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Twist-3 contributions in semi-inclusive DIS with transversely polarized target

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ABSTRACT

We study semi-inclusive DIS with a transversely polarized target in the approach of collinear factorization. The effects related to the transverse polarization are at twist-3. We derive the complete result of twist-3 contributions to the relevant hadronic tensor at leading order of α_s, and construct correspondingly experimental observables. Measuring these observables will help to extract the twist-2 transversity distribution, twist-3 distributions and twist-3 fragmentation functions of the produced unpolarized hadron. A detailed comparison with the approach of transverse-momentum-dependent factorization is made.

Experiments of lepton–hadron collisions with large momentum transfers have played an important role in exploring the inner structure of hadrons. Typical examples are DIS and Semi-Inclusive DIS (SIDIS) processes. Based on collinear factorizations of QCD, the differential cross sections of DIS and SIDIS at the leading power are predicted with parton distributions of the initial hadron and fragmentation function of the produced hadron. These distributions and fragmentation functions are defined as matrix elements of QCD twist-2 operators. In this letter we study the contributions involving twist-3 operators in SIDIS.

We will assume that the polarization of the produced hadron is not observed. The twist-3 contributions in SIDIS appear only in the case that the initial hadron is transversely polarized. The contributions contain not only twist-3 matrix elements of the initial hadron introduced in [1,2], but also the twist-2 transversity distribution introduced in [3], combined with chirality-odd twist-3 fragmentation functions. Therefore, they contain rich information about the inner structure of hadrons. Experimentally, the twist-3 contributions can be measured through asymmetries caused by the transverse spin. Hence, those twist-3 distributions and fragmentation functions can be extracted from the asymmetries. The relevant studies in experiments planned in the future can be found in [4–6] and references therein.

SIDIS with transversely polarized target has been studied in [7–9]. In these works one assumes that the initial lepton is unpolarized and the hadron in the final state has large transverse momentum. The obtained Single transverse-Spin Asymmetry (SSA) starts at order of α_s. We will derive the complete result of the twist-3 hadronic tensor of SIDIS at order of α_s^0. With the complete results we construct spin-dependent observables at α_s^0. Through measuring these observables one can extract relevant parton distributions and fragmentation functions. It is interesting to note that the obtained twist-3 hadronic tensor at tree-level can be expressed completely with the parton distributions and fragmentation functions defined with two-parton correlations.

With the employed approach of collinear factorization one can only derive the twist-3 hadronic tensor as a distribution tensor of the transverse momentum. From the tensor one can only obtain physical predictions in which the transverse momentum is integrated. At tree-level the produced hadron has a small transverse-momentum at order of Λ_{QCD}. In this kinematical region one can employ the approach of Transverse-Momentum-Dependent (TMD) factorization studied in [10–12]. The complete angular distribution of SIDIS at tree-level has been derived with the approach in [13–15] and in [16]. We will discuss the difference between the two approaches in detail after giving our results.

We consider the SIDIS process:

$$e(k, \lambda_e) + h(P, s) \to e(k') + h'(P_h) + X \qquad (1)$$

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where the initial hadron is of spin-1/2 with the spin vector s. The initial electron can be polarized with the helicity λ_e. The polarization of the hadron in the final state is not observed. At leading order of QED, there is an exchange of one virtual photon with the momentum $q=k-k'$ between the electron and the initial hadron. The relevant hadronic tensor is:

$$W^{\mu\nu}=\sum_X\int\frac{d^4x}{(2\pi)^4}e^{iq\cdot x}\langle P,s|J^\mu(x)|P_h,X\rangle\langle X,P_h|J^\nu(0)|P,s\rangle. \tag{2}$$

The standard variables for SIDIS are:

$$x_B=\frac{Q^2}{2P\cdot q},\quad y=\frac{P\cdot q}{P\cdot k},\quad z_h=\frac{P\cdot P_h}{P\cdot q}. \tag{3}$$

We will neglect the masses of hadrons and leptons.

It is convenient to use the light-cone coordinate system, in which a vector a^μ is expressed as $a^\mu=(a^+,a^-,\vec{a}_\perp)=((a^0+a^3)/\sqrt{2},(a^0-a^3)/\sqrt{2},a^1,a^2)$. Two light-cone vectors are introduced as $l^\mu=(1,0,0,0)$ and $n^\mu=(0,1,0,0)$. With these two vectors one can define two transverse tensors: $g_\perp^{\mu\nu}=g^{\mu\nu}-n^\mu l^\nu-n^\nu l^\mu$ and $\epsilon_\perp^{\mu\nu}=\epsilon^{\alpha\beta\mu\nu}l_\alpha n_\beta$. With these notations we introduce the relevant parton distributions and fragmentation functions. In this work we will use Feynman gauge.

