel to the HRR line. This is because the damage at the initiation distance is found to accumulate completely under HRR control. In Region 2, the initiation angle increases from $\theta_i = 0^\circ$ to that predicted by the RR line ($\theta_i = 60^\circ$ for $n = 10$).

6.2.2.1. Sensitivity to critical strain. The sensitivity of the $K$-RR and HRR-RR initiation time predictions to the critical strain, $\dot{\varepsilon}_{\text{crit}}$, is shown in Figs. 7 and 8, respectively for $d/a = 2.2 \times 10^{-3}$. Note that when presented in the form shown in theses figures the RRss and HRR predictions are independent of $\dot{\varepsilon}_{\text{crit}}$ (see Eqs. (26) and (33)). Predictions are shown in Figs. 7 and 8 for values of $\dot{\varepsilon}_{\text{crit}}$ equal to 7% and 50% (which cover the range of expected values for this class of material). As can be seen, the general trends remain unaltered with increas-
6.2.2. Sensitivity to crack tip distance. The sensitivity of the predictions to normalised initiation distance \( d/a \) is shown in Fig. 9 for the HRR and HRR-RR predictions (with \( \varepsilon_{\text{crit}} = 7\% \)). A change in \( d/a \) could imply either a change in specimen size or a change in critical distance for a fixed specimen size. Note that, as implied in Section 5.3, the \( t_i^{\text{HRR}}/\dot{\varepsilon}_0/\varepsilon_{\text{crit}} \) prediction lines become dependent on \( d/a \) when plotted against \( C^*/\dot{\varepsilon}_0\sigma_0d \), but are independent of \( d/a \) when plotted against \( Jf_{\varepsilon}\varepsilon_0\sigma_0d \). Two crack tip distances are examined \( d/a = 4.4 \times 10^{-3} \) and \( d/a = 1.1 \times 10^{-3} \), which for \( a/W = 0.45 \) corresponds to \( d = 0.05 \) mm for a half-sized \( (W = 25 \) mm) and large \( (W = 100 \) mm) specimen, respectively. It may be seen that increasing \( d/a \) has a similar effect to increasing \( \varepsilon_{\text{crit}} \), as the time required for creep strain to accumulate to a critical value will increase with both crack tip distance and \( \varepsilon_{\text{crit}} \). It can also be seen from Figs. 7–9, that as \( d/a \) or \( \varepsilon_{\text{crit}} \) is increased, the effect of
transient creep becomes more important over the region of practical relevance, as indicated by the separation between the $K$-RR and the $K$ line in Fig. 7 and between the HRR-RR and HRR line in Figs. 8 and 9.

7. Finite element model

In the analytical solution, the value of $C(t)$ and $J$ have been estimated and in evaluating the triaxiality, $h$, it has been assumed that the crack tip fields are characterised by a single parameter. Some discrepancy is thus expected between the analytical predictions and results from real cracked geometries. The magnitude of the discrepancy is assessed by comparing the previous results with finite element (FE) predictions which are assumed to be ‘exact’ solutions.

A 2-D plane strain FE analysis has been performed using the commercial software package ABAQUS 6.4 (ABAQUS, 2003). The damage parameter has been evaluated within a user defined subroutine (USDFLD). A small geometry change analysis was performed employing four noded ‘hybrid’ continuum elements (CPE4H). Half of the specimen has been modelled and symmetry conditions employed. A focused mesh, containing 3104 elements and 2787 nodes, and an infinitely sharp crack tip, modelled using collapsed quadrilateral elements has been used. $J$ and $C^*$ values have been obtained from the FE analysis, from a contour integral averaged over 41 contours. The value of $C(t)$ is taken at the third contour from the crack tip where $d/a = 2.8 \times 10^{-4}$.

The value of $J$ at load-up has been obtained from an elastic–plastic analysis for each load considered and the value of $C^*$ taken from the analysis at long times where there appeared to be no further tendency for $C^*$ to change with time. A range of loading conditions has been considered using predominantly elastic to widespread plastic conditions.

