



# Platonic gravitating skyrmions

Theodora Ioannidou<sup>a</sup>, Burkhard Kleihaus<sup>b,\*</sup>, Jutta Kunz<sup>b</sup>

<sup>a</sup> *Mathematics Division, School of Technology, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece*

<sup>b</sup> *Institut für Physik, Universität Oldenburg, Postfach 2503, D-26111 Oldenburg, Germany*

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## Abstract

We construct globally regular gravitating skyrmions, which possess only discrete symmetries. In particular, we present tetrahedral and cubic skyrmions. The  $SU(2)$  Skyrme field is parametrized by an improved harmonic map ansatz. Consistency then requires also a restricted ansatz for the metric. The numerical solutions obtained within this approximation are compared to those obtained in dilaton gravity.

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## 1. Introduction

Nonlinear field theories coupled to gravity lead to globally regular gravitating configurations [1]. Moreover, also black hole solutions with nonlinear hair arise [1]. These black hole solutions are asymptotically flat and possess a regular event horizon. Outside their horizon they retain the features of the corresponding gravitating solitons, and may thus be viewed as bound states of solitons and Schwarzschild black holes [2].

In the Einstein–Skyrme model the nonlinear chiral field theory describing baryons and nuclei in terms of solitons (so-called skyrmions) is coupled to gravity. Static spherically symmetric  $SU(2)$  gravitating skyrmions and black holes with skyrmion hair [3–6] exhibit a characteristic dependence on the coupling parameter: Two branches of solutions merge and end at a maximal value of the coupling parameter, and only the solutions on the lower branch are classically stable. Spherically symmetric  $SU(N)$  gravitating skyrmions and black holes have been analyzed similarly [7,8].

Recently, static axially symmetric  $SU(2)$  gravitating skyrmions and black holes with skyrmion hair have been constructed numerically [9], while approximations to axially sym-

metric gravitating  $SU(3)$  skyrmions have been obtained in [10]. The hairy black holes represent further examples demonstrating that Israel's theorem does not generalize to theories with non-Abelian fields [11].

On the other hand, in flat space also skyrmions with no rotational but only discrete symmetries have been constructed [12]. Among them are solutions with the symmetries of the platonic solids, to which we refer as platonic skyrmions. Besides the exact numerical solutions, also approximate solutions have been obtained. Such approximate skyrmion solutions with baryon number  $B$  have been constructed using rational maps of degree  $B$  between Riemann spheres [13], as well as an improved harmonic map ansatz [14].

In this Letter we consider gravitating skyrmion configurations with only platonic symmetries. In particular, we focus on configurations with tetrahedral and cubic symmetry, possessing baryon number  $B = 3$  and  $B = 4$ , respectively.

Recall that the  $SU(2)$  Einstein–Skyrme action reads

$$S = \int \left[ \frac{R}{16\pi G} + \frac{\kappa^2}{4} \text{Tr}(K_\mu K^\mu) + \frac{1}{32e^2} \text{Tr}([K_\mu, K_\nu][K^\mu, K^\nu]) \right] \sqrt{-g} d^4x, \quad (1)$$

where  $R$  is the curvature scalar,  $g$  denotes the determinant of the metric, the  $SU(2)$  Skyrme field  $U$  enters via  $K_\mu = \partial_\mu U U^{-1}$ ,  $G$  represents Newton's constant, and  $\kappa$  and  $e$  are the Skyrme model coupling constants.

\* Corresponding author.

*E-mail addresses:* ti3@auth.gr (T. Ioannidou), kleihaus@theorie.physik.uni-oldenburg.de (B. Kleihaus), kunz@theorie.physik.uni-oldenburg.de (J. Kunz).

In order for finite-energy configurations to exist the Skyrme field must tend to a constant matrix at spatial infinity:  $U \rightarrow \mathbb{1}$  as  $|x^i| \rightarrow \infty$ . This effectively compactifies the three-dimensional Euclidean space into  $S^3$  and implies that the Skyrme fields can be considered as maps from  $S^3$  into  $SU(2)$ .