Assuming that the initial hadron moves in the z-direction with the momentum $P^\mu=(P^+,0,0,0)$ and it is transversely polarized with $s^\mu=(0,0,s_\perp^1,s_\perp^2)$, the transversity distribution is defined as [3]:

$$h_1(x)s_\perp^\mu=\int\frac{d\lambda}{4\pi}e^{-ix\lambda P^+}\times\langle P,s_\perp|\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma^+\gamma_\perp^\mu\gamma_5\mathcal{L}_n(0)\psi(0)|P,s_\perp\rangle \tag{4}$$

where $\mathcal{L}_n(\xi)$ is the gauge link starting from ξ to ∞ in space–time. The transversity distribution is of twist-2. At twist-3 one can define the three twist-3 distributions from two-parton correlations:

$$\begin{aligned}
q_T(x)s_\perp^\mu&=P^+\int\frac{d\lambda}{4\pi}e^{-ix\lambda P^+}\\
&\quad\times\langle P,s_\perp|\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma_\perp^\mu\gamma_5\mathcal{L}_n(0)\psi(0)|P,s_\perp\rangle,\\
-iq_\partial(x)s_\perp^\mu&=\int\frac{d\lambda}{4\pi}e^{-ix\lambda P^+}\\
&\quad\times\langle P,s_\perp|\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma^+\gamma_5\partial_\perp^\mu(\mathcal{L}_n\psi)(0)|P,s_\perp\rangle,\\
-iq'_\partial(x)\tilde{s}_\perp^\mu&=\int\frac{d\lambda}{4\pi}e^{-ix\lambda P^+}\\
&\quad\times\langle P,s_\perp|\bar\psi(\lambda n)\mathcal{L}_n^\dagger(\lambda n)\gamma^+\partial_\perp^\mu(\mathcal{L}_n\psi)(0)|P,s_\perp\rangle.
\end{aligned} \tag{5}$$

One may replace in the second line of Eq. (5) γ_5 with I to define another twist-3 distribution. But one can show that it is zero [17]. The three distributions are real. From three-parton correlations one can define two twist-3 distributions:

$$\begin{aligned}
&T_F(x_1,x_2)\tilde{s}_\perp^\mu\\
&\quad=g_s\int\frac{d\lambda_1 d\lambda_2}{4\pi}e^{-i\lambda_2(x_2-x_1)P^+-i\lambda_1x_1P^+}\\
&\quad\times\langle P,\vec{s}_\perp|\bar\psi(\lambda_1 n)\gamma^+G^{+\mu}(\lambda_2 n)\psi(0)|P,\vec{s}_\perp\rangle,\\
&T_\Delta(x_1,x_2)s_\perp^\mu\\
&\quad=-ig_s\int\frac{d\lambda_1 d\lambda_2}{4\pi}e^{-i\lambda_2(x_2-x_1)P^+-i\lambda_1x_1P^+}\\
&\quad\times\langle P,\vec{s}_\perp|\bar\psi(\lambda_1 n)\gamma^+\gamma_5G^{+\mu}(\lambda_2 n)\psi(0)|P,\vec{s}_\perp\rangle,
\end{aligned} \tag{6}$$

where we have suppressed the gauge links for short notations and $\tilde{s}^\mu=\epsilon_\perp^{\mu\nu}s_{\perp\nu}$. Corresponding to the two distributions in Eq. (6) one can define additionally two twist-3 distributions by replacing the field strength tensor $g_sG^{+\mu}(x)$ with $P^+D_\perp^\mu(x)$, where $D^\mu(x)$ is given by $D^\mu(x)=\partial_\mu+ig_sG^\mu(x)$. These two functions will not appear in our calculation. In fact they can be expressed with the distributions given in Eqs. (5), (6) as shown in [7]. Among the introduced five twist-3 distributions one can show:

$$\begin{aligned}
&\frac{1}{2\pi}\int dx_1 P\frac{1}{x_1-x_2}\Big[T_F(x_1,x_2)-T_\Delta(x_1,x_2)\Big]\\
&\quad=-x_2q_T(x_2)+q_\partial(x_2),\qquad T_F(x,x)=-2q'_\partial(x),
\end{aligned} \tag{7}$$

where P stands for the principle-value prescription. The first relation has been derived in [18]. The second relation is obtained by examining the relation between T_F and that obtained from T_F by the mentioned replacement. It should be emphasized that the second relation is for SIDIS. We note here that the distribution $q'_\partial(x)$ is defined with the gauge links in Eq. (5) pointing to the future to factorize the effects of final-state interactions in SIDIS. In Drell–Yan processes there are no final-state interactions. But there are initial-state interactions. Hence, the distribution $q'_\partial(x)$ in Drell–Yan processes is defined with gauge links pointing to the past. With the symmetries of time-reversal and parity one can show that there is a sign-difference between the two distributions. For Drell–Yan processes, the $-$-sign in the relation should be replaced with $+$. We notice here that the two distributions with different gauge links have been studied in [19], where it has been shown that the difference between the two distributions is proportional to $T_F(x,x)$. The twist-3 distribution $q'_\partial(x)$ can also be defined as a transverse-momentum-moment of Sivers function. Such moments are in general related to parton distributions at high twists, as discussed in [20].

