



## Diphoton isolation studies

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### Abstract

We consider the effects of the photon isolation on the production of a pair of photons in hadron collisions. We study in detail advantages and disadvantages of the standard and ‘smooth’ cone isolation criteria, concerning the theory and the experiment. We put special interest in those kinematic configurations related to recent Higgs boson studies and searches, and finally we show the set of isolation parameters proposed by the Les Houches accord which serves as a guide to understand the comparison of the theoretical predictions with data.

**Keywords:** NLO QCD calculations, NNLO QCD calculations, Diphoton production, Higgs

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### 1. Introduction

The measurements of the production cross section of two energetic isolated photons (diphotons) in high energy hadron collisions are important for testing Standard Model (SM) predictions in the domain of searches for undiscovered particles and new physics. Describing the reaction mechanisms in such complicated environment, produced in such collisions, is a challenge for perturbative Quantum Chromodynamics (pQCD) calculations. The photons which have their origin in hadron collisions (“prompt”<sup>1</sup> photons) are an ideal probe for testing these calculations because they do not interact with other final state particles, and their energies and directions can be measured with high precision in modern electromagnetic calorimeters. Hadronic production of isolated diphotons is not only important for QCD studies. Prompt diphoton production creates an irreducible background to the diphoton decay channel of the Higgs boson [1, 2], in spite of its relatively low branching fraction, since this channel benefits from a clean signature,

provided that a sufficiently high-resolution electromagnetic calorimeter is used. An improved knowledge of the SM background will help the development of more powerful search strategies and studies for this particle. The collider experiments at the Tevatron and the LHC do not perform *inclusive* photon measurements. The background of secondary photons coming from the decays of  $\pi^0$ ,  $\eta$ , etc., overwhelms the signal by several orders of magnitude, and the experimental selection of prompt diphotons requires *isolation* cuts, to reject this background. The prompt-diphoton cross section itself is affected by the isolation cuts (or criteria) which reject the background of secondary photons, in particular by reducing the effect of fragmentation.

The standard cone isolation and the “smooth” cone isolation proposed by Frixione [4] are two of these criteria. The case of the standard cone isolation is easily implemented in experiments, but it only suppresses a fraction of the fragmentation contribution. The smooth cone isolation (formally) eliminates the entire fragmentation contribution, but its experimental implementation (at least in its original form) is complicated<sup>2</sup> by the finite granularity of the LHC and Tevatron detectors.

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<sup>1</sup>The expression ‘prompt photons’ means that these photons do not come from the decay of hadrons, such as  $\pi^0$ ,  $\eta$ , etc., produced at large transverse momenta. Prompt photons can be produced according to two possible mechanisms, one of them being a fragmentation mechanism and the other arises directly from the hard part of the interaction.

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<sup>2</sup>There is activity in the experimental implementation [3, 5, 6] of the discretized version of the Frixione isolation criterion. An experi-

The complexity of the calculations can be greatly increased including fragmentation contributions of photon production, while the application of appropriate isolation cuts can effectively remove those fragmentation contributions. How to apply these isolation criteria to the theoretical tools and how to include the fragmentation contributions consistently by the theory side is one of the subjects of this proceeding.

Besides the differences between these two isolation criteria, it is possible (*i.e.* it has physical meaning) compare theoretical descriptions obtained using the smooth cone isolation criterion and data taken with the standard criterion, because a cross section obtained using the Frixiene isolation criterion is always a lower bound for a cross section in which the standard criterion was implemented<sup>3</sup>. Furthermore, as we show in the next sections, this bound turns out to be an excellent approximation for the cross section calculated with the standard criterion, with an accuracy of the order of the 1%, if tight cuts are imposed.

Given these results, and the fact that in general it is not possible to exactly match the experimental isolation conditions to the theoretical implementation and viceversa, we propose a pragmatic accord to perform a more precise comparison between the data and the fixed order calculations, that allows to extend the TH computation up to NNLO in some cases<sup>4</sup>.

The following studies concern Monte Carlo integrators (as DIPHOX [7], JetFOx [12] or 2 $\gamma$ NNLO [8], etc) for which the fragmentation component is a purely collinear phenomenon. For Monte Carlo generators (parton–shower Monte Carlo), in which the fragmentation photons are emitted off quarks at non–zero angle during the showering process, we recommend the reference [11].