Variation of action (1) with respect to the metric  $g^{\mu\nu}$  leads to the Einstein equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}, \quad (2)$$

with stress–energy tensor

$$\begin{aligned} T_{\mu\nu} = & -\frac{\kappa^2}{2} \text{Tr} \left( K_\mu K_\nu - \frac{1}{2} g_{\mu\nu} K_\alpha K^\alpha \right) \\ & - \frac{1}{8e^2} \text{Tr} \left( g^{\alpha\beta} [K_\mu, K_\alpha] [K_\nu, K_\beta] \right. \\ & \left. - \frac{1}{4} g_{\mu\nu} [K_\alpha, K_\beta] [K^\alpha, K^\beta] \right). \end{aligned} \quad (3)$$

Variation with respect to the Skyrme field leads to the field equation

$$\nabla_\mu \left( \kappa^2 K^\mu + \frac{1}{4e^2} [K_\nu, [K^\mu, K^\nu]] \right) = 0. \quad (4)$$

The Einstein–Skyrme system has a topological current

$$B^\mu = \frac{1}{\sqrt{-g}} \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} \text{Tr}(K_\nu K_\alpha K_\beta), \quad (5)$$

which corresponds to the baryon current, yielding the baryon number  $B$

$$B = \int \sqrt{-g} B^0 d^3x. \quad (6)$$

For comparison, we also consider Skyrme-dilaton theory with action

$$\begin{aligned} S = & \int \left[ -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{\kappa^2}{4} \text{Tr}(K_\mu K^\mu) \right. \\ & \left. + \frac{1}{32e^2} e^{-2\gamma\phi} \text{Tr}([K_\mu, K_\nu][K^\mu, K^\nu]) \right] d^4x, \end{aligned} \quad (7)$$

with  $\gamma = \sqrt{4\pi G}$ , and indices are lowered and raised by the flat Minkowski metric.

The idea of the harmonic map ansatz for  $SU(N)$  skyrmions [15] (which is the generalisation of the rational map ansatz of Houghton et al. [13]) involves the separation of the radial and angular dependence of the fields as

$$\begin{aligned} U = & e^{2ih(r)(P-\mathbb{1}/N)} \\ = & e^{-2ih(r)/N} [\mathbb{1} + (e^{2ih(r)} - 1)P]. \end{aligned} \quad (8)$$

Here  $h(r)$  is the corresponding profile function and  $P$  is a  $N \times N$  Hermitian projector, which depends only on the angular variables  $(\xi, \bar{\xi})$ , where  $\xi$  is the Riemann sphere variable given by  $\xi = e^{i\varphi} \tan(\theta/2)$  in terms of the usual spherical coordinates  $r, \theta, \varphi$ . Note that the matrix  $P$  can be thought of as a mapping from  $S^2$  into  $CP^{N-1}$ . Thus,  $P$  can be written as

$$P(V) = \frac{V \otimes V^\dagger}{|V|^2}, \quad (9)$$

where  $V$  is a  $N$ -component complex vector (depending on  $\xi$ ).

The improved harmonic map ansatz is obtained by allowing the profile function  $h$  to depend on all spatial coordinates  $r, \xi, \bar{\xi}$ . As has been shown in [14], this ansatz leads to a better approximation of the skyrmion energy in flat space.

For  $SU(2)$  skyrmions the harmonic map ansatz can be related to rational maps  $R(\xi)$  by

$$V = (R(\xi), 1)^t, \quad (10)$$

where  $R(\xi) = p(\xi)/q(\xi)$ , and  $p(\xi), q(\xi)$  are polynomials of  $\xi$ . Defining the unit vector  $\hat{n}_R$  by

$$\hat{n}_R = \frac{1}{1+|R|^2} (R + \bar{R}, -i(R - \bar{R}), 1 - |R|^2), \quad (11)$$

the Skyrme field  $U$  takes the simple form

$$U = \cos(h)\mathbb{1} + i \sin(h)\hat{n}_R \cdot \vec{\tau}, \quad (12)$$

with the vector of Pauli matrices  $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ .

## 2. Metric ansatz

To obtain static solutions without rotational symmetries let us first consider the following ansatz for the metric

$$ds^2 = -f dt^2 + \frac{1}{f} (m_1 dr^2 + m_2 r^2 d\theta^2 + l r^2 \sin^2 \theta d\varphi^2), \quad (13)$$

where  $f, m_1, m_2$  and  $l$  are functions of  $r, \theta$  and  $\varphi$ . In the case of axial symmetry,  $m_1 = m_2 = m$  [16].

Insertion of the Skyrme field (12) and the metric (13) in the action (1), and subsequent variation of the action with respect to the Skyrme profile function  $h$  yields a second-order partial differential equation (PDE) for  $h$ . Similarly, PDEs for the metric functions are obtained, which are equivalent to those obtained from the general Einstein equations (2) after insertion of the Skyrme field and the metric.