To define fragmentation functions, we assume that the produced hadron moves in the $-z$-direction with the momentum $P^\mu=(0,P^-,0,0)$. From two-parton correlations we define:

$$\begin{aligned}
&zP^-\int\frac{d\xi}{2\pi}e^{-i\xi P^-/z}\sum_X\\
&\quad\times\Big[\langle 0|\mathcal{L}_l^\dagger(0)\psi(0)|PX\rangle_i\langle XP|\bar\psi(\xi l)\mathcal{L}_l(\xi l)|0\rangle_j\Big]\\
&\quad=\Big(\gamma\cdot P\hat{d}(z)+\hat{e}(z)+\frac{i}{2}\sigma_{\alpha\beta}\gamma_5\epsilon_\perp^{\alpha\beta}\hat{e}_I(z)\Big)_{ij}+\cdots,
\end{aligned} \tag{8}$$

where ij stand for Dirac indices and color indices. $\mathcal{L}_l(\xi)$ is the gauge link along the direction l^μ starting from $-\infty$ to ξ in space–time. $\hat{d}(z)$ is the standard twist-2 fragmentation function [21]. $\hat{e}$ and $\hat{e}_I$ are twist-3 fragmentation functions introduced in [22]. Besides these two twist-3 fragmentation functions, there are another two twist-3 fragmentation functions defined as:

$$\begin{aligned}
&\hat{E}_F(z_1,z_2)\\
&\quad=-\frac{z_2g_s}{4N_c}\int\frac{d\lambda_1d\lambda_2}{(2\pi)^2}e^{-\lambda_1P^-/z_1-i\lambda_2P^-(1/z_2-1/z_1)}\epsilon_{\perp\mu\nu}\\
&\qquad\times\sum_X\mathrm{Tr}\langle 0|\gamma^-\gamma^\nu\gamma_5\psi(0)|hX\rangle_i\langle hX|\bar\psi(\lambda_1 l)G^{-\mu}(\lambda_2 l)|0\rangle,\\
&\hat{e}_\partial(z)=-i\frac{z}{4N_c}\int\frac{d\lambda}{2\pi}e^{-\lambda P^+/z}\sum_X\mathrm{Tr}\langle 0|\gamma^-\gamma^\nu\gamma_5\mathcal{L}_l^\dagger(0)\psi_i(0)|hX\rangle\\
&\qquad\times\partial_\perp^\mu\langle hX|\bar\psi(\lambda l)\mathcal{L}_l(\lambda l)|0\rangle\epsilon_{\perp\mu\nu}.
\end{aligned} \tag{9}$$

Similarly one can define an additional fragmentation function $\hat{E}_D$ by replacing $g_sG^{-\mu}(\lambda_2 l)$ with $P^-D_\perp^\mu(\lambda_2 l)$. But this function is

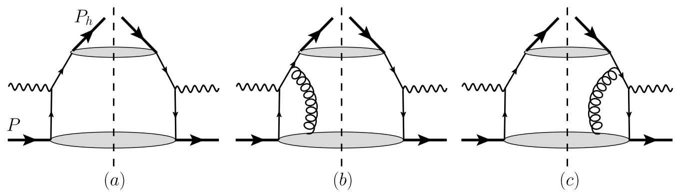

Fig. 1. Diagrams for contributions in SIDIS.

completely determined by $\hat{E}_F$ and $\hat{e}_\partial$ [23]. All introduced twist-3 fragmentation functions are chirality-odd. The functions $\hat{e}$, $\hat{e}_I$ and $\hat{e}_\partial$ are real, while $\hat{E}_F$ is complex in general. It is shown in [23] that there are relations among these four twist-3 fragmentation functions. In our notations they are:

$$z_2^2 \int \frac{dz_1}{z_1} P \frac{1}{z_2 - z_1} \operatorname{Im} \hat{E}_F(z_1, z_2) = z_2 \hat{e}_\partial(z_2) - \hat{e}_I(z_2),$$
$$\hat{e}(z_2) = z_2^2 \int \frac{dz_1}{z_1} P \frac{1}{z_2 - z_1} \operatorname{Re} \hat{E}_F(z_1, z_2). \tag{10}$$

In [24] it is shown that $\hat{E}_F(z, z) = 0$. This implies that there will be no soft-gluon pole contributions represented by $\hat{E}_F(z, z)$.

For deriving the twist-3 contribution to $W^{\mu\nu}$ for the process in Eq. (1), it is convenient to take the frame, in which the initial hadron moves in the z-direction with $P^\mu = (P^+, 0, 0, 0)$ and the final hadron moves in the $-z$-direction with $P_h^\mu = (0, P_h^-, 0, 0)$. Then the virtual photon has the momentum $q^\mu = (q^+, q^-, q_\perp^1, q_\perp^2)$. We call this frame as $\mathcal{C}_0$-frame. The obtained result is covariant. It can be conveniently transformed into the frame called $\mathcal{C}_1$-frame, in which the virtual photon moves in the $-z$-direction and the initial hadron moves in the z-direction. The produced hadron in $\mathcal{C}_1$-frame can then have nonzero transverse momentum.

The twist-3 $W^{\mu\nu}$ can be divided into two parts: One consists of contributions with nonperturbative quantities defined with chirality-odd operators, another one consists of contributions with nonperturbative quantities defined with chirality-even operators. The chirality-even part can only contain the twist-2 fragmentation function and twist-3 parton distributions. At tree-level, it receives contributions from diagrams given in Fig. 1. It is rather standard to calculate contributions at different twists from Fig. 1 by collinear expansion. We take Fig. 1a as an example to illustrate this.