The focused mesh consist of 41, rings spanning a radial distance $0 < d/a < 0.1$. The initiation time at a distance $d$ from the crack tip is taken to be the time for first failure of any of the elements in the appropriate ring. A typical crack tip damage distribution is illustrated in Fig. 10. The darkest regions indicate failure, i.e. $\omega = 1$. As shown in Fig. 10, one of the elements in the ring of radial distance $d$, has attained $\omega = 1$ over its entire area. The initiation time at $d$, is therefore the time to failure of this element and the initiation angle, $\theta_i$, is as defined in Fig. 10.

7.1. Predictions under elastic–plastic–creep conditions

In Fig. 11 normalised initiation time predictions from the FE analysis are compared to the $K$-RR and HRR-RR estimates for a range of $C^*/\dot{\varepsilon}_0 \sigma_0 d$ at a load corresponding to $L_t = 0.1$. As the load, and thus $C^*$,
and material properties are fixed in the analysis, $C^*/\dot{\varepsilon}_0\sigma_0 d$ is increased by reducing the initiation distance, $d$ (a single FE analysis provides results for a range of $d$ values). At low values of $C^*/\dot{\varepsilon}_0\sigma_0 d$ (distances far from the crack tip for a given load) the FE solution follows the $K$-RR and RRss line and for $C^*/\dot{\varepsilon}_0\sigma_0 d < 1 \times 10^{-17}$ the $K$-RR prediction is in very good agreement with the FE solution. For high values of $C^*/\dot{\varepsilon}_0\sigma_0 d$ (i.e. close to the crack tip for a given load) the slope of the FE prediction follows that of the HRR-RR line. However the HRR-RR line is significantly conservative relative to the FE prediction.

In Fig. 12 normalised initiation time predictions from the FE analysis are compared to the HRR, HRR-RR and RRss predictions at the load corresponding to $L_r = 0.4$. For large normalised $C^*$ values, initiation is conservatively estimated by the HRR solution at this load, and at lower $C^*/\dot{\varepsilon}_0\sigma_0 d$ values the initiation time is better represented by the HRR-RR estimate. The RRss prediction is found to be non-conservative for the range of $C^*/\dot{\varepsilon}_0\sigma_0 d$ shown in Fig. 12.

The finite element initiation time predictions are re-plotted in non-normalised form in Fig. 13 for a $C(T)$ specimen of width $W = 26$ mm ($a/W = 0.45$ and $B/W = 0.5$) at the initiation distance $d = 0.05$ mm for four loads giving rise to significant plasticity at this distance ($L_r = 0.2, 0.4, 0.8, 1.1$). The FE predictions are compared to the HRR, HRR-RR and RRss predictions. As previously shown, the HRR and HRR-RR estimates provide conservative estimates of the finite element prediction for the range of $C^*$ considered, at this initiation...
The HRR prediction is on the order of four times less than the FE prediction (conservative). The RRss prediction however is non-conservative relative to the FE prediction—the CCI times from the RRss prediction are on the order of 450 times greater than the FE predictions for a given $C^*$. The FE predictions are in closest agreement with the HRR-RR line within the range of $C^*$ values presented and a reasonable prediction is also obtained from the HRR line over the relevant range.

The difference between the FE solution and the HRR-RR line is contributed to by the errors associated with the definitions of $h$ and $C(t)$ in the analytical solutions. As shown in Fig. 2, the MSF factor employed is highly sensitive to $h$. Therefore, a relatively small overestimate in $h$ leads to a significant underestimate of MSF and $v_{crit}^*$ (through Eq. (4)) and thus, from Eq. (31), an overestimate of the damage accumulation rate, leading to conservative predictions of the CCI time. It has been observed in the FE analyses that the triaxiality does not directly switch between that of the HRR (or $K$) field to that of the RR field as implemented in the

![Figure 12](image12.png)

**Fig. 12.** Comparison of FE initiation time predictions for $L_t = 0.4$ with HRR, HRR-RR and RRss estimates.

![Figure 13](image13.png)

**Fig. 13.** Comparison HRR, HRR-RR and RRss initiation time predictions with the FE solutions for a $C(T)$ specimen of $a/W = 0.45$ at $d = 0.05$ mm, $W = 26$ mm, under plane strain conditions.
analytical method, but gradually increases with time. The triaxiality may therefore be significantly overesti-
mated by the RR field during the initiation period.