It now turns out, that this coupled system of PDEs does not possess a solution in general. While surprising at first, this fact has the following reason: The Einstein tensor  $G_{\mu\nu}$  is defined so that its covariant divergence vanishes,

$$\nabla_\mu G^{\mu\nu} = 0. \quad (14)$$

Consequently, the Einstein equations require

$$\nabla_\mu T^{\mu\nu} = 0, \quad (15)$$

provided there exists a solution. Insertion of the Skyrme field (12) and the metric (13) yields, however,

$$\begin{aligned} \nabla_\mu T^{\mu 0} = 0, & \quad \nabla_\mu T^{\mu r} = 0, \\ \nabla_\mu T^{\mu \theta} \neq 0, & \quad \nabla_\mu T^{\mu \varphi} \neq 0. \end{aligned} \quad (16)$$

Consequently, not all Einstein equations can be satisfied. In the axially symmetric case, when  $m_1 = m_2$  and the Skyrme and metric functions do not depend on  $\varphi$ , we can achieve  $\nabla_\mu T^{\mu\varphi} = 0$ , but still  $\nabla_\mu T^{\mu\theta} \neq 0$ . Only in the spherically symmetric case, when  $m_1 = m_2 = l$  and the Skyrme and metric functions depend only on  $r$ , all equations in (15) are satisfied.

In this case, however, the ansatz for the skyrmion field leads to an exact solution.

This problem can be traced back to the fact that the harmonic map ansatz for the Skyrme field is too restrictive. Indeed, for a general ansatz for the Skyrme field  $U$  involving three functions  $h_i(r, \theta, \varphi)$

$$U = \cos(h_1)\mathbb{1} + i \sin(h_1)[\sin(h_2)(\cos(h_3)\tau_1 + \sin(h_3)\tau_2) + \cos(h_2)\tau_3] \quad (17)$$

all equations in (15) are satisfied.

While aiming at the numerical construction of exact platonic gravitating skyrmions, in view of the complexity of the coupled Einstein–Skyrme equations we first want to obtain simpler approximate solutions, based on the improved harmonic map ansatz.

We therefore argue as follows: Since we employ only an approximate ansatz for the Skyrme field, we should restrict also to an approximate ansatz for the metric, compatible with the ansatz for the Skyrme field. An appropriate ansatz for the metric is given by

$$ds^2 = -f dt^2 + \frac{l}{f}(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2), \quad (18)$$

where we allow only for two metric functions  $f$  and  $l$  (as in the spherically symmetric case), which, however, depend on all three coordinates, like the Skyrme function  $h$  of the improved harmonic map ansatz.

We then derive the set of three coupled PDEs as variational equations from the Einstein–Skyrme action (1), after the second order derivatives of the metric functions have been eliminated by integration by parts. We refer to this approximation as “ $f$ - $l$ -approximation”.

We note that a further restriction of the metric obtained by setting  $l = 1$ , leads to solutions of the Skyrme-dilaton model with action (7). In this case the dilaton can be expressed by the metric function  $f$ ,

$$\phi = -\frac{1}{2\gamma} \log(f), \quad \gamma = \sqrt{4\pi G}. \quad (19)$$

We refer to this approximation as “dilaton-approximation”.

To obtain an estimate for the quality of the approximation, we can substitute the approximate solutions in the full set of Einstein and matter equations, to see how strongly these are violated.

### 3. Numerical solutions

#### 3.1. Parameters and boundary conditions

Introducing the dimensionless radial coordinate  $x = \kappa r$  and coupling parameter  $\alpha = 4\pi G\kappa^2$ , the action (1) becomes

$$S = \frac{\kappa}{e^2} \int \left[ \frac{R}{4\alpha} + \frac{1}{4} \text{Tr}(K_\mu K^\mu) + \frac{1}{32} \text{Tr}([K_\mu, K_\nu][K^\mu, K^\nu]) \right] \sqrt{-g} d^4x, \quad (20)$$

and the Einstein equations read  $G_{\mu\nu} = 2\alpha T_{\mu\nu}$ . Thus the solutions depend only on the coupling parameter  $\alpha$  and the chosen rational map. Likewise, in Skyrme-dilaton theory, the solutions depend only on the dimensionless coupling parameter  $\alpha = \gamma^2\kappa^2$  and the chosen rational map.

For skyrmions with axial symmetry rational maps of degree  $B$  are simply given by  $R_B = \xi^B$ , while for skyrmions with platonic symmetries these maps are more complicated. In particular, for tetrahedral  $B = 3$  skyrmions and cubic  $B = 4$  skyrmions these maps are given by

$$R_{\text{tetra}} = \frac{\sqrt{3}\xi^2 + 1}{\xi(\xi^2 - \sqrt{3})} \quad (21)$$

and

$$R_{\text{cube}} = \frac{\xi^4 + 2\sqrt{3}i\xi^2 + 1}{\xi^4 - 2\sqrt{3}i\xi^2 + 1}, \quad (22)$$

respectively.

In order to map the infinite range of the radial variable  $x$  to the finite interval  $[0, 1]$  we introduce the compactified radial variable  $\bar{x} = x/(1+x)$ .