Within the power accuracy considered here, one can already neglect the $+$-components and the transverse components of momenta carried by the parton lines entering into the upper bubble in Fig. 1. One can also neglect the $-$-components of momenta carried by the parton lines entering into the lower bubble. Projecting out the twist-2 part related to the final hadron, the contribution from Fig. 1a can be written as:

$$W^{\mu\nu}\Big|_{1a} = \int dk_B^- dk_A^3 \left[\delta^4(q + k_A - k_B) \frac{1}{z} \hat{d}(z) \left(\gamma^\mu \gamma^+ \gamma^\nu \right)_{ij} \right]$$
$$\cdot \int \frac{d^3\xi}{(2\pi)^3} e^{ik_A \cdot \xi} \langle h(P) | \bar{q}_i(0) q_j(\xi) | h(P) \rangle,$$
$$\xi^\mu = (0, \xi^-, \vec{\xi}_\perp), \quad k_A^\mu = (k_A^+, 0, \vec{k}_{A\perp}),$$
$$k_B^\mu = (0, k_B^-, 0, 0) = (0, P_h^-/z, 0, 0), \tag{11}$$

where ij stand for Dirac indices and color indices. k_A is the momentum carried by the quark line leaving the lower bubble in the left of Fig. 1a, k_B is the momentum carried by the quark line entering into the upper bubble. If we neglect $k_{A\perp}$ in $[\cdots]$ in Eq. (11), we obtain the twist-2 contribution. One needs to expand the $[\cdots]$ in Eq. (11) in $k_{A\perp}$ and to make corresponding projections of the quark density matrix to obtain the twist-3 contribution. After the expansion and projections we have:

$$W^{\mu\nu}\Big|_{1a} = -i\delta^2(q_\perp) \frac{1}{z_h} \hat{d}(z_h) \epsilon^{\mu\nu\alpha\beta} n_\alpha$$
$$\times \int \frac{d\xi^-}{2\pi} e^{i\xi^- x P^+} \langle h(P) | \bar{q}(0) \gamma_{\perp\beta} \gamma_5 q(\xi^- n) | h(P) \rangle$$
$$- \frac{\partial}{\partial q_\perp^\rho} \delta^2(q_\perp) \frac{1}{z_h} \hat{d}(z_h) \epsilon_\perp^{\mu\nu}$$
$$\times \int \frac{d\xi^-}{2\pi} e^{i\xi^- x P^+} \langle h(P) | \bar{q}(0) \gamma^+ \gamma_5 \partial_\perp^\rho q(\xi^- n) | h(P) \rangle$$
$$- \frac{\partial}{\partial q_\perp^\rho} \delta^2(q_\perp) \frac{1}{z_h} d(z_h) g_\perp^{\mu\nu} i$$
$$\times \int \frac{d\xi^-}{2\pi} e^{i\xi^- x P^+} \langle h | \bar{q}(0) \gamma^+ \partial_\perp^\rho q(\xi^- n) | h \rangle + \cdots, \tag{12}$$

where $\cdots$ stand for contributions at twist-2 or beyond twist-3. The three correlation functions of quark fields in Eq. (12) look like the three distributions q_T, q_∂ and q'_∂ defined in Eq. (5) without the gauge links. If one considers the contributions from Fig. 1b and 1c and those with exchanges of more than one gluon, one can realizes that parts of contributions from exchanges of gluons can be summed into gauge links. Adding these parts to the contributions in the above, the results are simply obtained by replacing the three correlation functions in Eq. (12) with q_T, q_∂ and q'_∂, respectively.

It is straightforward to calculate the contributions from Fig. 1b and Fig. 1c. Parts of the contributions will be added to the contribution of Fig. 1a as discussed in the above. The remaining contributions can be easily found as:

$$W^{\mu\nu}\Big|_{1b+1c} = -\frac{1}{z_h} \hat{d}(z_h) \delta^2(q_\perp) \frac{i}{\pi P \cdot q} \left(P^\mu \tilde{s}_\perp^\nu - P^\nu \tilde{s}_\perp^\mu \right)$$
$$\times \int dx_1 P \frac{1}{x_1 - x_B} \Big[T_F(x_1, x_B) - T_\Delta(x_1, x_B) \Big]$$
$$+ \frac{1}{z_h} d(z_h) \delta^2(q_\perp) \frac{1}{k_B^-} \left(l^\mu \tilde{s}_\perp^\nu + l^\nu \tilde{s}_\perp^\mu \right) T_F(x_B, x_B)$$
$$+ \cdots, \tag{13}$$

where $\cdots$ stand for contributions beyond twist-3. The symmetric part is obtained by the absorptive part of the quark propagator between the photon and gluon vertex in Fig. 1b and Fig. 1c. This gives the so-called soft-gluon pole contribution. We note here that the results in Eqs. (12), (13) can be simplified with the relation in Eq. (7). Adding every contribution we have the total chirality-even part of $W^{\mu\nu}$:

$$W^{\mu\nu}\Big|_{even} = \frac{2}{z_h} \hat{d}(z_h) \delta^2(q_\perp) \frac{i}{P \cdot q} \left(P^\mu \tilde{s}_\perp^\nu - P^\nu \tilde{s}_\perp^\mu \right)$$
$$\times \Big(x_B q_T(x_B) - q_\partial(x_B) \Big)$$
$$- i\delta^2(q_\perp) \frac{2}{z_h P \cdot P_h} \hat{d}(z_h) \epsilon^{\mu\nu\alpha\beta} P_{h\alpha} s_{\perp\beta} q_T(x_B)$$
$$+ i \frac{2}{z_h} \hat{d}(z_h) \epsilon_\perp^{\mu\nu} q_\partial(x_B) s_\perp^\rho \frac{\partial}{\partial q_\perp^\rho} \delta^2(q_\perp)$$
$$+ \frac{1}{z} \hat{d}(z) T_F(x_B, x_B) \Bigg[\delta^2(q_\perp) \frac{1}{P \cdot q} (P^\mu \tilde{s}_\perp^\nu + P^\nu \tilde{s}_\perp^\mu)$$
$$+ g_\perp^{\mu\nu} \tilde{s}_\perp^\rho \frac{\partial}{\partial q_\perp^\rho} \delta^2(q_\perp) \Bigg]. \tag{14}$$

