A brief history of negation

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A R T I C L E   I N F O

A B S T R A C T

The history of scholarship on negation tracks and illuminates the major developments in the history of metaphysics, philosophy of language, and philosophy of mind, from Parmenides, Plato and Aristotle through Frege, Russell, and Wittgenstein to contemporary formal theorists. Our perspective focuses on the catalytic role played by the 20th century philosopher of language Paul Grice, whose views on negation serve as a fulcrum for his attempt to bridge the (neo-)Traditionalist and Formalist traditions in logical thought. Grice's remarks on negation and speaker meaning and the elaboration of his ideas by subsequent neo-Griceans are summarized and situated within a broader picture of the role of contradictory and contrary negation in the frameworks of Aristotelians, Medievals, early modern schoolmaster-logicians, 19th and early 20th century neo-Idealists and Formalists, Oxford ordinary-language analytics, practitioners of classical and non-classical logics, and a range of other philosophers and linguists. Particular attention is paid to the relations between negation and the other operators of propositional and predicate calculus. Implications for accounts of the semantics and pragmatics of natural language are also pursued and extensive references to related work are provided.

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> "I wyl not deny my Greecian ofspring."
> Stanyhhurst, Æneis II. (Arb.), 1583, p. 46.

1. Introduction: Grice as a catalyst

The American Heritage Dictionary's entry for Grice identifies as a 'British logician', which for the purposes of this contribution is what he was. (The entry goes on to acknowledge that he is "best known for his studies of the pragmatics of communication and his theory of conversational maxims".) We shall take Grice as a catalyst, since he represents a breakthrough in a rivalry between two groups of philosophers in the history of logic. We hope to demonstrate that he was more of a logician than the history of logic typically recognizes.

In choosing Grice as a catalyst and foundation stone, we open with a discussion of Formalism (or Modernism). This we present as giving a "System" for the logic of negation – notably with a syntactic and a semantic component. In the second part, we briefly discuss Neo-Traditionalism (or Informalism) which Grice saw as presenting a challenge to Formalism. We propose, with Grice, that most of the observations made by the Informalists pertain to the pragmatic component of the System, and characterize pragmatic rather than logical inference.

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We will try to show that our choice of Grice as a catalyst is general enough to provide a basis for the History of Logic and the treatment of one of its central concepts. We center on the ideas of Grice as an example of a logical treatment of negation, but also as a memorial to a specific chapter in logical historiography. Our focus will be one particular logical feature of negation as it has been conceived in the history of logic as a 'unary' truth-functor. We shall now formulate the Modernist-Neotraditionalist debate.

We take as starting point Grice's opening passage in his epoch-making 'Logic and Conversation' (the second William James lecture), where negation is first cited:

> It is a commonplace of philosophical logic that there are, or appear to be, divergences in meaning between, on the one hand, at least some of what I shall call the formal devices – $\sim$, &, v, $\supset$, $(\forall x)$, $(\exists x)$, (ιx) (when these are given a standard two-valued interpretation) – and, on the other, what are taken to be their analogues or counterparts in natural language – such expressions as 'not', 'and', 'or', 'if', 'all', 'some' (or 'at least one'), 'the'.   (Grice [32, p. 22])

A close reading of the passage will shed light on much of the time-honored debate in the history of logic. Note for example that the contrast Grice makes is between a 'formal device' ("$\sim$") and its vulgar counterpart ("not").

The underlying assumption – which Grice states in the second passage is shared by Modernism, Neo-Traditionalism, and Grice's own Post-Modernism alike – seems to be that there *is* a formal counterpart (in our case, "$\sim$", the squiggle) to a vernacular expression ("not"). This assumption is notably not challenged by Grice's Post-Modernism. He may be a skillful heretic, but his heresy was still of the conservative kind, and he 'can always be relied to upon to rally to the defense of an 'under-dogma'' [32, p. 297]. In this case, the 'under-dogma' is the doctrine concerning the identity between "$\sim$" and "not" ("Some logicians may at some time have wanted to claim that there are in fact no such divergences; but such claims, if made at all, have been somewhat rashly made, and those suspected of making them have been subjected to some pretty rough handling").

What Grice then takes as a commonplace incorporates the idea that "$\sim$" 'formalizes' "not", or Latin *non* (for the Scholastics), or Greek ου. Interestingly, English 'not' shows a complexity that seems absent in the simplicity of "$\sim$". Old English had *na*, and Middle and Modern English 'not' incorporates Old English *na* plus the emphatic *with* (cf. Jespersen [58], Horn [47]). English was not, however, the language of the schools, and observations by other scholars of the highest intelligence, such as Ælfric, are hardly credited in the history of logic. "Sume [propositions] syndan *abnegativa*, thaet synd, withsacendlice, mid tham we withsacath", as in "Ic *ne* dyde").

As for the squiggle ("$\sim$"), the contention behind its use seems to be the old Pythagorean idea that negation parallels subtraction, whence the minus sign ("$\sim$" being a variant of "-"). The correspondence between "$\sim$" and "not" poses problems of a categorical type. In grammar and scholastic logic, the received opinion seems to have switched from the idea that 'not' is an adverb (*adverbium negandi* [quod] *denotat negationem*, as Christopher Cooper has it in his *Grammatica Linguae Anglicanae* (1685)) and a *syncategorematon*. A good compromise appears in Robert Bacon's *Sumule dialecticis* with his talk of *"syncategorematon adverbialium"*:

> *Non* est adverbium quod prius et principaliter determinat ipsum, erit primum adverbium: tale autem est hec dictio *non*. Per modum affectus significatur per han dictionem *non* qualiter autem hoc sic intelligendum, videtur sic: cum anima accipit duo incomplexa disconvenientia, ut *hominem* et *asinum*, afficitur quadam dissensione, et huic dissensione, que est intra, respondet hec dictio *non* in sermone extra. Unde illius dissensions que afficit animam nota est hec dictio *non*.

In Modernist Logic, on the other hand, "$\sim$" seems to be the only unary truth-functor worth discussing.

A further observation on Grice's use of "$\sim$" as a 'formal device': The idea seems to be not only that "$\sim$" will do duty for "not", which is already a complex assumption. In a narrow interpretation, "$\sim$" is a formal device if (and only if) it can be expressed in the logical form of a natural language counterpart containing "not". On this view, 'formal' would apply, strictly, to a 'formalized' calculus. On a broad view, however, 'formal device' may refer more to logical 'form' understood as an abstraction which may be the result of 'formalization'. ("Not" is still a 'formal device' in, say, Winston Churchill's speeches, even if they never get 'formalized' into first-order predicate logic.)

As it happens, while Grice goes on to invoke the debate between the rival groups of the Modernists and the Neo-Traditionalists, he is mainly interested in the latter (as represented by Strawson) and their caveats on 'reading' "$\sim$" as 'not'. For one thing, Strawson had urged a more faithful reading of "$\sim$" as "it is not the case that . . .", reflecting the sentence-initial external maximal scope position that the sentential operator "$\sim$" necessarily takes in propositional logic ("The identification of "$\sim$" with 'it is not the case' is to be preferred to its identification with 'not' simpliciter" [96, p. 79]). Grice ignores this and translates "$\sim$" merely as "not". (As a note of interest, Grice does list both 'some' and 'at least one' as the 'translations' of 'Ex' in the above-quoted passage. But also note that while "at least one" and "some" do not include each other, "not" is incorporated in the longer "it is not the case".)

In any case, Grice found the commonplace to be wrong, and resulting from a blurring (committed by Modernists and Neo-Traditionalists alike) of logical and pragmatic inference. (Here we rely on Grice's unpublished lectures on Negation; cf. Chapman [13, p. 87].)

Grice refers to Modernism and Neo-Traditionalism as "two rival groups . . . which I shall call the formalist and the in-formalist groups" [32, p. 22]. Grice alludes to "the Modernists, spearheaded by Russell" [32, p. 372], and it is an interesting

project to investigate to what extent works like *Principia Mathematica* commit the mistake that Grice attributes to Modernism. In any case, it seems obvious that Grice was parochially interested in the effect that tenets of Modernism had received in the rather conservative atmosphere of the Oxford of his time, especially as the target of attack by Strawson. (Strawson's *Introduction to Logical Theory* had been welcomed by ordinary-language logicians (e.g. Warnock and Urmson) as providing a more faithful characterization of the 'logical behaviour' of certain English expressions.)

Indeed, while Grice qualifies a position such as Strawson's as 'Neo-Traditionalist' (and not just 'Traditionalist') the same could hold for 'Modernism'. What Russell spearheaded was indeed a Neo-Modernism historically, if we recall that it was authors like Ockham in Oxford who had fought for a *logica moderna* to oppose to the traditionalist *logica vetusta* of his predecessors. But Ockham was perhaps not modern enough. The venerable inceptor whetting his razor (prefiguring Grice's Modified Ockham' Razor here in the form of a dictum "Do not multiply senses of 'not' beyond necessity") had considered whether a proposition featuring 'non' (a *terminus syncategorematicus*) was atomic or molecular, and concluded that the negation of a categorical proposition was still categorical. Thus, in a passage apparently flouting his own razor, he distinguished between propositional negation and term negation. The distinction, he thought, surfaced in cases of 'privation' (Aristotle's ονωμα αριστον): "Every S is non P" was rendered as "Every S is of a kind K that is naturally P, and no S is P".

A more neutral label for Modernism would be Classicism (as when we speak of first-order predicate logic as 'classical' [30, p. 67]). The issue arises as to what really is "(Neo-) Traditionalist" about Strawson's *Introduction to Logical Theory*. It cannot be Aristotelianism, when Strawson [95, p. 344] is eager to grant that neither Modernist ('Russellian') nor 'Aristotelian' logic faithfully represents the logic of ordinary language, which "has no exact logic". Rather, it seems that Aristotle was rather a pre- or proto-Gricean (see Horn [44] on "Greek Grice"). The central source is *De Interpretatione*, which divides indicative-mode declarative sentences into assertion and denial (negation, απόφασις, from αποφανειν 'deny, say no'), which respectively affirm or deny something about something [17a25]. As Grice observes, this division of indicative-mode sentences into affirmative and negative "may suggest that the notion of the exhibition of a subject-predicate form enters into the definition of the very concept of a [indicative-mode] declarative sentence or proposition" ([31, p. 178]; cf. *Cat.* 11b17).

And what are we to do with an author such as Cook Wilson [102], situated somewhat between the Modernists and the Neo-Traditionalists, who wonders if negation "is a different *species*" from affirmation, "or whether the latter is in some sense the form of all statements, and again whether the negative symbol belongs to the so-called copula". Wilson's target of attack is the 'Modernist' Mill, who listed judgments as being affirmative or negative, "without troubling to find the genus of which they are species – the elementary fallacy of defining by enumeration of species instead of a statement of the genus". Wilson's own view is that 'affirmative' and 'negative' "are not co-ordinate in the strict sense of the term" [102, p. 264] – the elementary fallacy of defining a thing by what it is not. Symmetricalism is best represented by Ralph Lever when attempting to replace the logician's talk of 'negation' by 'naysay': "every simple shewasay is eyther a yeaysay or a naysay" (*Arte of Reason*, [64]). (Cf. Horn [47, Chapter 1] for a comprehensive chronicle of the "(A)symmetricalist Wars".)

2. The background: Negation and opposition in Aristotelian logic

The genus of opposition (*apophasis*), as introduced in Aristotle's *Categories* (11b17), is divided into four distinct species: CONTRARIETY (between two CONTRARIES), e.g. *good* vs. *bad*, CONTRADICTION or αντίφασις (AFFIRMATIVE to NEGATIVE), e.g. *He sits* vs. *He does not sit*, CORRELATION (between two RELATIVES), e.g. *double* vs. *half*, and PRIVATION (PRIVATIVE to POSITIVE), e.g. *blind* vs. *sighted*. Aristotle proceeds to offer detailed diagnostics for distinguishing "the various senses in which the term 'opposite' is used" (11b16–14a25). Crucially, contradictory opposites (*All pleasure is good*, *Some pleasure is not good*) are mutually exhaustive as well as mutually exclusive, while contrary opposites (*All pleasure is good*, *No pleasure is good*) do not mutually exhaust their domain. Contraries cannot be simultaneously true, though they may be simultaneously false. Members of a contradictory pair cannot be true OR false simultaneously; contradictories "divide the true and the false between them". So too, contradictory terms (*black/non-black*, *odd/even*, *male/female*) exclude any middle term, an entity satisfying the range of the two opposed terms but falling under neither of them, a shirt which is neither black nor not-black, an integer which is neither odd nor even. Contrary terms, by definition, admit a middle: my shirt may be neither black nor white (but gray), my friend neither happy nor sad (but just blaah). (See Horn [52] for an overview of contradictory opposition and its relatives.)

Privatives and positives always apply to the same subject and are defined in terms of the presence or absence of a default property for that subject:

We say that which is capable of some particular faculty or possession has suffered privation when the faculty or possession in question is in no way present in that in which, and at the time in which, it should be naturally present. We do not call that toothless which has not teeth, or that blind which has not sight, but rather that which has not teeth or sight at the time when by nature it should.