Beyond the standard and “smooth” cone criteria, another way to define direct photons is the so-called democratic approach” [10]. In this criterion the photons and QCD partons are treated on the same footing when being clustered into jets. Direct photons are then defined by jets containing a photon which carries a large fraction (typically more than 70%) of the jet energy. A detailed study of this approach in the context of matrix element to parton shower merging has been performed recently in [3]. As in the case of the standard cone isolation criterion, the democratic approach also requires the use of

the fragmentation contribution in order to define an infrared safe cross section.

## 2. Isolation Criteria

In this section we present the standard and “smooth” isolation criteria, its advantages and problems concerning the theoretical and experimental implementations.

### 2.1. The standard cone isolation criterion

The standard cone isolation prescription is the criterion used by collider experiments. Schematically it can be described as follows. A photon is said to be isolated if, in a cone of radius  $R$  in rapidity and azimuthal angle around the photon direction, the amount of deposited hadronic transverse energy  $\sum E_T^{had}$  is smaller than some value  $E_{Tmax}$  chosen by the experiment:

$$\sum E_T^{had} \leq E_{Tmax} \quad \text{inside} \quad (y - y_\gamma)^2 + (\phi - \phi_\gamma)^2 \leq R^2. \quad (1)$$

$E_{Tmax}$  can be either a fixed value<sup>5</sup> or a fraction of the transverse momentum of the photon ( $p_T^\gamma \epsilon$ , where typically  $0 < \epsilon \leq 1$ ).

The theoretical (TH) description of the production of a pair of isolated photons is complicated by the occurrence of collinear singularities that appear in the final state, when a photon becomes collinear with a parton. A physical (infrared finite) cross section is only obtained when these singularities are absorbed into the fragmentation functions. As a result, the only quantity theoretically well-defined (if we don’t use the Frixiene criterion) is the sum of the direct and fragmentation contributions. Once these two contributions are included, one can isolate the photon using the cuts of Eq. (1) in an infrared safe way [12].

In addition, a tight isolation cut also has the undesirable effect of making the theoretical prediction unstable [12]. This is due to the restriction of the available phase-space for parton emission. When the size of the cone used is in the limit of the narrow cone ( $R \ll 1$ ,  $R \sim 0.1$ ), earlier studies reveals potential problems. This lead to a collinear sensitivity in the form of a fairly large dependence on  $\ln(1/R)$ , which could make the prediction unreliable<sup>6</sup> unless these logarithms were

mental implementation of the smooth isolation criterion was done by the OPAL collaboration [9].

<sup>3</sup>If we use the same isolation parameters for both criteria

<sup>4</sup>It not only concerns diphoton production at NNLO [8], but also diphoton production with up to three jets at NLO [18, 19, 21].

<sup>5</sup>This requirement was typically used at the Tevatron and was motivated by the fact that most of the energy in the isolation cone results from the underlying events (and pile-up), and so is independent of the photon energy [3].

<sup>6</sup>This could even lead to an unphysical result such as an isolated cross section larger than the inclusive one, thereby violating unitarity.

resummed<sup>7</sup>. In a recent calculation [13] these large logarithmic terms were resummed restoring the reliability of the calculation.

## 2.2. The Frixione or “smooth” isolation criterion

The isolation criterion proposed by Frixione [4] (see also Ref. [14, 15]) represents an alternative to the standard isolation prescription. This criterion modifies Eq. (1) in the following way

$$\sum E_T^{had} \leq E_{Tmax} \chi(r),$$

inside any  $r^2 = (y - y_\gamma)^2 + (\phi - \phi_\gamma)^2 \leq R^2$ , (2)

with a suitable choice for the function  $\chi(r)$ . This function has to vanish smoothly when its argument goes to zero ( $\chi(r) \rightarrow 0$ , if  $r \rightarrow 0$ ), and it has to verify  $0 < \chi(r) < 1$ , if  $0 < r < R$ . One possible choice is

$$\chi(r) = \left( \frac{1 - \cos(r)}{1 - \cos R} \right)^n, \quad (3)$$

where  $n$  is typically chosen as  $n = 1$ . This condition implies that, closer to the photon, less hadronic activity is allowed inside the cone. At  $r = 0$ , when the parton and the photon are exactly collinear, the energy deposited inside the cone is required to be exactly equal to zero, and the fragmentation component (which is a purely collinear phenomenon in perturbative QCD) vanishes completely. Since no region of the phase space is forbidden, the cancellation of soft gluon effects takes place as in ordinary infrared-safe cross sections. This is the advantage of this criterion: it kills all the fragmentation component in an infrared-safe way.