The symmetric part was first derived in [17]. Before we turn to the chirality-odd part, we discuss the $U(1)$-gauge invariance of our result. From the invariance one always has $q_\mu W^{\mu\nu} = 0$. But our

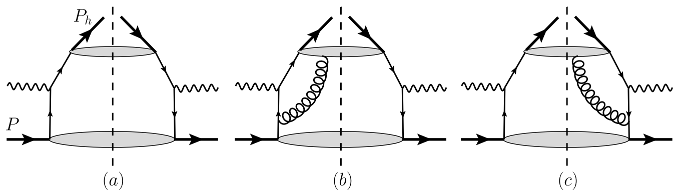

Fig. 2. Diagrams for contributions in SIDIS.

$W^{\mu\nu}$ is singular in $q_\perp$. It can only be taken as a distribution tensor. Then the $U(1)$-gauge invariance implies that for any test function $\mathcal{T}(q_\perp)$ one has:

$$\int d^2 q_\perp \mathcal{T}(q_\perp) q_\mu W^{\mu\nu} = 0. \tag{15}$$

It is easy to check that our result in Eq. (14) is $U(1)$-gauge invariant.

The chirality-odd contribution involves the transversity distribution and twist-3 fragmentation functions. It receives the contributions from diagrams given in Fig. 2. The calculation of Fig. 2 is similar to that of Fig. 1. Here we omit the details of derivation and give the results directly:

$$\begin{aligned}
W^{\mu\nu}\Big|_{2a} &= \Big[i\epsilon^{\mu\nu\alpha\beta} P_\alpha s_{\perp\beta} \hat{e}(z_h) - (P^\mu \tilde{s}_\perp^\nu + P^\nu \tilde{s}_\perp^\mu)\hat{e}_I(z_h) \Big] \\
&\quad \times \delta^2(q_\perp) \frac{2}{z_h P\cdot P_h} h_1(x_B) \\
&\quad + \Big[g_\perp^{\mu\nu}\tilde{s}_\perp^\rho - g_\perp^{\mu\rho}\tilde{s}_\perp^\nu - g_\perp^{\nu\rho}\tilde{s}_\perp^\mu \Big] h_1(x_B)\hat{e}_\partial(z_h) \\
&\quad \times \frac{1}{z_h}\frac{\partial}{\partial q_\perp^\rho}\delta^2(q_\perp), \\
W^{\mu\nu}\Big|_{2b+2c} &= -\frac{2}{x_B P\cdot P_h}\Big(P_h^\nu \tilde{s}_\perp^\mu + P_h^\mu \tilde{s}_\perp^\nu \Big)\delta^2(q_\perp) h_1(x_B) \\
&\quad \times \int \frac{dz_1}{z_1} P\frac{1}{z_1 - z_h}\,\mathrm{Im}\,\hat{E}_F(z_1, z_h) \\
&\quad + \frac{2i}{x_B P\cdot P_h}\Big(P_h^\nu \tilde{s}_\perp^\mu - P_h^\mu \tilde{s}_\perp^\nu \Big)\delta^2(q_\perp) h_1(x_B) \\
&\quad \times \int \frac{dz_1}{z_1} P\frac{1}{z_1 - z_h}\,\mathrm{Re}\,\hat{E}_F(z_1, z_h).
\end{aligned} \tag{16}$$

In the above only twist-3 contributions are given explicitly. Parts of contributions from Fig. 2b and 2c are added into the contributions to Fig. 2a as we have done for Fig. 1. The contribution from Fig. 2b and 2c can be simplified with the relation in Eq. (10). With the relation we obtain the chiral-odd contribution:

$$\begin{aligned}
W^{\mu\nu}\Big|_{odd} &= \frac{2i}{z_h P\cdot P_h} h_1(x_B)\hat{e}(z_h)\delta^2(q_\perp) \\
&\quad \times \Big[\epsilon^{\mu\nu\alpha\beta} P_\alpha s_{\perp\beta} - \frac{1}{x_B z_h}\Big(P_h^\nu \tilde{s}_\perp^\mu - P_h^\mu \tilde{s}_\perp^\nu \Big) \Big] \\
&\quad - \frac{2}{z_h P\cdot P_h}\delta^2(q_\perp) h_1(x_B) \Big[(P^\mu \tilde{s}_\perp^\nu + P^\nu \tilde{s}_\perp^\mu)\hat{e}_I(z_h) \\
&\quad - \frac{1}{x_B z_h}\Big(P_h^\nu \tilde{s}_\perp^\mu + P_h^\mu \tilde{s}_\perp^\nu \Big)\Big(z_h \hat{e}_\partial(z_h) - \hat{e}_I(z_h) \Big) \Big] \\
&\quad + \Big[g_\perp^{\mu\nu}\tilde{s}_\perp^\rho - g_\perp^{\mu\rho}\tilde{s}_\perp^\nu - g_\perp^{\nu\rho}\tilde{s}_\perp^\mu \Big] \\
&\quad \times h_1(x_B)\hat{e}_\partial(z_h)\frac{1}{z_h}\frac{\partial}{\partial q_\perp^\rho}\delta^2(q_\perp).
\end{aligned} \tag{17}$$

It is easy to find with Eq. (15) that the above result is $U(1)$-gauge invariant. The total twist-3 contribution of $W^{\mu\nu}$ is then the sum of the chirality-even part in Eq. (14) and the chirality-odd part in Eq. (17).