For axially symmetric solutions the Skyrme and metric functions depend only on the coordinates  $\bar{x}$  and  $\theta$ . Due to the reflection symmetry,  $z \leftrightarrow -z$ , it is sufficient to construct solutions for  $0 \leq \bar{x} \leq 1$ ,  $0 \leq \theta \leq \pi/2$ . The boundary conditions at the origin are

$$h(0) = \pi, \quad \partial_x f|_0 = 0, \quad \partial_x l|_0 = 0. \quad (23)$$

Asymptotically the Skyrme field tends to the unit matrix and the metric approaches the Minkowski metric, i.e.

$$h(\infty) \rightarrow 0, \quad f(\infty) \rightarrow 1, \quad l(\infty) \rightarrow 1. \quad (24)$$

On the  $z$ -axis ( $\theta = 0$ ) and in the  $xy$ -plane ( $\theta = \pi/2$ ) the boundary conditions follow from regularity and reflection symmetry, respectively,

$$\partial_\theta h = 0, \quad \partial_\theta f = 0, \quad \partial_\theta l = 0. \quad (25)$$

For solutions with discrete symmetries the Skyrme and metric functions depend on all three coordinates  $x, \theta, \varphi$ . The tetrahedral symmetry of the  $B = 3$  solution allows to restrict to the domain of integration to  $0 \leq \bar{x} \leq 1$ ,  $0 \leq \theta \leq \pi$ ,  $0 \leq \varphi \leq \pi/2$ . Similarly, the cubic symmetry of the  $B = 4$  solution allows to restrict to the domain of integration to  $0 \leq \bar{x} \leq 1$ ,  $0 \leq \theta \leq \pi/2$ ,  $0 \leq \varphi \leq \pi/2$ . The boundary conditions at the origin, at infinity, on the  $z$ -axis and in the  $xy$ -plane are the same as for the axially symmetric solutions. The remaining boundary conditions at  $\varphi = 0$  and  $\varphi = \pi/2$  follow from the platonic symmetries, i.e.

$$\partial_\varphi h = 0, \quad \partial_\varphi f = 0, \quad \partial_\varphi l = 0. \quad (26)$$

#### 3.2. Numerical results

Solutions are constructed with help of the software package FIDISOL [17] based on the Newton–Raphson algorithm. Typical grids contain  $70 \times 30$  points for the axially symmetric solutions and  $70 \times 25 \times 25$  points for the platonic solutions.

The estimated relative errors are approximately  $\approx 0.1\%$ , except close to  $\alpha_{\max}$ , where they become as large as  $1\%$ .

We have constructed gravitating skyrmions with baryon number  $B = 2, 3, 4$  in the “ $f$ - $l$ -approximation” and “dilaton-approximation”, and studied their dependence on the coupling parameter  $\alpha$ . The  $\alpha$ -dependence of the axially symmetric ( $B = 2, 3, 4$ ) and platonic ( $B = 3, 4$ ) skyrmions is completely analogous to the  $\alpha$ -dependence of the spherically symmetric  $B = 1$  skyrmions [5–7].

Gravitating skyrmions exist only up to a maximal value of the coupling parameter,  $\alpha_{\max}$ , which depends on the specific rational map and on the approximation (see Table 1). When  $\alpha$  is increased from zero a branch of gravitating skyrmions emerges

Table 1  
The maximal value of the coupling parameter,  $\alpha_{\max}$ , for axial ( $R_2, R_3, R_4$ ) and platonic ( $R_{\text{tetra}}, R_{\text{cube}}$ ) skyrmions

Rat. map	$\alpha_{\max}$	
	“ $f$ - $l$ ”	“dilaton”
$R_2$	0.0318	0.0294
$R_3$	0.0266	0.0246
$R_{\text{tetra}}$	0.0267	0.0248
$R_4$	0.0231	0.0214
$R_{\text{cube}}$	0.0234	0.0218

from the corresponding flat space skyrmion solution. This first (lower) branch extends up to the maximal value  $\alpha_{\max}$ , where it merges with a second (upper) branch of solutions. The upper branch then extends back to  $\alpha = 0$ .

In Fig. 1 we present the mass per baryon number as a function of  $\alpha$  (left), for axially symmetric  $B = 2$  and platonic  $B = 3$  and  $B = 4$  skyrmions. On both branches the mass decreases with increasing  $\alpha$ . But whereas the mass remains finite in the limit  $\alpha \rightarrow 0$  on the lower branch, it diverges in this limit on the upper branch. Thus on the upper branch the limit  $\alpha \rightarrow 0$  does not correspond to a flat space limit, where gravity decouples.