(*Categories* 12a28-33)

On this understanding, a newborn kitten is no more blind than is a chair, and a baby is not toothless. In later work, privation is taken to be a subcase of contrariety.

One more species of opposition is worth mentioning. Aristotle's early commentators Apuleius and Boethius, in structuring the Aristotelian system in the form of a Square of Opposition (see Parsons [76]), define an additional relation of subcontraries, so called because they are located under the contraries in the geometry of the Square:
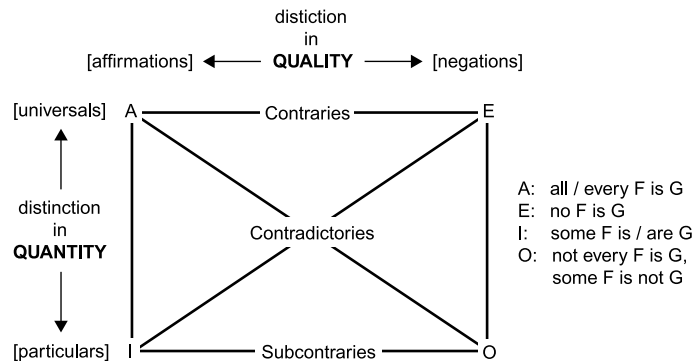
As the contradictories of the two contraries, the subcontraries (e.g. *Some pleasure is good*, *Some pleasure is not good*) can both be true, but cannot both be false. For Aristotle, this was therefore not a true opposition, since subcontraries are "merely verbally opposed" (*Prior Analytics* 63b21-30).

The Aristotelian categories of opposition held sway through generations of logic handbooks. Here, for example, is Edward Bentham [6, pp. 40–41]:

> An universal affirmative and an universal proposition are termed *Contrary*; They may be both false, but can not be both true. A particular affirmative and particular negative are termed *Subcontrary*; They may be both of them true, but cannot in any instance be both of them false. An universal affirmative, and particular negative, as also an universal negative and particular affirmative are termed *Contradictory*. They can in no instance be both of them true, nor both of them false. The difference between *contrary* and *contradictory* propositions should be the more carefully observed, as it is common enough to find the two contending parties in a dispute to be both of them mistaken, while they maintain *contrary* positions; which may be both of them false. So likewise as to subcontrary propositions. Men expressing themselves indefinitely sometimes grow angry with each other, supposing that their assertions are inconsistent; when if rightly explained, they may be both of them found to be very true.

In this connection, Bentham provides *Some faith does justify* and *Some faith does not justify* as an example of what ought to be non-fighting words.

3. Modernists and Neo-Traditionalists in post-Gricean hindsight

It should be easier to catalog logicians as either Modernist or Neo-Traditionalists post-'Logic and Conversation'. A few instances show that this is not so easy. Thus, "'∼' "is pronounced 'not'". Hodges [39, p. 92] puts it: "Given any proposition *p*, one can form from it another proposition which is its negation. In English this is usually done by inserting *not* in some appropriate place in the sentence expressing it, though ambiguity is better avoided by tediously writing out *It is not the case that* at the front of the sentence" (cf. Bostock [8, p. 17]). He goes on to pose the following exercise: "Discuss that ∼ and 'not' mean the same" (Bostock [8, p. 20]). More emphasis comes from Cambridge: "By all means, read "∼" as "not", but remember that it shouldn't be thought as a mere abbreviation of its ordinary-language reading. For a mere abbreviation would inherit the ambiguity of the original, and then what would be the point of introducing "∼"? "∼" is best thought of as a cleaned-up replacement for the vernacular *not*, which unambiguously expresses its original core meaning" (P. Smith, *Logic* (2003) 59).

What Grice saw as the little war between Neo-Traditionalism and Modernism seems to be alive and well. Oxford has now two different chairs for the two kinds of logic: the Wykeham chair of logic (New College, Faculty of Arts) and the chair of "Mathematical Logic" (Merton, and The Mathematical Institute at St. Giles). Other universities have adopted other ways of dissociating the two groups, in a latter-day version of C.P. Snow's nightmare of the 'two cultures'.

4. Modernism

It was said of Grice that he could always be relied upon to rally to the defense of an underdogma. Modernism was one such underdogma in Oxford [32, p. 297]. Modernism claims that the logic of *not* worth preserving is that expressed by '∼'. Any implicature should be seen as a 'metaphysical excrescence' [32, p. 23]. What we propose is to outline what Modernism

offers as the logical behaviour of 'not' (and '∼'): the syntactic and semantic components of a system that will define the class of valid inferences featuring negation.

It makes sense to start the discussion with Modernism not just because it is the first of the rival groups that Grice mentions. It shows that Strawson wasn't really refuting Modernism by bringing attention to the divergence between the formal device and the vulgar counterpart. (As Grice notes, that there is a divergence is a commonplace shared by both Modernism and Neo-Traditionalism.) Second, in adding a pragmatic apparatus to the system in the form of conversational implicature (as well as in proposing a bracketing device to provide a "conventional regimentation" of a pragmatic distinction, a point to which we return below; cf. Grice [32, Chapters 4 and 17]), Grice is ultimately seeking a defense of Modernism.

Curiously, the strawperson here is not Strawson but Quine. Quine had sat in on the seminars on logical form given by Grice and Strawson. Grice indeed presents his System Q as a tribute to Quine. What this System does is incorporate the constraints that the formal device "∼" is supposed to have. Logic may have all started with Aristotle, but many argue that we wouldn't be studying it now if it were not for its 'modern' developments. The term 'modern', as applied to logic, is of course regularly recycled – it was used to refer to the logic of William of Ockham, for example. In the more recent understanding, it applies to classical two-valued systems as found in Whitehead/Russell's *Principia Mathematica*; it is this approach that Grice classifies as 'Modernism' in his 'Retrospective Epilogue' [32, p. 372]. In the vademecum of classical logic, Whitehead and Russell famously introduce the "∼" operator as the 'contradictory function', to be read as having always maximal sentential scope, in the tradition of the Stoics and Frege. "The contradictory function with argument p, where p is any proposition, is the proposition which is the contradictory of p, that is, the proposition asserting that p is not true. This is denoted by '∼p'. Thus '∼p' is the contradictory function with 'p' as argument and means the negation of the proposition p. It will also be referred to as the proposition 'not-p'. Thus '∼p' means 'not-p,' which means the negation of 'p'" [99, p. 6].

If Grice is right, Modernism must find some 'divergence' between 'not' and "∼". It's not easy to identify one particular feature that may count as an example of this 'divergence' (but cf. Cohen [14]). In any case, the Modernist approach, as viewed by Grice, would consist of dealing with any such divergence as a 'metaphysical excrescence' of "not" – from which "∼" is by definition detached. Russell's particular interest was the role of "∼" in association with a definite description, which he took to yield an essentially scopal ambiguity, with "The king of France is not bald" corresponding either to '∃$x(Kx$ & ∀$y(Ky → y = x)$ & ∼$Bx)$' or to '∼∃$x(Kx$ & ∀$y(Ky → y = x)$ & $Bx)$', of which only the latter is a contradictory of "The king of France is bald".

Boole uses "-" for contradictory negation, alluding to arithmetical subtraction. "Whatever class of objects is represented by the symbol x, the contrary class will be expressed by '$1 − x$'" ("man, *not*-man"). Boolean negation is classical negation. However, it has been argued that Boole's is Modernism gone wrong, arguably leading to logically uninterpretable expressions in the course of calculating logical equations according to the model of arithmetic.

The Modernist program then is to provide a standard for the class of valid inferences concerning a formal device. This is the idea of a system, such as first-order predicate calculus. The range of valid inferences will be defined by the syntactic and semantic components. We shall proceed accordingly. (With the addition of a pragmatic component, the system can be integrated into a complete 'semiosis'.)

5. Semiosis for negation

The epitome of a first-oder formal predicate calculus (as proposed by the Formalists) is something like System G (System G_{HP}). (We take the idea from George Myro [73] and offer this 'highly powerful'/'hopefully plausible' version of his system G.) What a system like System G does is to provide a syntactic and semantic component for various formal devices, including "∼".

6. Syntax for negation

By syntax is understood both the 'formation' rule and the 'inference' rules (introduction and elimination) before they get a semantic interpretation. (Grice's own system relies heavily on Mates's *Elementary Logic*.) A syntax-sensitive formation rule for "∼" indicates the order in which the formal device is introduced with respect to the pre-radical. This will be of use in explaining away the alleged divergence between "∼" and "not" in terms of implicature and 'scopal ambiguity'. Grice [28, p. 126] mentions similar work on these scope indicators by Charles D. Parsons and George Boolos:

If $\phi_{[n]}$ is a formula, $\sim_{n+m} \varphi$ is a formula.

The apparently simple formation rule already seems to commit the system to a purely sentential account of negation. It suggests that by default negation will be assigned maximal scope. (Non-maximal scope would yield a subscript other than "$n + m$".)

What the formation rule does is transform a negation-free 'radical' into a radical containing negation. At this point Grice plays with algebraic and chemico-physical terminology. The "∼p" formula is construed as 'radical' (in fact symbolized as "$\sqrt{}\sim p$"; [34, p. 59]). Grice's analogy here is to chemistry, where a radical is "an atom or group of atoms regarded as a (or the) primary constituent of a compound or class of compounds, remaining unaltered during chemical reactions in which other constituents are added or removed... An individual atom or element as a constituent of a compound was formerly

termed a simple radical, as distinct from a group or compound radical" (OED). Keeping the chemical analogy, we may refer to "∼" as a *negaton*. As such, "∼", while having maximal scope within the formula, remains internal to any 'illocutionary force' operator that can be appended at a later stage ("⊢ $\sqrt{}$∼p", "!$\sqrt{}$∼p"). Again, having this clearly in mind at the 'formation-rule' stage allows us to explain some tricky interfaces between mode-operators and negation as being ultimately implicatural.

For visual clarification Grice uses here the algebraic radical sign ($\sqrt{}$) "⊢ $\sqrt{}$∼p". In Myro's terminology, "p" is the *negatum* of "∼p" and vice versa. The word had been used before to indicate one of the *modi* of the proposition. "Now affirmatum and negatum, verum, . . . are . . . words of art, for indeed they belong to logic. They call these modals, because the modus is the genus" (Richardson, *The Logician's Schoolmaster*, 1629, p. 261). With relation to "⊢ $\sqrt{}$∼p", Grice follows Moravcsik in referring to ⊢ as the mode (*modus*). The modus has been interpreted by some earlier logicians as involving "∼": "The former is termed the dictum; the latter the modus . . . And in general, modal propositions are affirmative or *negative*, according as the modus is affirmed, or denied of the dictum" [6, p. 45].

While the root sign obviously arises ultimately from algebra, the direct source is the analogy with the radical in chemistry, since it appears in more complex structures. Note that the Modernist 'formation rule' has "∼" as an external affix to a propositional complex. This is very much in the Stoic and Fregean (rather than the Aristotelian, or later the Montagovian) tradition; cf. Horn [47, Chapter 7].

The negated radical "∼p" is the ἀπόφασις, of the Stoics ("Not: it is day"), which they carefully distinguished from statements like "No one is walking" (ἀρνησις), and "This man is unkind" (στηρερις, *privatio*). (See Mates [67], Horn [47, §1.1.2] for more on the Stoic negation.) It is a good exercise to find the negatum of these other types of utterance to see whether they all reduce to a formula containing "∼".

The formation rule also leans towards Asymmetrism. A formula "φ", which does not contain negation, is the base for a new formula which does. "φ" is far from being a full-fledged 'affirmation'. That is, precisely, the whole point of invoking the notion of 'radical' instead. But the idea is there. In this, System G is traditional; cf. Coke [15]: "The affirmation is before, and more worthy then [sic] the negation. Denying or negative is which divideth the consequent subject from the antecedent predicate, as, "Good works do not justifie", "A man is not a stone". That a proposition may be a negative, it is necessary that the particle of denying be either set before the whole proposition: as "No elect are damned", or be immediately added to the coupler [sic], and verb adjective that hath the proce of the coupler or band; as: "Marriage is not a sacrament", "Works justifie not" ".

The formation rule presupposes that "∼" applies to a constituent, to form a 'radix', which is still 'mode-less', neither assertoric nor non-assertoric. Furthermore, there is no allusion in the formation rule to anything like the copula. The same formation rule of propositional calculus is used in the formation rule of predicate calculus. Modernism aims at simplifying some of the sempiternal worries of the Traditionalists. Thus, Hobbes famously held that 'not' attaches to the predicate. (For a view of Hobbes as a proto-Gricean, in the analysis of propositions like "That man is *not* a stone" in terms of utterer's intentions, see Speranza [91], following Hacking.)

When applying the mode operator to the radical (as in "⊢ $\sqrt{}$∼p" and "!$\sqrt{}$∼p"), Modernism also simplifies an account such as Frege's, for whom "⊢" is a complex sign, with "|" and "—" representing judgement and content respectively. For that matter, no such complexity is preserved either in systems with mode-operators like "⊣" that some have proposed for the sign of 'illocutionary denegation' – as in " ⊣ p" (as in "I deny that it's raining") or "¡p" for prohibition (as in "No parking"); cf. Searle [88] and related work, and Horn [47] for a critique of the "speech act" of negation.