To study how to construct experimental observables it is convenient to express our $W^{\mu\nu}$ in the introduced $\mathcal{C}_1$-frame where the produced hadron has nonzero transverse momentum. The transverse momentum $P_{h\perp}$ in the $\mathcal{C}_1$-frame is related to the transverse momentum $q_\perp$ in the $\mathcal{C}_0$-frame as:

$$q_\perp^\mu\Big|_{\mathcal{C}_0} = -\frac{1}{z_h} P_{h\perp}^\mu\Big|_{\mathcal{C}_1}. \tag{18}$$

It should be noted that the two tensors $g_\perp^{\mu\nu}$ and $\epsilon_\perp^{\mu\nu}$ are covariant. But they are defined differently in different frames. In the following we will use the same notations for momenta and spin without confusion. The two tensors in the $\mathcal{C}_1$-frame are defined as:

$$\begin{aligned}
g_\perp^{\mu\nu} &= g^{\mu\nu} - \frac{1}{P\cdot\bar{P}}\left(P^\mu \bar{P}^\nu + P^\nu \bar{P}^\mu\right), \\
\epsilon_\perp^{\mu\nu} &= \frac{1}{P\cdot\bar{P}}\epsilon^{\alpha\beta\mu\nu} P_\alpha \bar{P}_\beta, \quad \bar{P} = x_B P + q.
\end{aligned} \tag{19}$$

With these notations our twist-3 contribution of $W^{\mu\nu}$ in the $\mathcal{C}_1$-frame is given by:

$$\begin{aligned}
W^{\mu\nu} &= \frac{2}{x_B P\cdot q}\delta^2(P_{h\perp}) \\
&\quad \times \Big[-i\epsilon^{\mu\nu\alpha\beta} q_\alpha s_{\perp\beta}\Big(x_B z_h q_T(x_B)\hat{d}(z_h) + h_1(x_B)\hat{e}(z_h) \Big) \\
&\quad + \Big((2x_B P + q)^\mu \tilde{s}_\perp^\nu + (2x_B P + q)^\nu \tilde{s}_\perp^\mu \Big) \\
&\quad \times h_1(x_B)\Big(z_h \hat{e}_\partial(z_h) - \hat{e}_I(z_h) \Big) \Big] \\
&\quad + z_h^2 \Big(\frac{\partial}{\partial P_{h\perp}^\rho}\delta^2(P_{h\perp}) \Big) \Big[-2i q_\partial(x_B)\hat{d}(z_h)\epsilon_\perp^{\mu\nu} s_\perp^\rho \\
&\quad - T_F(x_B, x_B)\hat{d}(z_h) g_\perp^{\mu\nu}\tilde{s}_\perp^\rho \\
&\quad - \Big(g_\perp^{\mu\nu}\tilde{s}_\perp^\rho - g_\perp^{\mu\rho}\tilde{s}_\perp^\nu - g_\perp^{\nu\rho}\tilde{s}_\perp^\mu \Big) h_1(x_B)\hat{e}_\partial(z_h) \Big].
\end{aligned} \tag{20}$$

This expression is explicitly $U(1)$-gauge invariant because in the $\mathcal{C}_1$-frame q^μ is given by $q^\mu = (q^+, q^-, 0, 0)$. This result is our main result. The obtained $W^{\mu\nu}$ in Eq. (20) is in fact a tensor distribution of $P_{h\perp}$, physical predictions can only be obtained if $P_{h\perp}$ is integrated out. We notice that in our main result all nonperturbative quantities are defined with two-parton correlations with the relation between $T_F(x_B, x_B)$ and $q'_\partial(x_B)$ in Eq. (7). However, this is only true at tree-level. Beyond tree-level it is not the case as discussed in detail in [18].

We consider the experimental situation in which the initial hadron is transversely polarized with the spin vector $s_\perp^\mu$. This vector is transverse to the lepton beam direction. In fact, this vector is not exactly the transverse spin vector in the $\mathcal{C}_1$-frame. But in the kinematical region of large Q^2, the two vectors are approximately the same [25]. We will neglect the difference between the two spin vectors. The incoming and outgoing leptons span the so-called lepton plane. In the $\mathcal{C}_1$-frame the azimuthal angle between the spin vector and the lepton plane is denoted ϕ_s. Similarly, one defines the azimuthal angle ϕ_h for the produced hadron. The azimuthal angle of the outgoing lepton around the lepton beam with respect to the spin vector is denoted ψ. In the kinematical region of SIDIS, one has $\psi \approx \phi_s$ [25]. With this specification the differential cross section is given by [16,25]:

$$\frac{d\sigma}{dx_B dy dz_h d\psi d^2 P_{h\perp}} = \frac{\alpha^2 y}{4 z_h Q^4} L_{\mu\nu} W^{\mu\nu}. \tag{21}$$