Comparing Eqs. (1) and (2), we can notice also that both criteria coincide at the outer cone ( $r = R$ ,  $\chi(R) = 1$ ), and due to the presence of the  $\chi(r)$  function, which verifies  $0 \leq \chi(r) \leq 1$ , the smooth cone isolation criterion is always more restrictive than the standard one. For the same reason, we expect smaller cross sections when we use the Frixione criterion than when we implement the standard one (for both, theoretically and experimentally), if the same parameters<sup>8</sup> are used in both criteria,

$$\sigma_{Frix}\{R, E_{Tmax}\} \leq \sigma_{Stand}\{R, E_{Tmax}\}. \quad (4)$$

The smooth behaviour of the  $\chi(r)$  function is the main obstacle to implement the Frixione isolation criterion into the experimental situation. First, because of the finite size of the calorimeter cells used to measure the

electromagnetic shower, the smooth cone criterion must be applied only beyond a minimum distance of approximately 0.1 (in  $\{\Delta\eta, \Delta\phi\}$  plane). This allows a contribution from fragmentation in the innermost cone and we have to check to which extent the fragmentation component is still suppressed. In addition, the transverse energy in the experimental isolation cone is deposited in discrete cells of finite size. Thus concerning its experimental implementation, the continuity criterion, initially proposed by Frixione has to be replaced by a discretized version consisting of a finite number of nested cones, together with the collection of corresponding maximal values for the transverse energy allowed inside each of these cones.

Another feature of the smooth cone isolation criterion is that it's free of the problems associated with the narrow cone. In particular a diphoton NLO cross section obtained with the Frixione isolation criterion in this narrow cone limit ( $R \ll 1$ ,  $R \sim 0.1$ ) remains smaller than the inclusive cross section.

## 2.3. Theoretical issues

As it was previously stated, the inclusion of the fragmentation contributions complicates the calculation. And in some cases (as in the diphoton production at NNLO in pQCD) it is not available all the machinery to include it at the desired perturbative level of accuracy.

We can find in the literature theoretical calculations in which the fragmentation component is considered at one perturbative level less than the direct component (e.g. the  $\gamma\gamma$  and  $W/Z\gamma$  production at NLO in pQCD in MCFM [16],  $\gamma\gamma + \text{Jet}$  at NLO in pQCD [17], etc.). This procedure (which is a way to approximate the full consistent result in which the fragmentation component is included at the same perturbative level that the direct component) could introduce inconsistent results in the presence of the standard cone isolation criterion. The following exercise shows how these problems can easily appear.

Let's consider the diphoton production at the LHC ( $\sqrt{s} = 8$  TeV) calculated at NLO. First, we compare the calculation using the fragmentation at NLO with the results in which the fragmentation is considered only at LO (one perturbative order less than the direct NLO component). The acceptance criteria in this case require:  $p_T^{\text{harder}} \geq 40$  GeV and  $p_T^{\text{softer}} \geq 30$  GeV. The rapidity of both photons is restricted to  $|y_\gamma| \leq 2.5$  and  $100 \text{ GeV} < M_{\gamma\gamma} < 160$  GeV. The isolation parameters are set to the values  $n = 1$  (in the case of the Frixione criterion) and  $R = 0.4$ , and the minimum angular separation between the two photons is  $R_{\gamma\gamma} = 0.5$ . The remaining isolation parameter  $E_{Tmax}$  (or  $\epsilon$ ) is varied in or-

<sup>7</sup>As it was done in Ref. [13].

<sup>8</sup>I.e., the same  $(E_{Tmax}, R)$  or  $(\epsilon, R)$ .

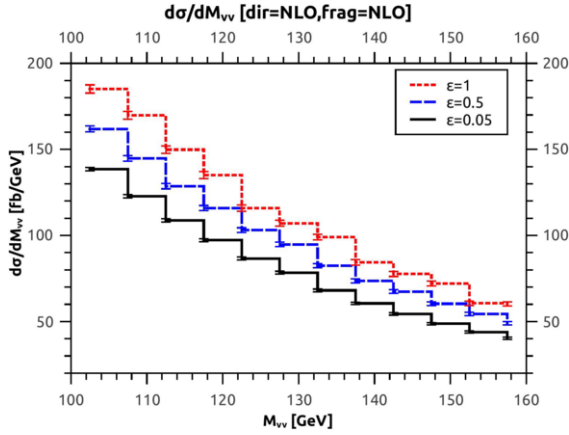


Figure 1: Diphoton cross section as a function of the invariant mass  $M_{\gamma\gamma}$  with NLO fragmentation component.

der to understand the cross section dependence on it. All the cross sections are obtained using the CTEQ6M set of parton distribution functions, We used for this analysis the DIPHOX code that includes the full NLO pQCD description.