But 'deny' is a mere 'expositive' used "naturally, but no longer necessarily, to refer to conversational interchange" [3, p. 162]. In Grice's terminology, 'deny' would constitute a 'central', rather than 'peripheral', speech act; indeed, so central that it's part of the radical (Austin's phatic or even rhetic: cf. Grice [32, p. 122]).

In a system like G, the interface between these mode-operators and their negated radicals must be accounted for implicaturally: "⊢ $\sqrt{}$∼$(p \ \& \ q)$", "⊢ $\sqrt{}$∼$(p \vee q)$", and "⊢ $\sqrt{}$∼$(p \rightarrow q)$". Their 'neustic' equivalents, "$\sqrt{}$ ⊣ $(p \ \& \ q)$", "$\sqrt{}$ ⊣ $(p \vee q)$", and "∼$(\sqrt{}$ ⊣ $(p \rightarrow q))$" are either ill-formed or misleadingly incorporate what is part of the implicatum into logical form. "I deny that she had a child and got married: she got married and had a child", etc.

The idea of a phrastic ϕ as a *negatum* in a more complex formula "∼φ" can be traced back at least to Richardson [85, p. 261]: "Now *negatum* is a word of art, for indeed it belongs to Logic". Cf. "A proposition is negative when the modus is denied of the dictum" [6, p. 45]. Grice's anatomy of negation as a sub-atomic particle thus has its logical historiography. As Bentham subtly expresses it, a proposition is negative not when the dictum is denied, but rather when "the modus is denied of the dictum" [6, p. 45]. (As an exercise, the reader may attempt to deny the dictum without denying the modus). Interestingly, the dictum is re-introduced in Oxonian parlance by R.M. Hare (cited by Grice, [30, p. 50] as a member of the Play-Group) to refer to the phrastic or radical. In other logical treatises, we find, as we do in Richardson's *Logician's School-Master*, negation characterized as a "mode" (*modus*) of a proposition, affecting its quality. The contrast is between the *modus* – what Grice will have as the mode-sign, and the early Hare will have as the 'dictor' – and the dictum, which Grice will have as the radical (the *negatum*). Keeping the chemical analogy, we may refer to "∼" as the *negatron*.

It is not transparent how the radix/modus distinction should apply in contexts like, 'The king of France is not bald'. Hare maintains that "not", as represented by "∼", is part of the radical. Most Formalists would reject as nonsense a formula such as "∼⊢ $\sqrt{}p$". Negation, rather, is 'part of what is asserted'; "that is, that we *assert* either that the cat is on the mat, or that the cat is not on the mat" (Hare [36, p. 25]; cf. Frege [21] for a precursor of this view). Grice's 'radical' connects with Hare's views. Hare had used dictum, dictor and dictive for what he later refers to as the 'phrastic' vs. the 'neustic' ("⊢ $\sqrt{}$∼$(\iota x)(Kx \ \& \ Bx)$").

A phrase containing an illocutionary verb in the first person present indicative active, such as "I *deny* that..." or "I *assert* that..." can be used as an illocutionary-force indicator, and can be negated. However, in a system like G, it cannot be negated *and* used as an illocutionary force indicator.

Formation rules for negation are bound to become more complicated in the predicate calculus by incorporating the copula. For Locke, *is not* disconnects the subject from the predicate, and is "the general mark of the mind, denying" (cf. Duncan [20]). By 'annexing the negative particle *not*' to the copula *is*, the mind "*disjoins* the subject idea from the predicate idea according to the result of its [the mind's] perceptions". The negative particle *not* is inserted after the copula, to signify the disagreement between the subject and the predicate. (Similar ideas date back to Peter of Spain [78,80,79]; Kretzmann [63]; O'Donnell [75].)

Note that the formation rule of System G gives the squiggle maximal scope, and the issue of whether *not* attaches to the 'copula' is not even raised. But the issue was raised by Duncan [20], whose solution involves the idea of a negative pregnant, complete with a proto-theory of implicature cancellation:

> Perhaps it may still appear a mystery how the copula can be said to be part of a negative proposition, whose proper business it is to disjoin ideas. This difficulty however will vanish, if we call to mind that a negation *implies* an affirmation. Affirmations are of two kinds, viz. of agreement or disagreement. Where perceptions disagree, there we must call in the negative particle *not*, and this gives us to understand, that the affirmation *implied* in the copula is not of any connection between the subject and predicate, but of their mutual opposition and repugnance.

JeLogicians working before the advent of Formalism seemed to have difficulties in deciding just what it is that *not* attaches to. Locke terms *not* a particle (*syncategorematon*), rather than an 'adverb'. Again, it's Duncan [20] who adds the proto-Gricean flavor of psychological intentions. A negation is the disjoining of the subject idea and the predicate idea ("The law is not an ass"): "But as this is the very reverse of *what is intended*, a negative mark is added, to shew that this union does not here take place".

The formation rule for "∼", as it stands, should be qualified for sub-clausal structures (e.g. "The cat which is not black is not on the mat"). Duncan [20] considers the entailments due to the embedding of *not*. His example: 'The man who departs not from an upright behaviour, is beloved by God". The predicate, *beloved of God*, is evidently affirmed of the subject, so that notwithstanding the negative particle in the subordinate, the proposition is still affirmative.

Logicians working with the idea of the copula explain the canonical use of 'not' as an element of 'repugnance' attaching to the copula or a functionally equivalent predicate. Thus Coke [15, p. 107]:

> That a Proposition may be a Negative, it is necessary that the Particle of denying be either set before the whole Proposition, as, No Elect are damned; or be immediately added to the Coupler [sic], and Verb adjective [sic] that hath the force of the Coupler, or Band, as, Marriage is not a sacrament; Works justifie not. Every true Negation, hangs on a true Affirmation: For it could not rightly be said, *Works justifie not* unless it were true, that Faith onely justifieth.

The second ingredient that System G incorporates in the syntax for *not* is the doublet of introduction and elimination rules. Since Gentzen, it has been the received view of Formalism and Modernism that these are the only two rules or procedures constraining the inferential behaviour of 'not'. These are syntactic procedures in that they precede a full-fledged model interpretation (e.g. via a truth-table for the propositional calculus). Any such pair of introduction and elimination rules should be in principle be harmonious and stable.

7. Reductio ad absurdum

System G standardly adopts *reductio ad absurdum* as the introduction of negation:

If $\varphi^1_{[m]}, \varphi^2, \ldots, \varphi^k \vdash \psi_{[n-l]} \,\&_n \sim_{[n-k]} \psi_{[n-k-l]}$, then $\varphi^2, \ldots \varphi^k \vdash \sim_{n+l} \varphi'$.
(Grice [28, p. 126]; cf. Myro [73, p. 89])

Reductio ad absurdum can be traced back to the Eleatics, particularly Zeno and his *epicheirema*. Aristotle has this as η εις το αδυνατον απαγογη – *reductio ad impossibilem*, rather than *ad absurdum* – and it was first used in (Latinized) English by Isaac Watts: "reducing [the respondent] to an absurdity ... is called reductio ad absurdum". (*The Improvement of Mind*, 1741.)

8. Duplex Negatio Affirmat

System G$_{HP}$ also adopts the standard rule for the elimination of "∼":

$\sim_{n+k} \sim_n \phi_{[n-m]} \vdash \varphi$.
(Grice [28, p. 126], cf. Myro [73, p. 62])

The source here is υπεραπόφασις, as used by the Stoics (υπεραφότικον [εστιν] αποφάτικοναποφάτικου (*Stoic* 2.66)), rather than Proclus.

The introduction and the elimination rules should provide for a harmonious system. Griss [35] has argued that mathematical discourse can do without negation. In other words, negation in mathematical discourse is either introduced in a stronger way than by *reductio ad absurdum* or it cannot simply eliminated via the law of double negation. Faced with what is argued to be a system both unharmonious and unstable, Griss opts for a 'negationless' system.

Dummett [19, p. 291] has explored the entailments of a system like G that includes RAA and DNE as introduction and elimination of negation: "Plainly, the classical rule [of elimination, DNE] is not in harmony with the introduction rule". So there is a requirement of harmony that the classical logician thinks she fulfills and the intuitionist logician thinks she does not. Dummett relates the harmony requirement with the stability requirement. "The negation-elimination rule, ... validates negation introduction, which, however, fails to validate negation-elimination. This was a situation we did not envisage when we discussed stability" [19, p. 293]. In view of the problems brought about by negation, some intuitionist logicians, such as Griss, speak of 'negationless' systems. We are treating together different accounts of 'anti-realist' negation, taking Griss as the paradigm. In contrast, Grice and system G is realist. "Since different notions of incompatibility are being used [by Griss and Grice], there is no sound objection to the claim that the semantic value of classical negation is determined" [77, p. 165].

9. Semantics for negation

The second component of a formal system for negation is the semantic one. The constraints that system G so far has incorporated are syntactical ones. They characterize *not* by its internal role in the system, regardless of any meaning it may be intended to contribute. The question arises as to whether a purely syntactical account is sufficient. In any case, narrowly construed, the semantic constraint for *not* should create no big metaphysical problem. It must support the classical truth-table for what Whitehead and Russell define as the 'contradictory function'.

It is with the semantic component at play that a system like G provides a defense of truth-functionality. Qua truth-functional operator, and 'when given a standard, classical two-valued interpretation' [32, p. 22], *not* is a toggling operator, reversing the truth-value of its constituent radical:

φ is Corr(1) on Z iff if $\varphi = \sim_n \psi$, ψ is Corr(0) on Z.
(Grice [28, p. 136], cf. Myro [73, p. 43])

('Corr(1)' and 'Corr(0)' are abbreviations, respectively for 'correlated with 1' and 'correlated with 0'.) The aim is the standard one, as in the comment "It seems a fair reflection of ordinary usage to identify the negation of *p* with any statement "$\sim p$" which is so related to p that if either is true it follows that the other is false". (Ayer [4, p. 42])

Note that System G allows for three other unary truth-functors: "φ is Corr(1) on Z iff if $\varphi = T_n \psi$, ψ is Corr(1) on Z", "φ is Corr(0) on Z iff if $\varphi = P_n \psi$, ψ is Corr(1) on Z", and "φ is Corr(0) on Z iff if $\varphi = Q_n \psi$, ψ is Corr(1) on Z". The choice of "$\sim$" over the other three unary 'formal devices' seems to be a matter of notational convenience or pragmatics.

Neo-Traditionalism has long understood negation in semantic terms, even when dealing with tricks like vacuous descriptors or names. Consider the implicatures (or entanglements) expressed by Oxford logic tutor Simon of Faversham (Simon de Daversiham):

"Ponatur in casu, et est possible, quod Socrates *non* sit, haec ergo est falsa 'Socrates est', et similiter haec erit falsa *Socrates non est* (*iustus*), Significaretur enim quod Socrates, qui est, *non* est; sed hoc est falsum; ergo et haec est falsa 'Socrates *non* est'; ergo haec sunt simul falsa 'Socrates est' et 'Socrates *non* est', et haec sunt *contradictoria*, ergo contradictoria essent simul falsa; hoc est impossibile" Ditto *Nullus homo est animal*, 'sive homo sit sive homo non sit' est falsa. Cuius probation est quia quaelibet universalis negativa includit in se tres habitudinem, quonian includit unam habitudinem quæst prædicati ad subiectum, et aliam quæst subiecti ad quodlibet suppositum, tertia quæst praedicata ad quodlibet suppositum. Prima habitudo negativa est, secunda affirmativa, tertiam negativa. Prima habitudo significatur per maiorem, secunda per minorem, tertia autem per conclusionem; et ideo in omni universali negativa virtute includitur forma syllogismi; et ideo qui dicit quod nullus homo est animal et quod quodlibet suppositum hominis est homo et quod quodlibet suppositum hominis *non* est animal, dicit in minore quod quodlibet suppositum hominis est homo. Esse significatum habent ea de quibus intellectus aliquid negat.

While Grice refers to both Modernism and Neo-Traditionalism as agreeing that there are (or more fastidiously put, 'appear to be') divergences between "$\sim$" (as defined in the calculus so far) and *not*, it is not to easy to come up with an example of a Modernist explicitly alleging so. The straightforward definition of negation as the contradictory function adopted by Whitehead and Russell and later by Quine offers the standard approach.

Grice colourfully ascribes to the Modernist the prejudice any such divergence will be a 'metaphysical excrescence' [32, p. 23]. It seems far easier to find illustrations of the alleged divergence from the rival group of Neo-Traditionalism. Historically, too, Grice's reply was explicitly provoked by the Neo-Traditionalist discourse, rather than the Modernist one.