To better understand the limit  $\alpha \rightarrow 0$  on the upper branch, we note that the coupling parameter  $\alpha = 4\pi G\kappa^2$  vanishes, either when  $G$  vanishes while  $\kappa$  remains constant, or when  $\kappa$  vanishes while  $G$  remains constant. The first case corresponds to the flat space limit of the lower branch, while the second case corresponds to the limit  $\alpha \rightarrow 0$  of the upper branch. Introducing the rescaled radial coordinate  $\tilde{x} = x/\sqrt{\alpha}$  and the rescaled mass  $\tilde{M} = M\sqrt{\alpha}$ , one observes, that the rescaled mass remains finite in the limit  $\alpha \rightarrow 0$  on the upper branch, as illustrated in Fig. 1 (right).

We note that the  $\alpha$ -dependence of the mass of the gravitating skyrmions solutions is almost the same in the “ $f$ - $l$ -approximation” as in the “dilaton-approximation”. In

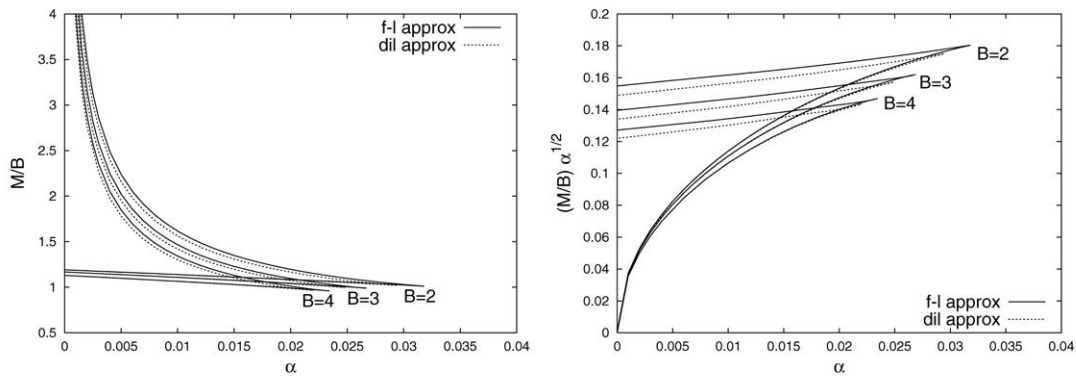


Fig. 1. The dimensionless mass per baryon number  $M/B$  (left) and the scaled mass per baryon number  $(M/B)\sqrt{\alpha}$  (right) are shown as functions of the coupling parameter  $\alpha$  for axial ( $B = 2$ ) and platonic ( $B = 3, 4$ ) skyrmions in the “ $f$ - $l$ -approximation” and the “dilaton-approximation”.

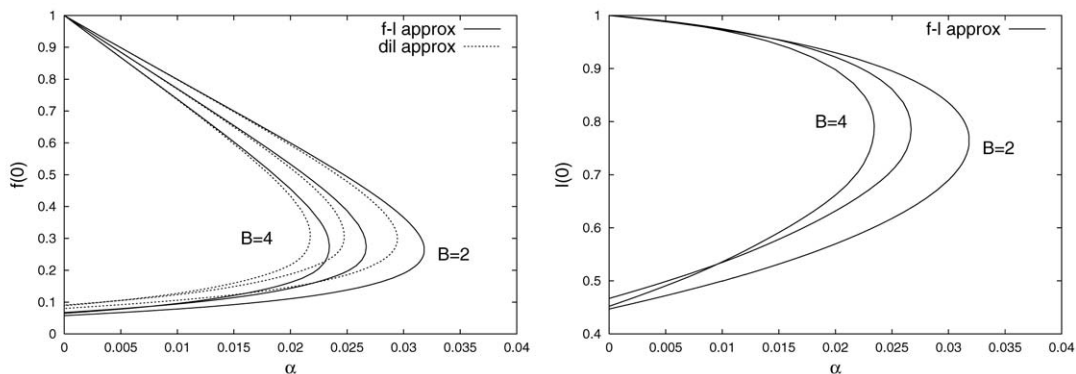


Fig. 2. Same as Fig. 1 for the value of the functions  $f$  and  $l$  at the origin. Note, that  $l = 1$  in the “dilaton-approximation”.