The results in Fig. 2 show that we obtain larger cross sections as we impose more severe isolation cuts, which is clearly inconsistent. The same behaviour was reported in Ref. [17] for  $\gamma\gamma + \text{Jet}$  at NLO, and we obtained the same for  $\gamma\gamma$ ,  $W/Z\gamma$  production at NLO in pQCD with MCFM. On the other hand, the Fig. 1 shows the correct behaviour when we consider the full and consistent result in which the fragmentation contribution is at the same perturbative level than the direct component (i.e., NLO in this case). The precedent comparison suggests that one has to be aware that approximating the fragmentation component at one order lower than the direct one can result in unphysical results. The situation can be even more serious when one looks at some extreme kinematical region where the cross section is dominated by higher order contributions.

In Table 1 we present the results for the corresponding cross section with different isolation prescriptions and parameters. The values presented there help to understand the unexpected behaviour in Fig. 2. The analysis contained in this table was made with the cuts used by CMS, in a recent measurement of the production cross section for pairs of isolated photons in pp collisions at  $\sqrt{s} = 7\text{TeV}$  [20]. We require the harder photon to have a transverse momentum  $p_T^{\text{hard}} \geq 40\text{ GeV}$  while for the softer we choose  $p_T^{\text{soft}} \geq 30\text{ GeV}$ . The rapidity of both photons is restricted to  $|y_\gamma| \leq 2.5$ . Finally, we constrain the invariant mass of the diphotons to lie in

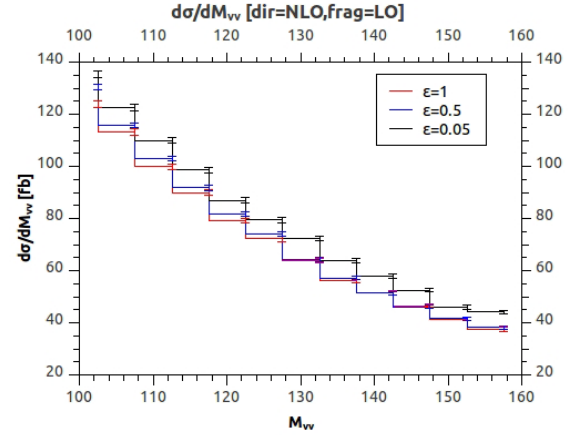


Figure 2: Diphoton cross section as a function of the invariant mass  $M_{\gamma\gamma}$  with LO fragmentation component.

the range  $100\text{ GeV} \leq M_{\gamma\gamma} \leq 160\text{ GeV}$ . And we simulate the CMS calorimeter crack, in the rapidity of both photons, imposing  $1.442 \leq y_\gamma \leq 1.556$ . In the cases in which the standard criterion was applied ( $a - f$  in Table 1) we observe that as the isolation criterion turns out to be “loose”, the direct component becomes smaller and the fragmentation component larger. The sum of them behaves as expected with respect to the isolation parameters, since the increase in the fragmentation component overcompensates the decrease of the direct one. We remind the reader that in the standard isolation the theoretical separation between direct and fragmentation components is not physical and the results presented in this note correspond to the conventional  $\overline{MS}$  subtraction. On the other hand, if only the LO calculation is used for the fragmentation contributions, for which the QCD corrections are quite large (with  $K$ -factors exceeding 2), the mismatch in the perturbative order spoils the compensation between the behaviour of the NLO direct and the LO fragmentation terms resulting in the unphysical behaviour observed in Fig. 2 (as one considers less stringent isolation parameters).