10. Neo-Traditionalism

If Modernism is concerned with classical logic, Neo-Traditionalism might be viewed as romantic, although in many ways the latter is more faithful to Aristotelian foundations than is the former, at least in spirit. The vademecum here is Strawson's *Introduction to Logical Theory* [96], best seen as a direct attack on Russell's Modernism (but also Quine's logical Puritanism). Interestingly, it was influenced by early views by Grice. In the preface, Strawson explicitly cites Grice as the person "from whom I have never ceased to learn about logic since he was my tutor in the subject" [96, p. v], and in an important footnote notes that it was "Mr. H.P. Grice" who demonstrated to him "the operation of a pragmatic rule" prefiguring Grice's own maxim of quantity: "One should not make the (logically) less, when one could truthfully (and with greater or equal clarity) make the greater claim" (Strawson [96, p. 179]). Strawson later recollected: "The nature of the logical constants of standard logic was a question that H.P. Grice and I used to discuss in the early 1950s, and I have no doubt that [some of Strawson's essays were] influenced by those discussions" [97, p. 10]. And again, later, "I had the pleasure of listening to Grice expounding the essentials of his view [on logical constants] in a paper read to the Philosophical Society in Oxford in the late 1950s" (Strawson [97, p. 16]).

While it is difficult to pinpoint to an explicit reference by Modernists on the alleged divergence between "not" and "∼", Grice's favorite Neo-Traditionalist is clear on this. Indeed, a sub-section of the *Introduction* (Part II, ch. 3, 2§7) bears the title, "'∼' and 'not'". Granted, Strawson's main caveats are not with "∼" but with some of the binary truth-functors (notably the conditional). However, his Neo-Traditionalism seems to adopt a focus on 'use' rather than 'meaning'.

By focusing on contradiction as one of the primary and standard uses of "∼", Strawson underlines the role of negation in discourse. The use or function of "∼" (and *not*) is to "exclude". It is "a device used when we wish to *contradict* a previous assertion". But also when we wish "to correct a possible false impression, or to express the contrast between what had been expected, feared, suggested, or hoped, and the reality". [...] "A standard and primary use of *not* is specifically to contradict, or correct; to cancel a suggestion of one's own or another" [96, p. 7]. And later: "A standard and primary *use* of *not* in a sentence is to assert the contradictory of the statement which would be made by the use, in the same context, of the same sentence without the word *not*" [96, p. 79].

Strawson does include a caveat aimed apparently at something like the formation rule of Modernism ("if φ is a formula, $\sim\varphi$ is a formula"): "Of course, we must not suppose that the insertion of *not anywhere* in any sentence always has this effect". His example, adapted from Aristotle: "Some bulls are not dangerous" is not the contradictory, but the subcontrary, of "Some bulls are dangerous". "This is why the identification of '∼' with *it is not the case that* is to be preferred to its identification with *not* simpliciter".

Strawson [96, p. 79] continues: "This identification [of *not* and "∼"], then, involves only those minimum departures from the logic of ordinary language which must always result from the formal logician's activity of codifying rules with the help of verbal patterns: viz., the adoption of a rigid rule when ordinary language permits variations and deviations from the standard use ("$\sim\sim p \rightarrow p$", "$p \lor \sim p$") and that stretching of the sense of 'exemplify' which allows us, e.g., to regard 'Tom is not mad' as well as 'Not all bulls are dangerous' as exemplifications of 'not-p'". Divergences notwithstanding, Strawson concludes the section, "So we shall call '∼' the negation sign, and read '∼' as 'not'".

Strawson goes on to attack the Modernist (chiefly Russellian) account of definite descriptions. With Grice, we shall focus our discussion on the negation of vacuous descriptors ("The King of France is not bald"). The attack is premised on the assumption that negation, normally or invariably, leaves the subject 'unimpaired'. For Modernism, as reflected in System G, the negation of a sentence containing a vacuous descriptor comes out as *true*. For Strawson, if what is presupposed is not satisfied, "The king of France is not bald" is not true. It's not false, either, but rather a pointless thing to say, the question of whose truth or falsity fails to arise. For Grice, Modernism is essentially correct: "The king of France is not bald", in the absence of the king of France, is true. (But cf. Grice [29] for a somewhat more complex picture.) What Strawson takes as *praesuppositum* is merely an *implicatum* (and never an entailment). Strawson does recognize the marginal existence of a non-presupposing negation, as in the exchange "Does he care about it?" "—He neither cares nor does not care. He's dead" [96, p. 18]. For the true Modernist, the only correct answer would be: "No, he does not care about it. He is dead". By and large, however, Strawson takes both affirmative and negative statements featuring vacuous descriptors to result in a 'truth-value gap' (in Quine's happy parlance).

It is not obvious that Grice's opposition between the Modernists and the Neo-Traditionalists is an exclusive one. For one, it is difficult to place in either camp an author such as A.J. Ayer, "at [some] time the *enfant terrible* of Oxford philosophy" [30, p. 48], later Wykeham Professor of Logic, and the author of an influential essay on 'Negation'. In any case, Ayer seems to assume that negation carries something of a metaphysical excrescence, and should not for this be eliminated from a system. "A statement is negative if it states that an object lacks a certain property rather than stating that it possesses the complementary property. A statement is negative if it states that a certain property is not instantiated, rather than stating that the complementary property is universally instantiated" [4, p. 61].

11. Robbing Peter to pay Paul

As noted, Grice's aim is a synthesis between Modernism and Neo-Traditionalism, and one crucial issue is what we may call the identity or isomorphism thesis, the claim that there is no divergence (in terms of entailment and valid inference)

between "∼" (as defined by System G) and "not". The challenge is to respond to those Modernists and Neo-Traditionalists who have held such a divergence to exist.

Strawson has been quite explicit as to the origin of his views on negation: his disputes with Grice at the Oxford Philosophical Society, and earlier, in the premises of St. John's, where Grice '– from whom I have never ceased to learn logic' – was his tutor. For Grice, the assumption that there is a divergence between "∼" and "not" "seemed to me to rest on a blurring of the logical/pragmatic distinction" [32, p. 374]. The "general rule of linguistic conduct" that Strawson [96, p. 179] borrows from Grice was developed by the latter in his William James lectures into a fully fledged set of maxims within an overarching Co-operative Principle ("Make your conversational contribution such as is required, at the stage at which it occurs"). These maxims are sorted into four Kantian categories: Quantity ("Make your contribution as informative as is required (for the current purpose of the exchange)", "Do not make your contribution more informative than is required"); Quality ("Try to make your contribution one that is true, Do not say what you believe to be false, Do not say that for which you lack adequate evidence"), Relation ("Be relevant"), and Manner ("Be perspicuous" – Avoid obscurity of expression, Avoid ambiguity, Be brief, Be orderly). In his posthumously published retrospective epilogue, Grice [32, p. 273] adds a tenth submaxim, 'Facilitate your reply', expanding the system into a conversational decalogue; cf. Speranza [92,93].

This program is presented within 'Logic and Conversation', the William James lectures Grice presented at Harvard in the spring term of 1967. It is in the 'Prolegomena' that Grice explicitly refers to Strawson [96] as an "A-philosopher", i.e., as a philosopher who would advance a non-classical semantic or conventional account for various phenomena for which Grice offers an independently required pragmatic, non-conventional approach to supplement the standard Modernist treatment. Among the "A-philosophers", i.e. the Oxford Neo-Traditionalists or Informalists ranged on Strawson's side, we could place other members of the Play-Group, such as H.L.A. Hart, J.O. Urmson, and G.J. Warnock, as well as (on occasion) the group leader J.L. Austin himself.

The dispute concerns the divergence between a given formal device (e.g. "⊃", the horseshoe) and its vulgar counterpart ('if') – a topic that was being hotly debated at Oxford and elsewhere (cf. Speranza [93] on J.F Thomson's 'In defence of the material conditional' and Horn [47, pp. 378–379] on the role of assertability in Grice's – and Dummett's – characterization of negated conditionals). But for Grice the battle has many fronts of equal importance, examples that "involve an area of special interest to me, namely that of expressions which are candidates for being natural analogues to logical constants and which may, or may not, 'diverge' in meaning from the related constants, considered as elements in a classical logic, standardly interpreted" [32, p. 8].

In the second lecture, as we have seen, Grice explicitly tackles the '∼'/not pair. His broadens the target of attack, though. It is not just Neo-Traditionalists like Strawson who have claimed that such a divergence exists, but some of the Modernists themselves. In this respect, Grice might be considered post-modern. Grice begins his discussion in 'Logic and Conversation' by challenging the acceptance of such a divergence on the part of both Modernism and Neo-Traditionalism as a "commonplace"; rather, he maintains, "the common assumption of the contestants that the divergences do in fact exist is (broadly speaking) a common mistake" [32, p. 24]. But this divergence is typically seen as relatively innocent in the case of negation, compared to that of the (material) conditional and the other binary connectives.

Over a century ago, Russell provided "∼" (borrowed from Peano) to be read as *not*. The common garden variety of logic manual will typically accept such an identification. Thus note, for example, Benson Mates's treatment of negation in his *Elementary Logic* [68], and back in Oxford, David Mitchell in *An Introduction to Logic*. In the section dealing with 'The interpretation of the constants' in Chapter 2, Mitchell [71, p. 59] agrees with Strawson that, in contrast with some of the binary truth functors, ""∼" raises no [major] difficulties as the sign for propositional negation, used in conjunction with either propositions, as '∼(Tom is Australian)', or propositional forms. It may be read as *not*".

Or compare Suppes on "∼(Sugar causes tooth decay)": "The usual method of asserting the negation is to attach *not* to the main verb. We shall use '∼'". He enlists with those who perceive a divergence here, and has a caveat regarding the ∼ /*not* identification. "Of course, the rich, variegated character of natural language guarantees that in many contexts ["not" is] used in delicately shaded, non-truth-functional ways. Loss in subtlety of nuance seems a necessary concomitant to developing a precise, symbolic analysis of sentences" [98, p. 4]. Suppes seems to be one logician who learned from the mistake, if he ever made it, and from Grice's treatment (see his contribution to the Gricean festschrift in Grandy and Warner [26]).

The strategy of the conversationalist manoeuvre is to explain away the divergence between '∼' and 'not' via conversational implicature. A requirement for this strategy, though, is that we possess a more or less definite account of what "∼" already means in a system like system G. System G is an axiomatic calculus of the type made familiar by Whitehead and Russell which validates the inferential roles that "∼" can take. In this, Grice proves a conservative dissident parting ways with the tradition in Oxford that had been, in matters logical, to follow in the footsteps of the neo-Hegelians or the Informalism of "no calculus" favored by Ryle and Strawson. The Modernism of Whitehead/Russell (and Quine's Logical Puritanism as an offspring) had never taken root in Oxford (cf. Urmson and Warnock), and it has done so only recently with the institution of the chair of "Mathematical Logic" (Merton College and the Mathematical Institute).

12. What the eye no longer sees the heart no longer grieves for

The defense of Modernism as providing an adequate account of the explicit logical force of "not" proceeds by drawing a distinction between a 'logical inference' and a 'pragmatic inference' [32, p. 374]. Pragmatic inferences thus fall outside the

domain logic, or if not, they pertain to the terrain of non-monotonic defeasible calculi. In any case, the account they provide concerns the implicit content of 'not' rather than its contribution to truth-conditions.

Overall, Grice's sympathies were with the Modernists. He acknowledges his attraction to 'the tidiness of Modernist logic' [32, p. 374]. More importantly, he notes that his additions are not meant to provide departures from the standard Modernist apparatus: "Even if it should prove necessary to supplement the apparatus of Modernist logic with additional conventional devices, such supplementation [should be regarded as] undramatic and innocuous". These will typically involve "bracketing or scope devices" ("What the eye no longer sees..."). The strategy is to enrich the syntax (rather than the semantics) by adding the formal device. In this way, the alleged divergence between "∼" and "not" will be one of pragmatic import, which can be eliminated once the scope device is put to use.

Grice's observation regarding the credo of both Modernism and Neo-Traditionalism as resting on a mistake may itself be interpreted in a similar way by reducing the alleged semantic property of natural language "not" to a complex of a syntactic property and a pragmatic property.

It should be noted that the terminology of 'pragmatic' is already used by Strawson ("Some will say these points are irrelevant to logic (are 'merely pragmatic')") [96, p. 179]. On this, and Grice's view, it's only the syntax and the semantics for 'not' as "∼" which account for the 'logical' inferences. Any further divergence will pertain to 'pragmatic inference'. The point is always to simplify the logic, even if the explanation must still be provided. As it has been said [66, p. 215], "Grice saves" – but there's no such thing as a free lunch.

At this point, we may provide an inventory of issues which have traditionally been considered part of the logic of negation, but which we can be recalibrated as pragmatic. We need some order for the inventory. We have already touched on those alleged divergences concerning the formal treatment of "∼". We continue with other inferences that do not pertain to the formal system of negation but which have nevertheless been included by some as regarding the logic of "not". In all cases, the idea is to provide an *equi*-vocal account of "not".

One issue arises with the negation of sentences containing vacuous descriptors. System G deems "The king of France is not bald" as true in the context of a French republic. This analysis contrasts with the Parmenidean, Platonic and neo-Platonic accounts. Parmenidean negation creates a Meinongian jungle, in which the non-being of the King of France is not directly referred to. The problem is not particularly solved by Plato and his proposal to analyze "not" as "other than". "The king of France is not bald, but other than bald" allows for the cancellation, "indeed he is non-existent" – but somehow one does not think that is what Plato had in mind.