As discussed before, from our result we cannot predict the differential cross section as a function of ϕ_h and $P_{h\perp}^2$. The predictions can only be made by integrating out $P_{h\perp}$. Using the result in Eq. (20) and integrating $P_{h\perp}$ out, we obtain the twist-3 contribution to the differential cross section:

$$\begin{aligned}
&\frac{d\sigma}{dx_B dy dz_h d\psi} \\
&= \frac{4\alpha^2}{z_h Q^3} |s_\perp| \sqrt{1-y} \\
&\times \Bigg[-\lambda_e x_B \left(x_B z_h q_T(x_B) \hat{d}(z_h) + h_1(x_B) \hat{e}(z_h) \right) \cos\psi \\
&+ \frac{2-y}{y} h_1(x_B) \left(z_h \hat{e}_\partial(z_h) - \hat{e}_I(z_h) \right) \sin\psi \Bigg].
\end{aligned} \tag{22}$$

In this angular distribution the terms in Eq. (20) with the derivative of $\delta^2(P_{h\perp})$ do not contribute. To extract information of these terms, one can construct weighted angular distributions defined as:

$$\frac{d\sigma\langle \mathcal{F} \rangle}{dx_B dy dz_h d\psi} = \frac{\alpha^2 y}{4 z_h Q^4} \int d^2 P_{h\perp} L_{\mu\nu} W^{\mu\nu} \mathcal{F}(P_h, k', s_\perp) \tag{23}$$

with $\mathcal{F}$ as the weight function. For $\mathcal{F} = 1$ one obtains the differential cross section given in Eq. (22). One can obtain the following weighted angular distribution for these derivative terms:

$$\begin{aligned}
&\frac{d\sigma\langle P_{h\perp}\cdot k'_\perp \rangle}{dx_B dy dz_h d\psi} \\
&= \frac{\alpha^2 z_h}{2 Q y^2} |s_\perp| \sqrt{1-y} \Bigg[-2\lambda_e y(2-y) q_\partial(x_B) \hat{d}(z_h) \cos\psi \\
&- \Big((1+(1-y)^2) T_F(x_B, x_B) \hat{d}(z_h) \\
&- 2(1-y) h_1(x_B) \hat{e}_\partial(z_h) \Big) \sin\psi \Bigg].
\end{aligned} \tag{24}$$

One can construct more spin-dependent observables by integrating over the azimuthal angle with different weight functions. Our $W^{\mu\nu}$ in Eq. (20) have five tensor structures. Correspondingly one can have five observables. We can obtain the five weighted differential cross sections:

$$\begin{aligned}
&\frac{d\sigma\langle P_{h\perp}\cdot \tilde{s}_\perp \rangle}{dx_B dy dz_h} = \frac{\pi\alpha^2}{Q^2} \frac{1+(1-y)^2}{y} |s_\perp|^2 z_h T_F(x_B, x_B) \hat{d}(z_h), \\
&\frac{d\sigma\langle P_{h\perp}\cdot s_\perp \rangle}{dx_B dy dz_h} = -\lambda_e \frac{2\pi\alpha^2(2-y)}{Q^2} |s_\perp|^2 z_h q_\partial(x_B) \hat{d}(z_h), \\
&\frac{d\sigma\langle P_{h\perp}\cdot k'_\perp k'_\perp \cdot \tilde{s}_\perp \rangle}{dx_B dy dz_h} \\
&= \pi\alpha^2 |s_\perp|^2 z_h \frac{(1-y)^2}{y^3} \\
&\quad \times \left[h_1(x_B) \hat{e}_\partial(z_h) - \frac{1+(1-y)^2}{2(1-y)} \hat{d}(z_h) T_F(x_B, x_B) \right], \\
&\frac{d\sigma\langle k'_\perp \cdot s_\perp \rangle}{dx_B dy dz_h} \\
&= \lambda_e \frac{4\pi\alpha^2}{z_h Q^2} \frac{1-y}{y} |s_\perp|^2 \left(x_B z_h q_T(x_B) \hat{d}(z_h) + h_1(x_B) \hat{e}(z_h) \right), \\
&\frac{d\sigma\langle k'_\perp \cdot \tilde{s}_\perp \rangle}{dx_B dy dz_h} \\
&= \frac{4\pi\alpha^2}{z_h Q^2} \frac{1-y}{y^2} (2-y) |s_\perp|^2 h_1(x_B) \left(z_h \hat{e}_\partial(z_h) - \hat{e}_I(z_h) \right).
\end{aligned} \tag{25}$$

There are corrections to our hadronic tensor given in Eq. (20). They are from higher orders of α_s and from power corrections suppressed by $1/Q$ or $1/Q^2$. Therefore, our results of observables in Eqs. (22), (24), (25) are subjects of these corrections. Parts of one-loop corrections to the weighted differential cross section with $P_{h\perp}\cdot\tilde{s}_\perp$ in Eq. (25) have been given in [28,29].

Now we are in position to discuss the difference in our case between TMD and collinear factorizations. In general, TMD factorization can be used in the kinematical region of $P_{h\perp} \ll Q$. If one has $Q \gg P_{h\perp} \gg \Lambda_{QCD}$, one can show that the TMD factorization and the collinear factorization are equivalent in this kinematical region [26,27]. At first look, one can use these two factorization approaches to calculate observables like those in Eq. (25), in which $P_{h\perp}$ is integrated out. However, this is not trivial in fact. We will take the contribution containing Sivers function as an example to show this.