Furthermore, from the cases in which the smooth cone criterion was applied ( $h - m$  in Table 1) we observed that, as expected, the result in the smooth cone case always provides a lower bound for the one obtained with the standard criterion when the same isolation parameters (energy in this case) are used. In the case of smooth cone isolation the (single and double) fragmentation component is identically null. Even more interestingly, and with one single exception, the results for the NLO cross sections computed using different iso-

	Code	$\sum E_T^{had} \leq$	$\sigma_{total}^{NLO}$ (fb)	$\sigma_{dir}^{NLO}$ (fb)	$\sigma_{one}^{NLO}$ (fb)	$\sigma_{two}^{NLO}$ (fb)	Isolation
a	DIPHOS	2 GeV	3756	3514	239	2.6	Standard
b	DIPHOS	3 GeV	3776	3396	374	6	Standard
c	DIPHOS	4 GeV	3796	3296	488	12	Standard
d	DIPHOS	5 GeV	3825	3201	607	17	Standard
e	DIPHOS	$0.05 p_T^\gamma$	3770	3446	320	4	Standard
f	DIPHOS	$0.5 p_T^\gamma$	4474	2144	2104	226	Standard
g	DIPHOS	incl	6584	1186	3930	1468	none
h	2 $\gamma$ NNLO	$0.05 p_T^\gamma \chi(r)$	3768	3768	0	0	Smooth
i	2 $\gamma$ NNLO	$0.5 p_T^\gamma \chi(r)$	4074	4074	0	0	Smooth
j	2 $\gamma$ NNLO	2 GeV $\chi(r)$	3754	3754	0	0	Smooth
k	2 $\gamma$ NNLO	3 GeV $\chi(r)$	3776	3776	0	0	Smooth
l	2 $\gamma$ NNLO	4 GeV $\chi(r)$	3795	3795	0	0	Smooth
m	2 $\gamma$ NNLO	5 GeV $\chi(r)$	3814	3814	0	0	Smooth

Table 1: Cross sections for the  $pp \rightarrow \gamma\gamma + X$  process at the LHC at NLO. All these values are at 1% of statistical accuracy level.

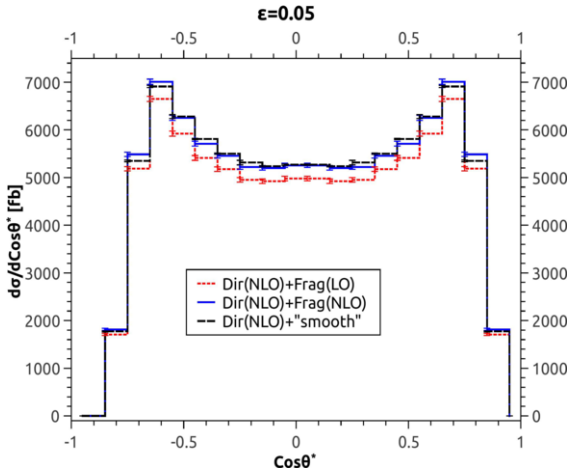


Figure 3: Diphoton cross section as a function of the  $\cos \theta^*$ . A comparison between the different isolation criteria is showed. The applied cuts are the same as in Figs. 1,2, and are described in the text.

lation prescriptions differ by less than 1%. This result indicates that using the smooth cone prescription for a theoretical calculation (even when the data is analyzed using the standard one) provides an approximation that it is far much better than the one consisting in the standard prescription with a lowest order calculation for the fragmentation component. The only case where one can observe larger differences (of the order of 10%) corresponds to the use a very loose isolation, as for  $\sum E_T^{had} \leq 0.5 p_T^\gamma$ , where the fragmentation component in the standard case amounts more than half of the total cross section. In all cases we have studied, the smooth cone provides an excellent approximation to the standard result as long as the isolation parameters are tight enough, i.e.  $\sum E_T^{had} \leq 0.1 p_T^\gamma$  or  $\sum E_T^{had} \leq 5$  GeV for the LHC at 7 TeV. Equivalently, one could define the isolation to be tight enough when the contribution from the fragmentation component does not exceed