A related problem which was emphasized by Ryle is the negation of category (or as Grice has them, 'eschatological') mistakes. While 'Virtue is square' is false, 'Virtue is not a square' comes out as trivially true, if only a redundant – if not downright irrelevant – thing to say in most contexts (but cf. 'Virtue is not square, but some windows are' vs. 'Virtue is not square, indeed virtue does not exist'). (See Horn [47, §2.3] for a review of some of the relevant literature.)

"System G" allows for two syntactic devices to signal scopal ambiguity and avoid lexical ambiguity. The central manoeuvre of the pragmatic component of System G is the introduction of the notion of implicature. Implicature contrasts with entailment but also with explicit content: "Whatever is implied ... is distinct from what is said" [32, p. 24]. When first (re)introduced into the philosophical literature, and thence into the consciousness of linguists, presupposition and implicature each appeared in turn as the meaning relation that dare not speak its name, as The Other: an inference licensed in a given context that cannot be identified with logical implication or entailment. Thus Strawson [95]:

> To say, "The king of France is wise" is, in some sense of "imply" to *imply* that there is a king of France. But this is a very special and odd sense of "imply". "Implies" in this sense is certainly not equivalent to "entails" (or "logically implies").

Two years later, this 'special and odd sense of "imply"' is recast as the neo-Fregean notion of presupposition. Similarly, when Grice first discusses (what would later be called) implicature, he begins by carving out a use of *imply:* "If someone says 'My wife is either in the kitchen or in the bedroom' it would normally be implied that he did not know in *which* of the two rooms she was" [27, p. 130]. Later, Grice makes a similar move to a more specialized and less ambiguous term: "I wish to introduce as terms of art, the verb 'implicate' and the related nouns 'implicature' (cf. implying) and 'implicatum' (cf. what is implied)" [32, p. 24]. (Latin *implicat* is ambiguous, and Suedonius had already used *implicatura* for 'entanglement' (Short/Lewis, *Latin Dictionary*). Other members of the Play-Group had employed similar notions under various names, e.g. Nowell-Smith [74] (and later Hungerland [56]) on "contextual implication". (See the discussion in relation to Grice and Strawson in Horn [48].)

The distinction among species of implication within the overall genus of inferential relations is of particular relevance for the study of negation. In his influential *Statement and Inference* (posthum., [102]), Cook Wilson considers the distinction between 'asserting' and 'implying' or 'presupposing': "A negative statement normally presupposes the existence of their subjects of attribution; this existence is not *asserted* by the negative statement as such" [102, p. 259]. Ryle [87] makes a distinction between 'asserting' and, not implying (or implicating) but presupposing. It is not altogether clear whether these logicians have identified what exactly it is that is implied, presupposed, asserted, or implicated in the issuing of a negation. "[E]limination may precede and not ensue upon the discovery of the required [positive] fact. But even when what enables me to assert 'A is not B' is the knowledge that A is C, the assertion 'A is not B' is different from the assertion 'A is C'" ... When I say the "The hat is not green (but some other colour)," I am (not stating but) presupposing that the hat is coloured ... It is known and presupposed that the Determinable 'being assertion coloured' characterizes the hat, [but] the

denial of Determinate B is not yet the assertion of F (say)', but presupposing that *some* Determinate is the colour of the hat, asserts of this it is *one* of the Determinates of the Determinable 'being coloured' other than the given Determate B" (all quotes from Ryle [87, pp. 85–87], emphasis added). This relates to the medieval doctrine of *suppositio* and the 'negative pregnant', discussed below. Part of the manoeuvre is to restate what has been already noted by medieval logicians. Insightful treatments of negative utterances in terms of their presuppositions or implicatures can be found in work by Peter of Spain ([78,79], Spruyt [94]) and William of Sherwood [75,63], among others.

Implicature serves to explain the medieval theory of the 'negative pregnant', "a negative implying or involving an affirmative. As is a man being impleaded, to have done a thing upon such a day, or in such a place, denyeth that he did it *modo & forma declarata*: which implyeth nevertheless that in some sort he did it" (Cowell [16]). As argued in Horn [47, Chapter 3], this notion of an affirmative supposition or ground underlying a negative statement must be analyzed as a Gricean implicature, not an entailment (or a Fregean or Strawsonian presupposition).

13. Negation as otherness

Following a suggestion by Cook Wilson, Grice [32, p. 75] looks for an analysis [or another term?] that can be specifically assigned to both "∼" and "not". The idea of 'not' as shorthand for 'other than' is taken up by Grice. "As regards *not*: if our language did not contain a unitary device, there would be many things we can now say which we should be then unable to say, unless (1) the language contained some very artificial-looking connective like one or other of the strokes [i.e. the Sheffer 'not both' stroke or the joint denial 'neither-nor'], or (2) we put ourselves to a good deal of trouble to find (more or less case by case) complicated forms of expression involving such expressions as *other than* or *incompatible with*" [32, p. 68].

In the case of 'incompatible with', consider the "reduction" of "∼p" to "$p \mid p$". Rather than "The king of France is not bald" we would have, "The king of France is bald is incompatible with the king of France being bald". In the case of 'other than', philosophers since Plato have tried without notable success to find the appropriate 'complicated' form. In Platonic parlance, it's το διαφορον, or το ετερον. Plato saw this as part of a problem for his ideationist theory of meaning. If for any object m, there is a meaning relation such that m means (idea) M, the issue is to find the correlation for a 'judgment' that denies an attribute to an object (*Sophist* 357B). There have been various attempts to formalize what Plato is up to. Wiggins [100] converts "∼FLY(Theaetetus)" into a Platonic rendering as 'There is a property P such that P△(FLY) & P(Theaetetus)', where △ stands for διαφορον; cf. also Bostock [8, p. 115]. The relationship between Parmenidean and Platonic conceptions of negations and non-being is examined in Pelletier [81].

Note that "Theaetetus is not big" cannot simply be reduced to the assertion 'Theaetetus is small' [δελουν]. For one thing, as Plato recognized (Sophist 257Dff.), he may be medium-sized, whence the position that 'Theaetetus is not big' *explicitly* means 'Theaetetus is *other than* big' (leaving aside those cases in which Theaetetus does not exist). Negation as otherness is invoked as well by Mill [69]: "Negative names are employed whenever we have occasion to speak collectively of all things *other than* some thing or class of things." While Grice, and before him, Wilson and Ryle, are more or less in agreement with an account of "not" that allows for its identification with "∼", with any mismatch handled via implicature, the question is trickier for the neo-Idealists or neo-Hegelians. The British neo-Idealist group included Bradley, Bosanquet, Joseph, and Joachim, but a particularly significant and often neglected figure here is the German Sigwart, who anticipated by two decades Frege's notion of presupposition (*Voraussetzung*) in discussing the problem of vacuous subjects.

For Strawson, as for his intellectual predecessor Frege [23], the notion of presupposition has semantic status as a necessary condition on true or false assertion, but more recent work has taken the commitment to existential import in such cases as constituting a pragmatic presupposition or an implicature (cf. e.g. Wilson [102]; Grice [32, Essay 17]). In fact, the earliest pragmatic treatments of the failure of existential presupposition predate Frege's analysis by two decades. Here is Christoph Sigwart [90] on the problem of vacuous subjects:

> **As a rule**, the judgement A is not B presupposes the existence of A in all cases when it would be presupposed in the judgement A is B... 'Socrates is not ill' presupposes in the first place the existence of Socrates, because **only on the presupposition** *[Voraussetuzung]* **of his existence can there be any question of his being ill**.
>
> (Sigwart [90]: 122, emphasis added)

Note in particular the contextual nature of the presupposition and the proto-Strawsonian flavor of the conclusion. Further, unlike either Frege or Strawson, Sigwart allows for wide-scope (presupposition-canceling) negation as a real, although marked, possibility, although to be sure a negative singular statement is 'commonly understood' as implying the existence of its subject referent.

> If we answer the question 'Is Socrates ill?' by yes or no, then – **according to our usual way of speaking** – we accept the presupposition *[Voraussetzung]* upon which alone the question is possible; and if we say of a dead man that he is not ill, we are guilty of using our words ambiguously. It may still, however, be claimed that, by calling such an answer ambiguous, we admit that the words do not, in themselves, exclude the other meaning; and that formally, therefore, **the truth of the proposition** [*Socrates is not ill*] **is incontestable** [if Socrates is not alive].
>
> (Sigwart [90]: 152, emphasis added)

The neo-Idealists are generally associated with the idea of 'positive negation', but this cannot be a truth condition (or more broadly a semantic requirement) on negative statements, which can perfectly well be true (if possibly odd) when their positive ground is lacking. Grice observes of a man lighting his cigarette in the ordinary way: "Now it is certainly the case that it would be false to say of the man using a match, 'He is now lighting his cigarette with a 20-dollar bill', and so it is true that he is *not* lighting his cigarette with a 20-dollar bill." He adds: "So far as I know no philosopher since the demise of the influence of Bradley has been in the least inclined to deny this" [32, p. 15]. (The mocking of Bradley seems to have been a favorite sport with British mid-century philosophers. For example, Ayer [4, p. 39], quoting from Bradley [10, p. 115], comments: "Any statement whatsoever which is seriously put forward may be picturesquely described as an attempt to qualify *reality*; and if the statement turns out to be false the attempt may be said to have baffled".)

Armed with this commentary by Grice, let us revisit Bradley's contribution to the history of negation. Formally, Bradley would have endorsed the unpacking of "$\sim Px$" as "$(\exists P')(P'x \& P \mid P')$". This is consistent with Grice's earlier observation that 'not' has the métier of 'incompatible with' (and not just 'other than'). Bradley's general view on negation focuses on the idea of 'positive ground'. This may again be seen as a development of the aforementioned medieval views on *suppositio* and the notion of a "negative pregnant". For Bradley [10, p. 200], any negative proposition presupposes a positive ground:

> Every negation must have a ground, and this ground is positive. Nothing in the world can ever be denied except on the strength of positive knowledge. We cannot deny without also affirming. We should never trust a negative judgement until we have seen its positive ground.

– a.k.a. its *negatum*. For denial to be possible, there must be, first, the "suggestion" of an "affirmative relation":

> If we suppose that, with reference to the tree the utterer has judged, 'This tree is *not* yellow', the judgment would have to be construed as *involving* the affirmative suggestion that the tree *is* yellow. In the negative judgment, the positive relation of 'yellow' to the tree must precede the exclusion of that relation. What gets denied must be something that already has a truth value.

The views of Bradley were influential enough in Oxford to merit a doggerel in *The Oxford Book of Oxford* [72]: "Thou positive negation! negative affirmation! thou great totality of every thing, that never is, but ever doth become, thee do we sing".

We turn to the second great British neo-Idealist (of whom Grice's assessment was somewhat unenthusiastic: "If we are looking at the work of some relatively minor philosophical figure, such as for example Bosanquet…" – [30, p. 66]. Bosanquet had his influence on Bradley himself; in his 'Terminal Essay' on negation, Bradley confesses "the chapter [on negation in *Principles of Logic*] contains some serious errors. I have since accepted in the main Dr. Bosanquet's account of negation". Bosanquet starts by citing Jevons, Mill, and Venn and arguing that "In negation, the work of affirmative belief appears to be performed by ignorance" [7, p. 277].

Of Sigwart's view that every negation presupposes an affirmation, so that 'S is not P' presupposes the affirmation 'S is P', Bosanquet declares: "I think it monstrous. I do not believe that you must find an affirmative standing before you can deny" [7, p. 277]. Further, "A negation is not a denial of an affirmative judgment, and therefore does not presuppose the affirmation of that which is denied". Yet "a negation does presuppose *some* affirmation" [7, p. 280]. Bosanquet refines Bradley's position by distinguishing the positive ground of a negative utterance (some *contrary* proposition, whose truth determines the truth of the negative proposition) from the positive consequent (the indeterminate proposition which logically follows from the negative). Thus, the positive ground of 'This shirt is not red' may be 'This shirt is (e.g.) blue', where *blue* is inconsistent with *red*. The positive consequent, however, is simply "There exists a colour P', $P' \neq$ red, such that this surface is P'" [7, p. 287]. The relation between a negation and its affirmative ground is one of contrariety, while a negation and its affirmative consequent are in contradictory opposition.

While Bradley leaned on Bosanquet, the direction of influence was in fact mutual. Bosanquet, citing Bradley's notion of 'a suggested affirmative relation' endorses the latter's view that 'in the beginning, a negation is a degree more remote from reality than is an affirmation'. But while an affirmation is epistemologically prior to negation, eventually, affirmation and negation alike become double-edged, each involving the other [7, p. 281]. For Bosanquet, only those negations which presuppose an affirmation can be significant (let alone true). A significant negative utterance "S is not P" can always be analyzed as "S is P′ which excludes P". "The surface is not red, but an undetermined colour". An apparently insignificant or bare negation (e.g. "The lion is not an elephant", "Virtue is not a square") does not posit a correct, true contrary, since it does not limit the sphere of negation [7, p. 289].

Bosanquet advocates a similar line on negative utterances with vacuous descriptors. Of his example, 'The house on the marsh is not burnt down', Bosanquet allows that the utterance is *true* when there is no house on the marsh, even if "reality excludes the burning down of any such house". Bosanquet confesses a 'strong sympathy' for the objection (straw or real) that the utterance may be said to have 'meaning only if there is a house, and the sentence presupposes [cf. Sigwart above] or asserts that there is one".