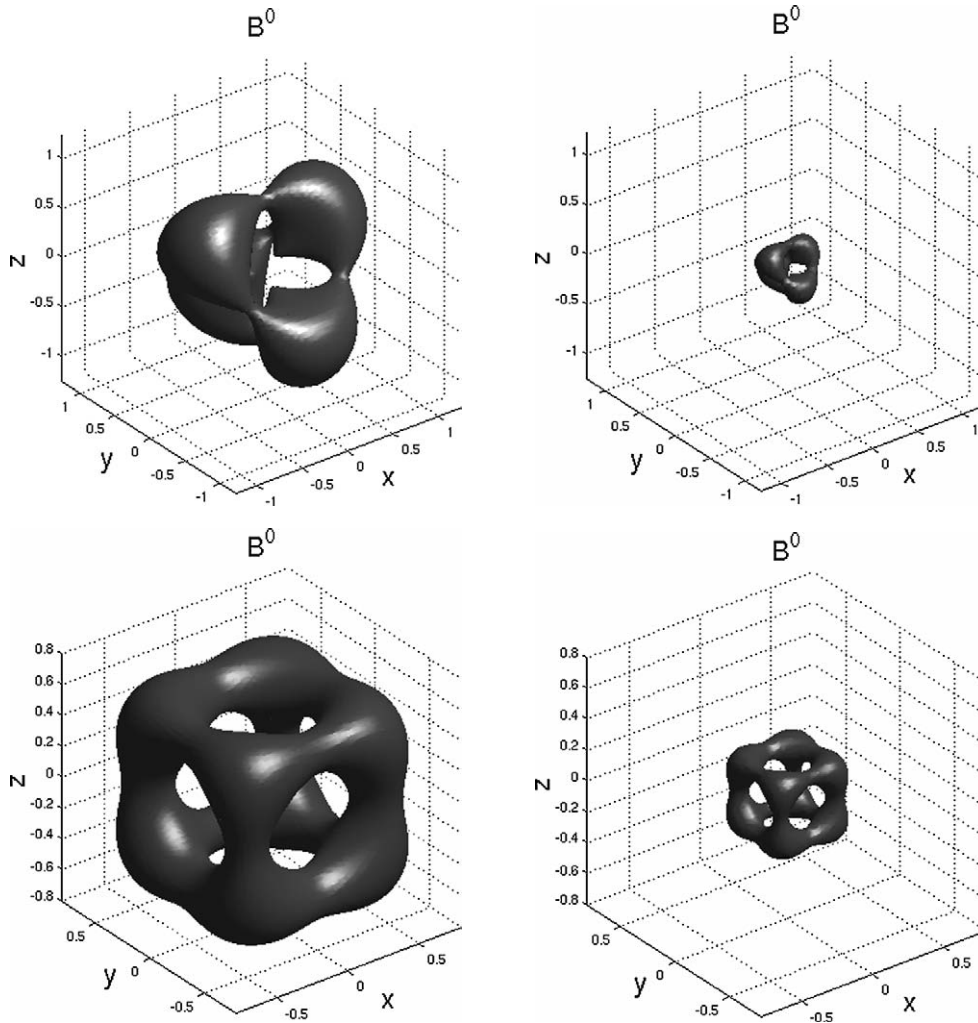


Fig. 3. Isosurface plot of the baryon density  $B^0(x, y, z) = 0.25 \times \max(B^0)$  for the  $B = 3$  skyrmion (upper row) and  $B^0(x, y, z) = 0.5 \times \max(B^0)$  for the  $B = 4$  skyrmion (lower row) in the “ $f$ - $l$ -approximation” for  $\alpha = 0.02$  on the lower branch (left column) and the upper branch (right column).

the “ $f$ - $l$ -approximation” the mass is slightly higher, in particular, along the upper branch. Also,  $\alpha_{\max}$  is slightly larger in the “ $f$ - $l$ -approximation”. The mass of the axially symmetric  $B = 3$  and  $B = 4$  skyrmions is always larger than the mass of the corresponding platonic skyrmions.

In Fig. 2 we exhibit the value of the metric functions  $f$  and  $l$  at the origin for these axial ( $B = 2$ ) and platonic ( $B = 3, 4$ ) skyrmions. We observe that  $f(0)$  and  $l(0)$  take finite values in the limit  $\alpha \rightarrow 0$  on the upper branch.

Focussing now on the platonic skyrmions, we exhibit in Fig. 3 surfaces of constant baryon density for tetrahedral  $B = 3$  and cubic  $B = 4$  skyrmion solutions on the lower branch (left) and upper branch (right). For a given rational map and coupling parameter  $\alpha$  the skyrmion on the upper branch is confined in a smaller volume than the skyrmion on the lower branch, while the shape of the baryon density is primarily determined by the rational map [12], analogous to the shape of the energy density [18].

In Fig. 4 we demonstrate that the metric functions  $f$  and  $l$  of the platonic  $B = 3$  and  $B = 4$  skyrmions possess the same symmetry as the baryon density. In fact, when gravity is weak the

function  $1 - f$  is proportional to the Newtonian gravitational potential.