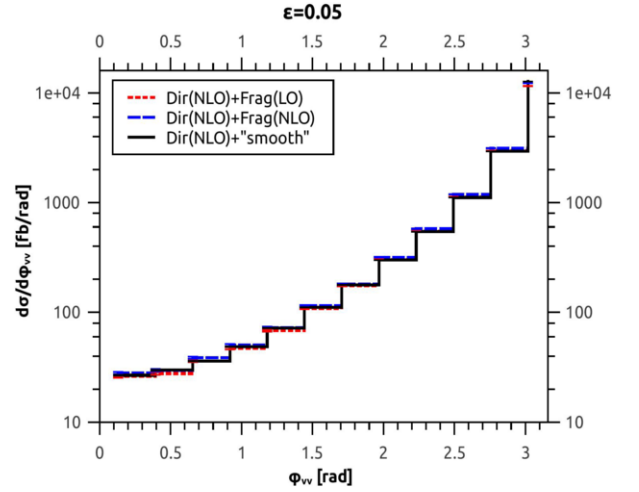


Figure 4: Angular separation between the photons  $\Delta\phi_{\gamma\gamma}$  (Right). A comparison between the different isolation criteria is showed. The applied cuts are the same as in Figs. 1,2, and are described in the text.

$\sim 15 - 20\%$  of the total cross section. While the previous analysis refers only to the fiducial cross section, it is known that the fragmentation contributions could be larger in kinematical regions far away from the back-to-back configuration<sup>9</sup>, and the approximation could in principle become less accurate for those distributions. In order to check that feature, in Figs. 3 and 4 we compare the distributions for the full NLO calculation with the standard prescription, the one obtained using only the LO fragmentation component and the result for the smooth cone with  $\sum E_T^{had} \leq 0.05 p_T^\gamma$  for  $\cos \theta^*$  and  $\Delta\phi_{\gamma\gamma}$  respectively. In both cases we observe that for all the bins the smooth cone provides the best approximation to the full result, always within a 2.5% accuracy. A more detailed analysis is presented in Fig. 5, for the diphoton invariant mass distributions with  $\sum E_T^{had} \leq 4$  GeV. Again, while using the LO fragmentation component fails to reproduce the full NLO result by up to 6%, the smooth cone approximation is always better than 1.5% in the same kinematical region.

In Fig. 4, the discrepancies between the full result (or the smooth approach) with the LO fragmentation approximation<sup>10</sup> are “hidden” in the  $\Delta\phi_{\gamma\gamma}$  distribution, in the bin corresponding to  $\Delta\phi_{\gamma\gamma} = \pi$  (the bin containing

<sup>9</sup>The low mass region in the invariant mass distribution, the low  $\Delta\phi_{\gamma\gamma}$  distribution and the kinematical regions near to  $\cos \theta^* = \pm 1$  belong to this case.

<sup>10</sup>The discrepancies evidently manifest in the invariant mass distribution (see Fig. 5) or in kinematical regions far away from  $\cos \theta^* = \pm 1$  (see Fig. 3).

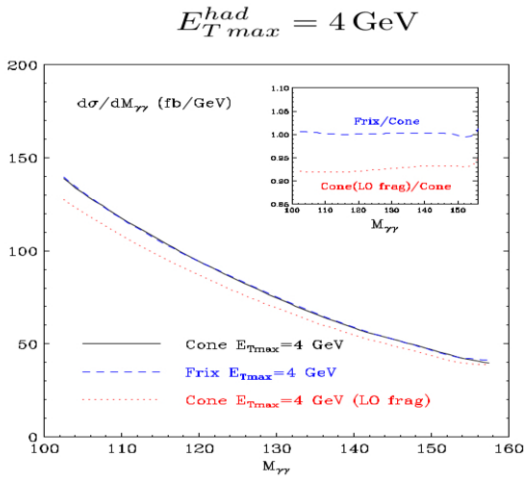


Figure 5: Diphoton cross section as a function of the invariant mass  $M_{\gamma\gamma}$ . Cross sections obtained with the standard cone isolation criterion (with LO and NLO fragmentation contributions) are compared with the cross section obtained with the Frixione criterion using the same isolation parameters.

the back-to-back configurations). Moreover in the low  $\Delta\phi_{\gamma\gamma}$  region we are dealing with events far away from the back-to-back configuration, and the only configuration that survives at NLO (in these kinematical regions) is the real emission at LO which is for the three cases effectively the same contribution under these conditions. If one relaxes the isolation and considers  $\epsilon = 1$  (which is equivalent to  $E_T^{max} > 40$  GeV, allowing for a huge amount of fragmentation contribution), the effects of fragmentation now strongly manifest at low  $\Delta\phi_{\gamma\gamma}$  values (see Fig. 7) and in the invariant mass distribution (Fig. 6). The full result considerably differs from both the LO fragmentation approximation and the smooth cone criterion. Also we can see from Fig. 7, in the bin corresponding to  $\Delta\phi_{\gamma\gamma} = \pi$  (the bin containing the back-to-back configurations) the cross section obtained with the smooth cone criterion provides a better approximation than the LO fragmentation one.