14. From contradiction to contrariety: Bosanquet, Anselm et al.

Bosanquet's insightful comment that "The essence of negation is to invest the contrary with the character of the contradictory" [7, p. 291] epitomizes his unfortunately overlooked discovery of a class of cases that would now receive a plausible Gricean account of contradictories in contrary clothing. (Cf. Horn [47, Chapter 5] and Horn [50] for elaborations of a variety of such pragmatic strengthening processes.) One example is "Jones is not good", which appears to represent contradictory negation on "p" ("Jones is good"), and thus being representable as "$\sim p$". However, the ordinary copular negative yields a relatively weak non-informative contradictory that tends to be strengthened, "so that from 'Jones is not good' one may be able to infer something more than that 'it is not true or the case that Jones is good'".

As a related illustration of the same tendency to enrich a formal occurrence of a contradictory "$\sim p$" to a contrary, given his premise that "the essence of formal negation is to invest the contrary with the character of the contradictory", Bosanquet offers an early account of the phenomenon of negative transportation or neg-raising, i.e. "The habitual use of such a phrases as 'I do not believe it', which refers grammatically to a fact of my intellectual state, but actually serves as a negation of something ascribed to reality. Compare Gk. ον φημι [lit. 'I don't say'], which means 'I deny', or our common phrase, 'I don't think that' – which is really equivalent to 'I think that – not'" [7, p. 319]. Again this phenomenon can best be accounted by an implicature-based account (see Horn [45], [47, Chapter 5]). Thus, instead of treating a negative judgment in Bradleyan fashion as aiming at a final affirmative, Bosanquet sees canonical contradictory negation as functioning to express a notional contrary, because when there are only two alternatives, the denial of one is equivalent to, and grounded on, the assertion of the other.

The phenomenon of strengthened contrary readings for apparent contradictory negation has long been recognized, dating back to classical discussions of the figure of litotes, in which an affirmative is indirectly asserted by negating its contrary has been recognized since the 4th century rhetoricians Servius and Donatus as a figure in which we which we say less and mean more (*minus dicimus et plus significamus*, cited in Hoffmann [41, pp. 28–29]), thus representing one of the first explicitly pragmatic analyses in the Western tradition (cf. Horn [49] for elaboration). Note, however, that litotic interpretations tend to be asymmetrical: it is more likely that calling someone "not happy" or "not optimistic" will convey a contrary (= rather unhappy/pessimistic) than that such virtual contrariety will be signaled by "not sad" or "not pessimistic", which tend to be understood as pure contradictories. Explanations for this asymmetry have been proposed by Ducrot [18] and Horn [47, Chapter 5].

For a more formal approach to the strengthening of contradictory negation to virtual contrariety is due to the fourteenth century logician Robert Bacon. Bacon begins the discussion in his *Syncategoreumata* by distinguishing three varieties of interaction between negation and its focus: the ordinary negative name (*nomen negatum*) 'isn't just' (*non est iustus*), the infinite name (*nomen infinitum*) 'is not-just' (*est non iustus*), and the privative name with incorporated negation (*nomen privatum*) 'is unjust' (*est iniustus*). Technically, he notes, the third of these unilaterally entails the second and the second the first, but ordinary usage is not always consistent with this:

> Ex hiis patet quod bene sequitur argumentum a privato ad infinitum, ut: 'est iniustus; ergo est non iustus'. Similiter: ab infinito ad negatum, ut: 'est non iustus; ergo non est iustus.' Econverso autem non tenet, sed est paralogismus consequentis.					(Braakhuis [9, Vol. I, pp. 144–145]; Spruyt [94, p. 252])
>
> From these it is apparent that the argument follows validly from the privative to the infinite, thus "s/he is unjust, therefore s/he is not-just". Similarly, from the infinite to the negative, thus "s/he is not-just, therefore s/he isn't just". However, the converse does not hold, but is the fallacy of consequence.

For Bacon, the move from the contradictory *X isn't just* to the contrary *X is unjust* is an instance of the fallacy of consequence, the deductively invalid but inductively plausible strengthening of a sufficient condition to a necessary-and-sufficient condition (see Horn [50] for an elaboration of this point within a pragmatic account of "conditional perfection").

As for the "neg-raised" reading of "I don't think that p" as "I think that not-p", while often dismissed as an incidental and deplorable ambiguity or (in Quine's terms) an "idiosyncratic complication" of one language –

> ... the familiar quirk of English whereby 'x does not believe that p' is equated to 'x believes that not p' rather than to 'it is not the case that x believes that p'
>
> [83, pp. 145–146]

> The phrase 'a does not believe that p' has a peculiarity ... in that it is often used as if it were equivalent to 'a believes that $-p$'.					[38, p. 15]

> 'I do not believe that p' can be unfortunately ambiguous between *disbelief* [$B_a\text{-}p$] and *not belief* [$-B_a p$].					[17, p. 55]

– its roots and significance for the study of negation go far deeper. The locus classicus for the phenomenon is St. Anselm's observation in the Lambeth fragments antedating Bosanquet by eight centuries; cf. Anselm [1] and the commentary in Henry [40, pp. 193–194], Hopkins [42, pp. 231–232], and Horn [45, p. 200], [47, p. 308ff.]. Anselm points out that '*non ... omnis qui facit quod non debet peccat, si proprie consideretur*' – not everyone who does what he *non debet* ('not-should') sins, if the

matter is considered strictly (i.e. with the contradictory reading of negation as suggested by the sentence structure); the problem is that we tend to use '*non debere peccare*' to convey *debere non peccare*, rather than its literal contradictory meaning ('it is not a duty to sin'). A man who does what is not his duty does not necessarily sin thereby, but it is hard to stipulate e.g. *non debet ducere uxorem*, the proposition that a man need not marry without seeming to commit oneself to the stronger *debet non ducere uxorem*, an injunction to celibacy (Henry [40, p. 193ff.]; cf. C.J.F. Williams [101], Horn [46, p. 200]).

For Henry [40, p. 193, §6.412], Anselm's observations on modal/negative interaction are "complicated by the quirks of Latin usage. He has become conscious of the fact that, according to that usage, '*non debet*', the logical sense of which is 'It isn't that he ought', is normally used not to mean exactly what it ways, but rather in the sense more correctly expressed by '*debet non*' ('he ought *not*')." In fact, rather than constituting a quirk of English and/or Latin usage, "neg-raising" – the lower-clause understanding of negation of a *believe-* or *ought-*type predicate – is distributed widely, although systematically, across languages and operators. The raised understanding is always **stronger** than the contradictory (outer) negation; it applies to a proper subset of the situations to which the contradictory applies (is true in a proper subset of possible worlds). Thus neg-raising, as Anselm recognized, always yields a virtual contrariety: the compositional meaning is true but too weak, and the addressee recovers a (short-circuited) conversational implicature to 'fill in' the stronger proposition.

In any event, Bosanquet seems to have been the first philosopher to see the general pattern represented by such tendencies in ordinary language, although other instances of his principle (e.g. the generalization that affixal negation in words like *unhappy* or *unjust* tend to develop contrary semantics cross-linguistically) could be cited. One way of putting the point is that contrariety tends to be maximized in natural language, while subcontrariety tends to be minimized.

15. Asymmetry revisited: the rise and fall of the neo-Idealists

Returning to the broader question of 'positive negation', we may count H.H. Joachim among its Idealist proponents. For Joachim, another holder of the Wykeham logic professorship, a negative utterance expresses some "knowledge in the making only" [60, p. 136]. Thus, an utterance like "The diagonal of the square is not commensurable with its side" is not really negative, but has a positive, real import, namely, "to constitute a problem for a certain level of geometrical *knowledge*. We have here a *real* disunion of elements in the *real* whole. The judicial separation expresses a *real* divorce". [60, p. 128] The Idealist account was the fare of the Oxford of the period, as in the logic manual by W.H. Joseph, of whom Grice observes that he "was dedicated to the Socratic art of midwifery; he thought to bring forth error and to strangle it at birth" [30, p. 62]. If "Dead nettles do not sting", they nevertheless should possess some positive quality or other. If no sentient being existed the utterance 'The wall is not blue' could not be true, since it can be uttered only because someone may suppose or believe the wall to be blue. We must accept the negative judgment as expressing the real limitation of things; but we must allow that it presupposes the affirmative. If dead-nettles do not sting, there must be some characteristic which they do possess, incompatible with stinging. There is always a positive character as the ground of negation. Snow is not hot because it is cold ... To know what anything is not is frequently a help to discovering what it is." [61, p. 172] But on this view, negation is subjective: if no sentient being existed the utterance 'The wall is not blue' could not be true, since it can be uttered only because someone may suppose or believe the wall to be blue. Note also that the square root of 4 would not be 3 even if no human ever existed to express this or any other negative proposition; see Horn [47, §1.2.2] on arguments for and against the putatively subjective nature of negation.

By the time Grice was writing, neo-Idealism had long since been effectively dispelled by Cook Wilson and others. Wilson [102] sketches a theory of negation that leaves room for presupposition, implicature, and their contextual cancellation. Most of his examples are implicature- or presupposition-carrying examples of negative utterances. "It is not an odd number", someone says, the implication being "It is an even number". Of course this is again cancellable, "In fact, it {is not a number at all/does not exist}". In the realm of non-binary and empirical propositions, the cancellability is more evident and the range of 'implications' fuzzier. With "This man is not a Mohammedan", Wilson [102, p. 250], "I cannot thereby determine his religion, nor even that he has any at all". (Due to the use of the demonstrative 'this' an actual cancellation will be awkward here: "In fact this man does not exist".) Wilson goes on to distinguish between the presupposition and the asserted content. When it comes to the latter, he adopts a neo-Platonic doctrine, foreshadowing Grice's later doctrine.

Wilson writes: "Although it is true that ordinary negative statements (e.g. "Nobody in the next room can read Greek") normally *presuppose* the existence of their subjects of attribution, this existence is not *asserted* by the negative statement as such" [102, p. 259], and it is thus cancellable as an implicature ("...since the next room is empty"). "When is the verbal form of negative statement natural and normal? When do we naturally say 'A is not B'? Clearly when our conception of Aness does not necessarily involve for us the distinction from Bness, or the absence of Bness" [102, p. 272]. He distinguishes two scenarios. "The statement may correspond either to the apprehension of something in A which excludes Bness, or to the mere observation of the fact that Bness is absent from A". This first case is of the form "A is C", where Cness excludes Bness. His example implicitly draws on the epistemological weakness that the felicitous use of negation typically suggests. "This substance does not show blue colour in the flame of the blowpipe" implies "This substance shows a colour *other than blue* in the flame of the blowpipe". We arrive at this by observing that the colour shown in the flame is, say, red. "Why then have the negative statement at all, and not the affirmative which tells us more and is fully adequate to the thought behind the expression? The negative is not adequate, for if I say "The colour is not blue", I do not say what colour it is and I omit besides something which I know, which also is the reason for what I say". (This argument works if I *know* that the flame is red, but not if I merely have sufficient information to rule out blueness.)

The second scenario for a natural negative utterance is "that in which A is merely observed to be without Bness, an attribute compatible with Aness" His example is "Private Atkins is not in the ranks" (implicating that a private *other than* Atkins is in the ranks). "To find out whether Atkins is in the ranks, we have to observe each rank and file and see that he is not Atkins".

Against the then flourishing neo-Hegelian, Idealist dogma, Wilson [102, p. 273] is eager to stress the *Realist* side to his account. In both scenarios, "the apprehension of the negation and of absence is after all the apprehension of two positive realities as different from one another".

The critique of neo-Idealism was redoubled with Ryle's appearance on the Oxford scene. What Ryle brings to the picture is a closer examination of colloquial cases. While Grice emphasizes that both Formalists and Informalists have failed to give adequate attention "to the nature and importance of the conditions governing conversation", he was presumably exempting Ryle, whose efforts he credited for spearheading "the rapid growth of Oxford as a world centre of philosophy" [30, p. 48].

With respect to negation, the aim of Ryle, as for Grice and Strawson, is to identify some of the "conversational features" of "not". Ryle's example is a familiar one: "Mrs. Smith's hat is not green", allowing the inference that "Mrs. Smith's hat is *other than* green", understood broadly enough to include those scenarios where Mrs. Smith's hat doesn't exist – and therefore it *is* other than green. But is the hat's other-than-greenness part of the content of the negative statement? Ryle writes:

> When I say 'Mrs. Smith's hat is not green', I can equivalently say 'but some other colour'. The 'but some other' is always there, sometimes explicitly, sometimes marked by tone of voice, or simply implied by the context. Without the but clause, negative sentences are elliptical, though still generally interpretable in context. When I say, 'Mrs. Smith's hat is not green but some other colour', I am not stating but presupposing that the hat is coloured. In general, for any 'The S is not P', what gets presupposed is that P belongs to a contextually assumed set, some other member P′ of which holds of S. The full explication of what is meant by a negative sentence takes the form of an assertion of otherness as specified or made determinate by mention of the particular disjunctive set to which the other belong to as members. (Ryle [87, p. 89])

Such presuppositions are regarded by Ryle, as they would be for Strawson and as they were for Frege, as pre-conditions for the truth or the falsity of a judgment, rather than merely as conditions on felicitous assertion. This presuppositional approach would deny that "Mrs. Smith's hat is not green" could be (trivially) *true* in the absence of Mrs. Smith's hat, whatever the implicature may be on that occasion. For transcategorial examples, the case is different; "Virtue is not square" is still *true*, even if the continuation, 'but some other shape' confuses rather than illuminates.