#### 4. Conclusions

While aiming at the numerical construction of exact platonic gravitating skyrmions, we have here obtained simpler approximate solutions, based on the improved harmonic map ansatz for the Skyrme field, thus avoiding the full complexity of the coupled Einstein–Skyrme equations. This ansatz for the Skyrme field involves a single function instead of three functions. Consequently, an appropriate restriction of the ansatz for the metric is required, involving either two functions (“ $f$ - $l$ -approximation”) or a single function (“dilaton-approximation”).

We have focussed on platonic gravitating skyrmions with tetrahedral ( $B = 3$ ) and cubic ( $B = 4$ ) symmetry. For comparison, we have also constructed gravitating axially symmetric skyrmions ( $B = 2, 3, 4$ ). The dependence on the coupling parameter  $\alpha$  of the axial and platonic skyrmions is completely analogous to the  $\alpha$ -dependence of spherical  $B = 1$  skyrmions.

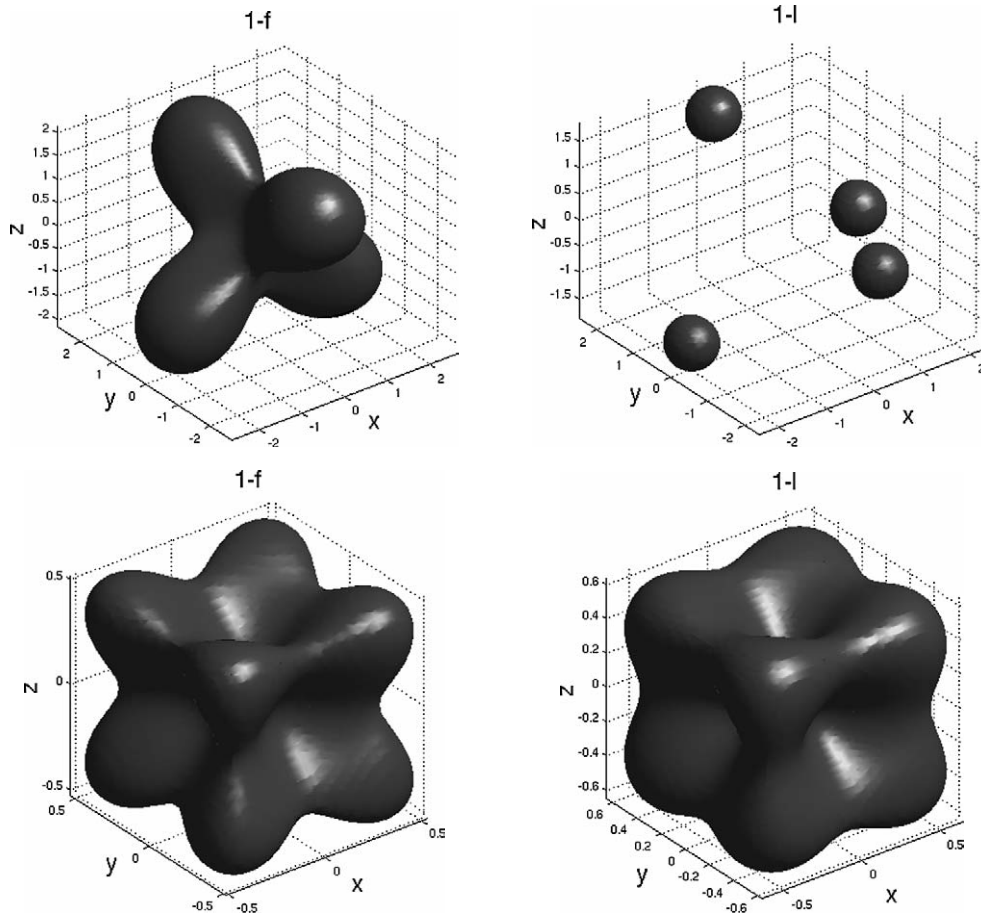


Fig. 4. Isosurface plots of the functions  $1 - f$  (left column) and  $1 - l$  (right column) for the  $B = 3$  skyrmion (upper row) and the  $B = 4$  skyrmion (lower row) in the “ $f$ - $l$ -approximation” for  $\alpha = 0.02$  on the lower branch.

When  $\alpha$  is increased from zero a lower branch of gravitating skyrmions emerges from the corresponding flat space skyrmion solution. This branch extends up to a maximal value  $\alpha_{\max}$ , where it merges with an upper branch of solutions, which extends back to  $\alpha = 0$ . Thus gravitating skyrmions exist only up to a maximal value of the coupling parameter,  $\alpha_{\max}$ , which depends on the specific rational map and on the approximation used.

The shape of the baryon density of platonic skyrmions is primarily determined by the rational map and analogous to the shape of the energy density. For a given rational map and coupling parameter  $\alpha$  the skyrmion on the upper branch is confined in a smaller volume than the skyrmion on the lower branch. The metric functions of the platonic skyrmions possess the same symmetry as the baryon density.