The \( C(t) \) estimate of Eq. (17), which was originally derived for elastic–creep behaviour, has been used here as an estimate for elastic–plastic–creep conditions. By comparing with the FE solution, the \( C(t) \) estimates have been found to be generally conservative during the transient creep period (Davies, 2006). Improved estimates of the \( C(t) \) parameter under elastic–plastic–creep conditions are expected to improve the agreement between predictions from the \( K \)-RR and HRR-RR lines and the finite element solutions.

8. Comparison of predictions with experimental data

The predictions are next compared to experimentally determined initiation times for tests performed on C(T) specimens of various dimensions. Some of these tests were pre-fatigued to provide a sharp crack tip, whilst others had a relatively blunt EDM notch (typical notch diameter, 0.25 mm). All the predictions shown are based on the initiation distance \( d = 0.05 \) mm and the experimentally determined test initiation times for the measured crack extension of \( \Delta a = 0.05 \) mm, determined using potential drop techniques. Further information on the experimental measurements is provided in Bettinson (2002) and Dean and Gladwin (2004). The crack tip parameters \( J \) and \( C^* \) have been evaluated for each specimen using the EPRI solution at the original crack length, which are in agreement with the FE values. As plane strain conditions are expected to prevail close to the crack tip the \( J \) and \( C^* \) solutions and the prediction lines are based on plane strain conditions.

In Fig. 14 the experimentally determined initiation times are plotted against \( J \) and the data are compared to the HRR prediction. As previously stated the HRR prediction is independent of specimen geometry when plotted in this way. Thus data from a variety of specimen sizes can be analysed together. (It has also been confirmed that the HRR and HRR-RR predictions are identical for the cases examined.) It is seen in Fig. 14 that the HRR prediction for a given \( J \), is generally conservative by two to three orders of magnitude.

The crack initiation angles for the data in Fig. 14 have not been recorded. However, during creep testing crack growth in the plane of the crack (\( \theta = 0 \)) is generally promoted by the presence of side-grooves. There is some evidence that crack growth may occur at a non-zero angle for smooth specimens (without side-grooves), see e.g. Dean and Gladwin (2004). The assumption made in this work that crack growth can take place at any angle \( \theta \), is expected to provide a conservative estimate of initiation times for side-grooved specimens.

In Fig. 15 the experimentally determined initiation times are plotted against the EPRI \( C^* \) and compared to the RRss prediction line. Note that the HRR and RRss lines cannot be shown on the same figure as the HRR

![Fig. 14. Comparison HRR initiation time predictions with experimental data for a C(T) specimen under plane stress and plane strain conditions.](image-url)
line is not unique when plotted against normalised $C^*$, as discussed in Section 5.4.1. The RRss line is expected to provide a very non-conservative prediction of experimental initiation times (see e.g. Fig. 6). However, in Fig. 15, some data points lie above the RRss line. This may be due to inaccuracies in measuring the experimental initiation times, which is particularly difficult for short initiation distances or to variations in creep properties from specimen to specimen. Note also that six of the seven points lying close to the RRss line had EDM notches rather than sharp fatigue cracks and thus would be expected to have relatively long CCI times. The method proposed is based on the assumption of a sharp crack and thus predictions are expected to be conservative compared to that obtained from EDM notched specimens, and for the pre-fatigued specimen data. The RRss prediction is however non-conservative for the majority of the data (i.e. the model predicts longer initiation times then is observed in the experiments).

9. Conclusions

An analytical method has been developed to predict creep crack initiation, CCI, based on the accumulation of a critical quantity of damage at an initiation distance, accounting for the redistribution of stresses from that experienced on initial loading from an elastic or elastic–plastic field to a steady state creep stress distribution via a transient creep region. The method has been applied to predict CCI times for a compact tension specimen of type 316H stainless steel at 550 °C. Reasonable and conservative predictions of CCI time can be obtained from the analytical solution relative to finite element solutions. Conservative predictions of experimental CCI times have been obtained when redistribution is accounted for. The CCI times obtained using the steady state creep distribution are, however, found to be non-conservative.

Acknowledgement

The authors would like to acknowledge helpful discussions with Dr R.A. Ainsworth of British Energy Generation Ltd.

References