16. Negation, presupposition, and the bracketing device

In the spirit of Modernism, Grice proposes – as we have seen in the formation, inference, and semantic rules of system G – some type of 'formal' indication of the systematic interaction of negation with other elements of the logic. Grice played with two 'scope' devices, and we shall consider them in turn, as they illustrate a pattern in the history of logic. The first is a subscript notation for scopal ambiguity. Quine, for one, found the system "forbiddingly complex" yet, he remarks, "on the whole I am for it" [84, p. 326]. The idea is that any constituent in a formula gets a subscript to mark its order of arrival. Thus, for the negation of a basic formula "$\sim p$", the device delivers two readings: "$\sim_2 p_1$" and "$\sim_1 p_2$". Consider one of Grice's early examples, "Jones has not left off beating his wife". The default 'logical' reading is the second. Hence the possibility of the cancellation, "He is not married". The default 'pragmatic' reading is the former, hence the paradoxical flavor.

Or consider, "The king of France is not bald". On one reading, the logical one, "$\sim$" has maximal scope: "$\sim_3 ((\exists x)K_1 x \ \& \ (\forall y)(K y \to x = y) \ \& \ (B_2 x))$". On the pragmatic reading, negation is internal: "$\sim_1 ((\exists x)K_2 x \ \& \ (\forall y)(K y \to x = y) \ \& \ (B_3 x))$", and gets the cancellation, 'There is no such king" ($\sim_1 ((\exists x)K_2 x \ \& \ (\forall y)(K y \to x = y) \ \& \ (B_3 x)) \models (\exists x)K_2 x$). In a notational variant mentioned by Grice, taking up a suggestion by Hans Sluga, the two readings are: $\sim_2 B_1 \iota x_3 K_1 x_2$ and $\sim_4 B_1 \iota x_3 K_1 x_2$. (Grice mentions a notational variant suggested by Charles Parsons: "$[Kx](\sim Bx)$" "$\sim [Kx](Bx)$".)

Grice refers to the two readings as the weak reading and the strong reading. The diagnostic is the role played by negation.

> If there were a clear distinction in sense (in English) between, say, 'The king of France is not bald' and 'It is not the case that the king of France is bald' (if the former demanded the strong reading and the latter the weak one), then it would be possible to correlate 'The king of France is bald' with the formal structure that treats the iota-operator like a quantifier. But this does not seem to be the case; I see no such clear semantic distinction. So it seems better to associate 'The king of France is bald' with the formal structure that treats the iota-operator as a term-forming device. (Grice [32, p. 272])

The second device is a bracketing device, yielding the representation: "$\sim [(\exists x)K x \ \& \ (\forall y)(K y \to x = y)] \ \& \ Bx$". Since the square-bracketed material is (normally) scopally immune to negation, this will as a default get rewritten as "$(\exists x)K x \ \& \ (\forall y)(K y \to x = y) \ \& \ \sim Bx$", with the externalization of the "presupposed" material.

What gets square-bracketed is not so much the 'positive ground' for a given negation, but what Grice calls the 'common-ground' status, or noncontroversiality. The square-bracketed material is only implicated, and thus cannot occur freely in monotonic inferences, since implicatures are cancellable (cf. Wilson [103] for a different implementation of a pragmatic account of existential "presuppositions" as conversational implicatures).

Consider "Jones does not regret Father is ill" [32, pp. 280–281]. A preferred pragmatic inference is again the implicature-carrying one ("Father is ill and Jones thinks Father is ill and Jones is not against Father being ill"), which is cancellable ("Jones does not regret Father is ill. Indeed Father is not ill"). What the square-bracket device does is raise the subordinate clause of 'regret' outside the scope of "∼": "[Father is ill &] Jones thinks Father is ill & ∼(Jones is ANTI (Father is ill))". (See Horn [51, pp. 74–76] for a new look at Grice's bracketing device.)

Another area for implicatural treatment concerns the scope of negation outside the radical ("∼ $\sqrt{p}$" rather than "$\sqrt{\sim p}$"). In discussing a 'general notion of satisfactoriness' [34, p. 83], while generalized versions for the binary truth-functors are unproblematic – 'ϕ & ψ' is satisfactory just in case ϕ is satisfactory and ψ is satisfactory, and so on, the unary truth-functor is not so easily dealt with: "The real crunch comes with negation. '$\sim \vdash p$' might perhaps [contra Frege [21]] be treated as equivalent to '$\vdash \sim p$'. But what about '$\sim !p$'?" – i.e. negation outside the scope of a directive speech act.

In particular, Grice asks,

"What do we say in cases like, perhaps, 'Let it be that I now put my hand on my head' or 'Let it be that my bicycle faces north', in which (at least on occasion) it seems to be that neither '!A' nor '!$\sim A$' is either satisfactory or unsatisfactory? What value do we assign to '$\sim !A$' and to '$\sim !A$'? Do we proscribe the forms altogether (for all cases)? But that would seem to be a pity, since '$\sim ! \sim A$' seems to be quite promising as a representation of 'you may (permissive) do A': that is, I signify my refusal to prohibit your doing A. Do we disallow embedding of these forms? But that (again if we use them to represent 'may') seems too restrictive".

The problem "would require careful consideration; but I cannot see that it would prove insoluble, any more than analogous problems connected with presupposition are insoluble; in the latter case the difficulty is not so much to find a solution as to select the best solution from those which present themselves". [34, p. 89]

17. Implicature and negation: scales, scopes, and metalinguistic negation

Grice characterizes the complications often introduced by negation in terms of a loss of (logical) innocence. "[I]f rational beings are equipped to assert a certain range of statements, they must also be supposed to be equipped to deny just that range of statements. In that case, the negations of the initial range of logically innocent statements may be supposed to lie within the compass of the speakers of the language; and these statements by virtue of their character as denials, may not wear the same guise of logical innocence". [32, p. 70] One illustration involves the interaction of negation with focus or contrast. Grice considers various conversational scenarios here, including one in which an utterer B says (apparently out of the blue), "JONES didn't pay the bill". Grice comments [32, p. 52]:

The remark is not prompted by a previous remark (it is volunteered), and we are inclined to say that the implicature is that someone thinks or might think that Jones did pay the bill. The maxim of Relation requires that B's remark should be relevant to something or other, and B, by speaking as if he would speak in reply to a statement that Jones paid the bill, shows that he has such a statement in mind.

A similar point had been made earlier by Ryle, concerning the example "Jones is not the secretary of the club", where stressing each constituent projects what, in Gricean terms, is a different implicature: "*Jones* is not the secretary of the club" (someone other than Jones is), "Jones is not the *secretary* of the club" (he holds an office other than secretary), "Jones is not the secretary of the *club* (he is the secretary of an institution other than the club), "Jones *is* not the secretary of the club (but he served in the role at some other time)" [87, p. 89].

Another much discussed set of examples involves an interchange related to a different exchange Grice presents in this section:

I KNEW that may be contrasted with *I believed that*, and the speaker may implicate not that he would deny *I believed that p*, but that he would not confine himself to such a weaker statement, with the implicit completion *I did not merely believe it*. (Grice [32, p. 52], cf. Proclus, *Parmenides* 913 on *hyperapophasis*.)

In such cases, a logically weaker or less informative utterance implicates that the speaker was not in position to have uttered a stronger alternative, salva veritate.

The operative principle is Grice's first submaxim of quantity (or the earlier "rule of strength" attributed by Strawson [96] to Grice; see discussion above). In fact, this rule and its epistemological constraints on its operation date back at least to Mill [70, p. 501]:

If I say to any one, 'I saw some of your children to-day', he might be justified in inferring that I did not see them all, not because the words meant it, but because, if I had seen them all, it is most likely that I should have said so: even though this cannot be presumed unless it is presupposed that I must have known whether the children I saw were all or not".

But as Mill goes on to observe, this cannot be a part of the content or, as we would now call it, the logical form of expressions with "some", contra Sir William Hamilton's arguments for doing just that: "No shadow of justification is shown...for thus adopting into logic a mere sous-entendu of common conversation in its most unprecise form". Similarly, while disjunctions are naturally taken exclusively – "When we say A is either B or C we imply that it cannot be both" – this too cannot be a logical inference: "If we assert that a man who has acted in a particular way must be either a knave or a fool, we by no means assert, or intend to assert, that he cannot be both" [70, p. 512]. (See Horn [43,47,48] and Speranza [91] for more on Mill and similar arguments from De Morgan.)

This scalar or Q-implicature induced here by the quantitative scale ⟨all, some⟩ arises in a wide range of cases involving both logical operators and ordinary predicates (see Levinson [65] for a comprehensive catalog). The one alluded to by Grice above involves the scale ⟨know, believe⟩, with the result that an assertion of belief generally (but non-monotonically) conveys absence of knowledge. In general, for a scale $\langle P_n, P_{n-1}, \ldots, P_2, P_1 \rangle$ and a constituent P_i, the Q-implicature is $\sim S(P_i/P_j)$ for all $P_j > P_i$ ($j \neq n$), where "$\varphi(P_i/P_j)$" denotes the result of substituting P_j for P_i within φ, $\sim S(Pi/Pn)$, and if $P_k > P_j > P_i$, $\sim S(P_i/P_j)$, $\sim S(P_i/P_k)$. (See Gazdar [24, pp. 55–62] for a comprehensive formulation.)

But now we are prepared to see that a speaker may choose to convey "I didn't merely believe that p" not by asserting "I KNEW that" (as in Grice's example of contrastive stress) by apparently denying that she believes that p, especially with the appropriate continuation or rectification: "I didn't believe that p, I KNEW it". This does not require rejecting the inference from knowledge to belief, if we acknowledge a specialized metalinguistic or polemic use of negation in such contexts, and at the same time such an account permits us to retain the "logical innocence" of ordinary negation. Note that while 'He believed it' is *true* if he also knew it, 'He merely believed it' is *false* in the same scenario. Essentially, this specialized use of negation targets not the propositional content (what is said) but the (potential) implicature.

The tacit principle which Mill invokes and which Grice later formulates as the Quantity submaxim, requiring the speaker to use the stronger *all* in place of the weaker *some* when possible and licensing the hearer to draw the corresponding inference when the stronger term is not used, is systematically exploitable to yield upper-bounding generalized conversational implicatures associated with scalar operators. Quantity-based scalar implicature – e.g. my inviting you to infer from my use of *some*... that for all I know *not all*... – is driven by our presumed mutual knowledge that I expressed a weaker proposition in lieu of an equally unmarked utterance that would have expressed a stronger proposition. Thus, what is said in the use of a weaker scalar value like those in *I saw some of your children* or *I believed that p* is the lower bound (...*at least n*...), with the upper bound (...*at most n*...) implicated as a cancellable implicature. The prima facie alternative view, on which a given scalar predication is lexically ambiguous between weaker and stronger readings, is ruled out by the general metatheoretical consideration that Grice dubs the Modified Occam's Razor principle: "Senses are not to be multiplied beyond necessity" [32, p. 47].

Negating such predications normally denies the lower bound: to say that something is not possible is to say that it's impossible, i.e. less than possible. When it is the upper bound that appears to be negated (*It's not possible, it's NECESSARY*), a range of linguistic and logical evidence indicates that what we are dealing here with is an instance of the metalinguistic (or echoic) use of negation, in which the negative particle is used to object to any aspect of an alternate (actual or envisaged) utterance, including its conventional and conversational implicata, register, morphosyntactic form or pronunciation (Horn [47, Chapter 6]; Carston [12]). If it's hot, it's (a fortiori) warm, but if I know it's hot, the assertion that it's warm can be mentioned and rejected as (not false but) insufficiently informative: "It's not WARM, it's HOT!", "You didn't see SOME of my children, you saw ALL of them", "I didn't BELIEVE that p, I KNEW it".

Such uses of negation effectively scope over other varieties of implicature. Thus, in "It is not the case that she got married and had a child" (after Grice [32, p. 8]), the utterer may be denying the implicature (generated by the "Be orderly" submaxim) that the events referred to occurred in the order in which they mentioned, conveying something like "Rather, she had a child and [then] got married". If this can be analyzed away as metalinguistic negation, there is no need to depart from the syntax and semantics of System G ("$\sim(p \& q)$").

Or with disjunction: "It is not the case that my wife is in Oxford or in London – she's in Oxford, as you well know". This too can be understood metalinguistically, with the negation scoping over the implicature responsible for the non-truth-functional condition on the felicity (but not the truth!) of *p or q* statements that the speaker should not be in a position to assert either disjunct individually. The logical form is again, however, the classical one: "$\sim(p \vee q)$".