Comparing the approximations, applied in the construction of the gravitating tetrahedral and cubic skyrmions, we note that their mass is slightly higher in the “ $f$ - $l$ -approximation” than in the “dilaton-approximation”. On the other hand, in flat space the exact skyrmion [12] has a slightly lower mass than the approximate skyrmion [14]. Therefore, it is an open question whether the mass of the exact gravitating skyrmion solutions remains lower than the mass of the approximate solutions for all values of the coupling parameter. Construction of the exact

platonic gravitating skyrmion solutions, however, remains currently still a numerical challenge.

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### References

- [1] M.S. Volkov, D.V. Gal'tsov, Phys. Rep. 319 (1999) 1.
- [2] A. Ashtekar, A. Corichi, D. Sudarsky, Class. Quantum Grav. 18 (2001) 919.
- [3] N.K. Glendenning, T. Kodama, F.R. Klinkhamer, Phys. Rev. D 38 (1988) 3226.
- [4] H. Luckock, I. Moss, Phys. Lett. B 176 (1986) 341; H. Luckock, Black hole skyrmions, in: H.J. de Vega, N. Sanchez (Eds.), Proceedings of the 1986 Paris-Meudon Colloquium, World Scientific, Singapore, 1987.
- [5] S. Droz, M. Heusler, N. Straumann, Phys. Lett. B 268 (1991) 371; M. Heusler, S. Droz, N. Straumann, Phys. Lett. B 271 (1991) 61; M. Heusler, S. Droz, N. Straumann, Phys. Lett. B 285 (1992) 21.
- [6] P. Bizon, T. Chmaj, Phys. Lett. B 297 (1992) 55.
- [7] B. Kleihaus, J. Kunz, A. Sood, Phys. Lett. B 354 (1995) 240.
- [8] Y. Brihaye, B. Hartmann, T. Ioannidou, W.J. Zakrzewski, Phys. Rev. D 69 (2004) 124035.
- [9] N. Sawado, N. Shiiki, gr-qc/0307115;

- N. Shiiki, N. Sawado, T. Torii, K. Maeda, *Gen. Relativ. Gravit.* 36 (2004) 1361;  
N. Shiiki, N. Sawado, gr-qc/0501025.
- [10] T. Ioannidou, B. Kleihaus, W.J. Zakrzewski, *Phys. Lett. B* 600 (2004) 116.
- [11] B. Kleihaus, J. Kunz, *Phys. Rev. Lett.* 79 (1997) 1595;  
B. Kleihaus, J. Kunz, *Phys. Rev. D* 57 (1998) 6138.
- [12] E. Braaten, S. Townsend, L. Carson, *Phys. Lett. B* 235 (1990) 147;  
C.J. Houghton, P.M. Sutcliffe, *Commun. Math. Phys.* 180 (1996) 343;  
R.A. Battye, P.M. Sutcliffe, *Phys. Rev. Lett.* 79 (1997) 363;  
R.A. Battye, P.M. Sutcliffe, *Phys. Lett. B* 416 (1998) 385;  
D.Yu. Grigoriev, P.M. Sutcliffe, D.H. Tchrakian, *Phys. Lett. B* 540 (2002) 146.
- [13] C.J. Houghton, N.S. Manton, P.M. Sutcliffe, *Nucl. Phys. B* 510 (1998) 507.
- [14] T. Ioannidou, B. Kleihaus, W.J. Zakrzewski, *Phys. Lett. B* 597 (2004) 346.
- [15] T. Ioannidou, B.M.A.G. Piette, W.J. Zakrzewski, *J. Math. Phys.* 40 (1999) 6353.
- [16] B. Kleihaus, J. Kunz, *Phys. Rev. Lett.* 78 (1997) 2527;  
B. Kleihaus, J. Kunz, *Phys. Rev. D* 57 (1998) 834.
- [17] W. Schönauer, R. Weiß, *J. Comput. Appl. Math.* 27 (1989) 279;  
M. Schauder, R. Weiß, W. Schönauer, *The CADSOL Program Package*, Universität Karlsruhe, Interner Bericht Nr. 46/92, 1992.
- [18] B. Kleihaus, J. Kunz, K. Myklevoll, *Phys. Lett. B* 582 (2004) 187;  
B. Kleihaus, J. Kunz, K. Myklevoll, *Phys. Lett. B* 605 (2005) 151;  
B. Kleihaus, J. Kunz, K. Myklevoll, *Phys. Lett. B* 632 (2006) 333;  
B. Kleihaus, J. Kunz, K. Myklevoll, hep-th/0601124.