Finally, considering the results presented in the preceding sections, it is clear that exists a set of isolation parameters (given a kinematic configuration) for which the standard and smooth cone isolation criteria are in agreement. Matching experimental conditions to theoretical calculations always implies a certain degree of approximation. Considering the large QCD corrections to processes involving photons (with NNLO essential to understand diphoton data [8]) and the agreement (typically at the % level for the diphoton case studied here) between the standard and smooth cone TH calculations,

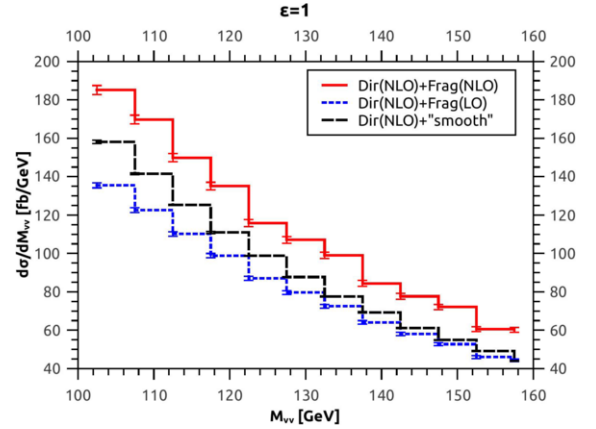


Figure 6: Diphoton cross section as a function of the invariant mass  $M_{\gamma\gamma}$ . Cross sections obtained with the standard cone isolation criterion (with LO and NLO fragmentation contributions) are compared with the cross section obtained with the same isolation parameters.

the use of the later for TH purposes is well justified.

The *Les Houches* or “pragmatic” accord that we proposed in ref. [21] is the corollary of the previous analysis. Pragmatic in the sense that we do not recommend the experiments to implement the smooth cone isolation, but to proceed to the analysis of the data with the usual standard isolation with cuts tight enough if the interesting observable needs to be an isolated cross section or distribution. While the definition of “tight enough” might slightly depends on the particular observable (that can always be checked by a lowest order calculation), our analysis shows that at the LHC isolation parameters as  $E_T^{max} \leq 5$  GeV (or  $\epsilon < 0.1$ ),  $R \sim 0.4$  and  $R_{\gamma\gamma} \sim 0.4$  are safe enough to proceed.

This procedure would allow to extend available NLO calculations to one order higher (NNLO) for a number of observables, since the direct component is always much simpler to evaluate than the fragmentation part, which identically vanishes under the smooth cone isolation. But it not concerns only NNLO calculations. Theoretical calculations for diphoton production in association with up to three jets at NLO [18, 19] also apply the smooth cone isolation prescription, because not all the matrix elements including fragmentation in this case are available.

We also refer to this approach as pragmatic in a numerical sense: we are certain that the smooth cone isolation applied for the TH calculation is NOT the one used in the experimental data, but considering that NNLO corrections are of the order of 50% for diphoton cross sections [8] and a few 100% for some distributions in

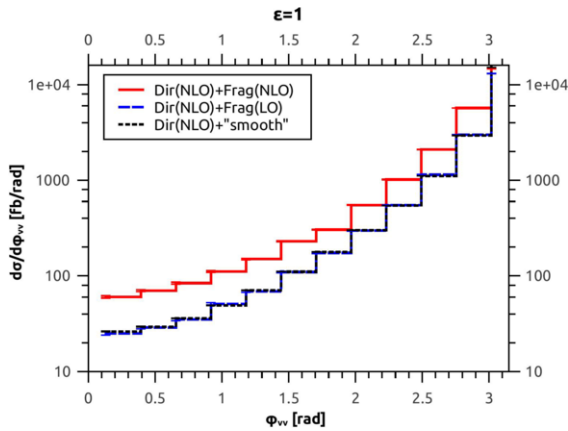


Figure 7: Diphoton cross section as a function of  $\Delta\phi_{\gamma\gamma}$ .

extreme kinematical configurations, it is far better accepting a few % error arising from the isolation (less than the size of the expected NNNLO corrections and within any estimate of TH uncertainties!) than neglecting those huge QCD effects towards some "more pure implementation" of the isolation prescription.

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