The relevant contribution in TMD factorization is [16]:

$$\begin{aligned}
&\frac{d\sigma}{dx_B dy dz_h d\psi d^2 P_{h\perp}} \\
&= \frac{\alpha^2(1+(1-y)^2)}{x_B y Q^2} |s_\perp| \sin(\phi_h - \phi_s) F_{UT,T}^{\sin(\phi_h-\phi_s)}(x_B, z_h, P_{h\perp}) \\
&\quad \cdot \left[1 + \mathcal{O}(P_{h\perp}^2/Q^2) \right] + \cdots, \\
&F_{UT,T}^{\sin(\phi_h-\phi_s)}(x_B, z_h, P_{h\perp}) \\
&= \frac{x_B}{|P_{h\perp}|} \int d^2 k_{A\perp} d^2 k_{B\perp} P_{h\perp}\cdot k_{A\perp} f_{1T}^\perp(x_B, k_{A\perp}) D_1(z_h, k_{B\perp}) \\
&\quad \cdot \delta^2(k_{A\perp} - k_{B\perp} - P_{h\perp}/z_h),
\end{aligned} \tag{26}$$

where $\cdots$ denote irrelevant contributions. The contribution given explicitly is relevant to the weighted differential cross section with $P_{h\perp}\cdot\tilde{s}_\perp$ in Eq. (25). We take the same notations here as those in [16]. $f_{1T}^\perp(x, k_\perp)$ is the Sivers function of the initial hadron and $D_1(z, k_\perp)$ is the TMD fragmentation function. Their definitions can be found in [16]. Formally, one can derive the relations:

$$\begin{aligned}
&\hat{d}(z) = z^2 \int d^2 k_\perp D_1(z, k_\perp), \\
&\int d^2 k_\perp |k_\perp|^2 f_{1T}^\perp(x, k_\perp) = -T_F(x, x).
\end{aligned} \tag{27}$$

The second equation is derived in [19]. With TMD factorization one is able to predict the distribution in ϕ_h. But the contribution given in Eq. (26) can only be used in the kinematical region for $P_{h\perp} \ll Q$. It has power corrections. If we neglect the power corrections, from Eq. (26) we have for the weighted differential cross section:

$$\begin{aligned}
&\frac{d\sigma\langle P_{h\perp}\cdot\tilde{s}_\perp \rangle}{dx_B dy dz_h} \\
&= -\frac{\pi\alpha^2}{Q^2} \frac{1+(1-y)^2}{y} |s_\perp|^2 z_h \\
&\quad \times \int d^2 P_{h\perp} d^2 k_{A\perp} d^2 k_{B\perp} |k_{A\perp}|^2 f_{1T}^\perp(x_B, k_{A\perp}) D_1(z_h, k_{B\perp}) \\
&\quad \cdot \delta^2(k_{A\perp} - k_{B\perp} - P_{h\perp}/z_h).
\end{aligned} \tag{28}$$

If we use the δ-function to perform the integration of $P_{h\perp}$, and then take the integration of $k_{A\perp}$ and that of $k_{B\perp}$ as two independent integrals, one can obtain the same result as that in Eq. (25) by using the relation in Eq. (27). However, in principle one cannot derive it in this way with TMD factorization. In fact the integration of $k_{A\perp}$ and that of $k_{B\perp}$ are not independent. They are correlated. Kinematically $P_{h\perp}$ is always finite. It cannot be infinitely large. Therefore $k_{A\perp} - k_{B\perp}$ is always finite. Since $P_{h\perp}$ here with TMD factorization is constrained at the order of Λ_{QCD} with $\Lambda_{QCD} \ll Q$, one always has the constraint $k_{A\perp} - k_{B\perp} \sim \Lambda_{QCD}$. The only way to derive the same result is to assume that one can neglect $k_{A\perp}$ and $k_{B\perp}$ in the δ-function. But with this assumption it implies that one actually uses collinear factorization at the beginning. It is interesting to note that at tree-level neglecting $k_{A\perp}$ and $k_{B\perp}$ in the δ-function is equivalent to relaxing the constraint $k_{A\perp} - k_{B\perp} \sim \Lambda_{QCD}$. Keeping these in mind, our results of observables can also be derived from TMD factorization. It is also worth to point out here that the formally derived relations in Eq. (27) are not exactly correct, as discussed in [30]. The reason is that in the integrations over transverse momenta there will be U.V. divergences and an U.V. subtraction needs to be implemented. This can be shown with the explicit calculation of $f_{1T}^{\perp}$ and $T_F(x,x)$ at the leading order of α_s with a multi-parton state in [31].

Before summarizing our work, we point out that from the derivation of our hadronic tensor in Eq. (20) one can realize that the virtual corrections to the terms with the derivative of $\delta^2(P_{h\perp})$ are determined completely by the quark form factor with certain subtractions of collinear divergences. This fact has been first noticed in the study of Drell–Yan processes in [17].

To summarize, we have derived the twist-3 part of the hadronic tensor in SIDIS at tree-level. This part depends on the transverse spin of the initial hadron. At tree-level, the obtained twist-3 part is completely expressed with nonperturbative quantities defined with two-parton correlations. Spin-dependent observables are constructed based on the obtained hadron tensor. A comparison of collinear factorization with TMD factorization is given in the studied case. Measurements of the various spin-dependent observables in SIDIS will be helpful to extract information about the transversity distribution at twist-2 and twist-3 parton distributions and fragmentation functions.

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