Grice considers a related phenomenon what he calls "substitutive disagreement", as opposed to truth-functional "*contradictory* disagreement" [32, p. 64]. Once again, we have a negation that does translate into "$\sim p$". In such cases, "I am not contradicting what you say. It is rather that I wish not to assert what you have asserted, but to substitute a different statement which I regard as preferable in the circumstances" [32, p. 64]. This situation arises with both disjunctions and conditionals. Thus,

> If you say "X or Y will be elected", I may reply "That's not so; X or Y *or* Z will be elected." Here . . . I am rejecting "X or Y will be elected" not as false but as unassertable.   (Grice [32, p. 82])

I do not thereby deny the proposition you have expressed (which would amount to a commitment to the electoral failure of both X and Y), but reject your assertion as epistemologically unwarranted (in ruling out candidate Z). Grice emphasizes that "the possibility of speaking in this way gives no ground for supposing that "or" is not truth-functional"; it does, however, introduce a subtler question on the truth-functionality of "not".

Similarly, to negate a conditional is typically not to assert '$\sim(p \to q)$': "to say 'It is not the case that if Jones is given penicillin, he will get better' might be a way of suggesting that the drug might have no effect on X at all" rather than committing the speaker to the truth of "Jones will be given penicillin" and the falsity of "Jones will get better".

> Sometimes the denial of a conditional has the *effect* of a *refusal to assert* the conditional in question, characteristically because the denier does not think that there are adequate *non*-truth-functional grounds for such an expression. In such a case, he denies, in effect, what the thesis represents as an implicature of the utterance of the unnegated conditional. (Grice [32, p. 81])

As Grice notes in his 'Retrospective Epilogue', this approach requires an acknowledgement of the possibility that conversational implicatures need not take wide scope, in particular with respect to negation, a possibility Grice endorses with some diffidence:

> When a sentence which used in isolation standardly carries a certain implicature is embedded in a certain linguistic context, for example appears within the scope of a negation-sign, must the negation sign be interpreted only as working on the conventional import of the embedded sentence, or may it on occasion be interpreted as governing not the conventional import but the non-conventional implicatum of the embedded sentence? Only if an embedding operator may on occasion be taken as governing not the conventional import but the non-conventional implicatum standardly carried by the embedded sentence can the first version of my account of such linguistic phenomena as conditionals and definite descriptions be made to work. The denial of a conditional needs to be treated as denying not the conventional import but the standard implicatum attaching to an isolated use of the embedded sentence.   (Grice [32, p. 375])

Neo-Traditionalists such as Strawson have remained unconvinced by the argument, refusing to concede that the issue of the divergence between "$\sim$" and "not" was now settled, especially with regard to negated conditionals: "The Gricean, though perhaps with a slight air of desperation, could reply that one who denies the condition has no interest in denying what it conventionally and literally means, but only in denying what it standardly and conversationally implies" [97, p. 15]. Nor is it clear that treating the negatum as a conventional implicatum rather than a conversational one would minimize the air of desperation in the eyes of a Strawson.

Another case that allows for reanalysis involving implicature, as we have noted, concerns the aforementioned case of the disappearing presupposition.

> As far as I can see, in the original version of Strawson's truth-gap theory, he did not recognized any particular asymmetry, as regards the presupposition that there is a king of France, between the two sentences, *The king of France is bald* and *The king of France is not bald*; but it does seem plausible to suppose that there is such an symmetry. I would have thought that the implication that there is a king of France is clearly part of the conventional force of *The king of France is bald*; but that his is not clearly so in the case of *The king of France is not bald*. . .An implication that there is a king of France is often carried by *[]*, but it tempting to suggest that this is implication is a matter of conversational implicature.
> (Grice [32, p. 270])

One key here is the application of the tests for implicature: cancellability (*The king of France isn't bald* – (*because*) *there is no king of France*) and non-detachability. To demonstrate the latter, Grice notes that whether we use the frame "It is not the case that the King of France is bald", "It is false that the king of France is bald", or "It is not true that the king of France is bald", "Many of what seem to be other ways of saying, approximately, what is asserted by ["The king of France is not bald"] also carry the existential implicature" (Grice [32, p. 271]). Such tests can be taken to support those proposals, including those employing Grice's bracketing device, concur in distinguishing the positive expression (in which existence is entailed) from the negative (in which it is non-monotonically implicated).

18. Implicature and negation: Subcontrariety and the three-cornered square

Scalar implicature plays another role in the expression of negation in natural language. A Gricean understanding of the relationship between strong and weak scalar values helps motivate a natural account of the lexicalization asymmetry of the

Square of Opposition, an asymmetry – a.k.a. the Story of ∗**O** – displayed in tabular form below (cf. Horn [43, Chap. 4]; Horn [47, §4.5]; Levinson [65]; Horn [53,54]).

	DETERMINERS/QUANTIFIERS	QUANT. ADVERBS	BINARY QUANTIFIERS	CORRELATIVE CONJUNCTIONS	BINARY CONNECTIVES
A: **I:** **E:**	all α, everyone some α, someone no α, no one (= all ¬/¬ some)	always sometimes never (= always ¬)	both (of them) one (of them) neither (of them) (= both ¬/¬ either)	both…and either…or neither…nor (= [both…and]¬)	and or nor (= and ¬)
O:	*nall α, *neveryone (= some ¬/¬ all)	*nalways (= ¬ always)	*noth (of them) (= either ¬/¬ both)	*noth…nand (= [either…or]¬)	*nand (= and ¬/¬ or)

In fact, this asymmetry was recognized for Latin by St. Thomas, who observed that whereas in the case of the universal negative (**A**) "the word 'no' *[nullus]* has been devised to signify that the predicate is removed from the universal subject according to the whole of what is contained under it", when it comes to the particular negative (**O**), we find that

> there is no designated word, but 'not all' *[non omnis]* can be used. Just as 'no' removes universally, for it signifies the same thing as if we were to say 'not any' [i.e. 'not some'], so also 'not all' removes particularly inasmuch as it excludes universal affirmation.

> (Aquinas, *in Arist. de Int.*, Lesson X, Oesterle, 1962, pp. 82–83)

The Gricean model offers a persuasive motivation for this asymmetry. Although *some* does not contribute the same semantic content as *some not* (= *not all*), the use of either of the two values typically results in a speaker communicating the same information in a given context, viz. 'some but not all'. The relation of mutual quantity implicature holding between the positive and negative subcontraries results in the superfluity of one of the two for lexical realization and the functional markedness of negation predicts that the unlexicalized subcontrary will always be the one with an **O** rather than **I** meaning. The existence of a lexicalized **O** form implies the existence of a lexicalized **E** counterpart but not vice versa. Additional evidence (see above sources) indicates that even when both forms are attested, as with the negative modalities *can't, mustn't, shouldn't* (**E**) vs. *needn't* (**O**), the lexicalized **E** form tends to be more opaque and semantically and distributionally less constrained than its **O** counterpart. This pragmatic, implicature-based account of the 'three-cornered square' is more general and more explanatory than rival theories that either dismiss the asymmetry as uninteresting or restrict it to the determiners and quantificational operators while bypassing other operator types (e.g. connectives, adverbs, and modalities) along with intermediate values that can be mapped onto the Square of Opposition. (See Horn [47] and Horn [54] for details and Jaspers [57] and Seuren [89] for alternative treatments and related discussion.)

19. Negation and denial

A topic that concerned mathematicians such as Griss [35] is whether weak negation can be made sense of. He thought not, whence his 'negationless system'. System G is not like that. But the issue of how denial and negation interact is a further problem that must be handled via implicature.

For Grice, one key step occurs when the logical squiggle is internalized into a psychological attitude operator. Grice sets the question in dealing with how negation interacts with the scope of various psychological attitudes in his presidential address to the American Philosophical Association, 'Method in Philosophical Psychology', where Grice explores what an account of negation in terms of the psychological attitude of denial might look like. This has a connection with topics that concerned Austin and specifically with the pattern of inferences in psychological contexts (e.g. If one believes that not-not-*p*, does one believe that *p*?). Undertaking the exploration of "not" and '∼' in connection with propositional attitudes is an attempt to examine negation in models of belief- and knowledge-based reasoning. Grice views this as a metaphysical issue. "References to such psychological states will be open to logical operations such as negation" [33, p. 146].

The internalization of the logical operation of negation within the scope of a psychological attitude operator may be seen as involving various stages. Grice distinguishes four:

> At the first stage we have some initial concept, like that expressed by 'not'. We can think of it as, at this stage, an intuitive or unclarified element of our conceptual vocabulary. At the second stage, we reach a specific mental state, in the specification of which it is possible, though maybe not necessary to use the name of the initial concept as an adverbial modifier, 'not-thinking' (or 'rejecting', or 'denying'). This specific state may be thought of as bound up with, and indeed as generating, some set of responses to the appearance on the scene of an instantiation of the initial concept. At the third stage, a reference to this specific state is replaced by a more general psychological verb, together with an operator corresponding to the particular specific stage which appears within the scope of a general verb ['accept'], but is still allowed only maximal scope within the complement of the verb, and cannot appear in sub-clauses ['thinking not-*p*', 'accepting not-*p*']. At the fourth stage, the restriction imposed by the demand that the operator at stage three should be scope-dominant within the complement of the accompanying verb is removed [...]. [30, p. 98]

While Grice uses "He is not lighting his cigarette with a 20-dollar bill" to make an indirect reference to Bradley on negation, he is positively concerned elsewhere with the ontology and logic of non-events (cf. Gale [22], Horn [47, pp. 54–55]):

In many cases, what are to be counted as actions are realized, not in events or happenings, but in non-events or non-happenings. What I do is often a matter of what I do not prevent, what I allow to happen, what I *refrain from* or *abstain from* bringing about – what, when it comes about, I ignore or disregard. I do not interrupt my children's chatter; I ignore the conversational intrusions of my neighbour; I omit the first paragraph of the letter I read aloud; I hold my fire when the rabbit emerges from the burrow, and so on... Such omissions and forbearances might lead to the admission of negative events, or negative happenstances, with one entity filling the 'event slot' if on a particular occasion I go to Hawaii, or wear a hat, and another entity filling that slot if on that occasion I do not go to Hawaii, or do not wear a hat. (Grice [30, p. 22])

It seems it would be again the conversational context that would advise us as to whether and how to apply the "∼" in a given formalization.

One simple set of implicatures concern *duplex negatio affirmat*. Following Bishop Lowth, Edward Bentham recites the standard principle: "According to the idiom of some languages (Latin and English), two negative particles destroy each others force, and make the proposition affirmative" (cf. Horn [49] for additional references and elaboration). But if, as Jevons avers, "Negatives signify the absence of a quality" [59, p. 22], a double negative would signify a double absence, which does not obviously yield a presence. The point had been noted by Strawson: "This identification [of *not* and "∼"], then, involves only those minimum departures from the logic of ordinary language which must always result from the formal logician's activity of codifying rules with the help of verbal patterns: viz., the adoption of a rigid rule when ordinary language permits variations and deviations from the standard use ("$\sim \sim p \supset p$", "$p \vee \sim p$")".

The issue was again raised by L.J. Cohen [14], who argues from the existence of negative concord (in languages like Italian, present-day non-standard dialects of English, or earlier standard English, in which e.g. "I don't want nothing" expresses the same proposition as that expressed in standard English by "I don't want anything") to the incoherence of Grice's identification of "∼" with "not". In such cases *Duplex negatio negat*, although here again, the question arises for the grammarian (if not the logician) as to why a speaker would go out of her way to employ double negation rather than expressing a simple negative (or, in the *Duplex negatio affirmat* cases, a simple affirmative) directly; cf. Horn [54]. In any case, Cohen's objection seems ill-taken; the natural Gricean rebuttal is to treat negation as being an abstract syntactic constituent or element of semantic representation, rather than to limit the focus to the superficial occurrence of equivalents of "not" in ordinary language (cf. Gazdar [24, pp. 63–64]).

Implicature also plays a role in motivating the choice of the standard "∼" operator, yielding a truth array of ⟨0 1⟩, over the three other possible unary truth-functors, those yielding arrays of ⟨1 0⟩, ⟨1 1⟩, and ⟨0 0⟩. The choice is based on the fact that "∼" is the only one-place operator that, in collaboration with the Co-Operative Principle and its maxims, yields an intuitively plausible system. The other unary functors can be shown to be either redundant or semantically incoherent [24, p. 76]. System G narrows propositional negation to the contradiction function, while deriving the strengthened reading of contrariety, where appropriate, as an implicature (along the lines of Horn [47, Chapter 5]).

20. Negation and falsity

One favorite issue for the Oxford Play-Group was the extent to which negation could be identified with falsity. While Grice himself often took "It is false that *p*" and "It is not the case that *p*" to count as moves in the same game [32, p. 271], the situation in reality is somewhat more complex.

For centuries, one popular method for eliminating negation has proceeded by identifying it with and "reducing" it to falsity. One question is whether such a "reduction", if it could be accomplished, would really accomplish anything. But there are in any case strong grounds for rejecting the proposed identification in the first place, without even considering its role within a reductionist program.

That negation and falsity might be conflated, and eventually confused, with each other should not be surprising. Aristotle discusses 'being in the sense of true and non-being in the sense of false' (*Met*. 1027b18), and he seems to explicitly link the negated copula with falsity (as the affirmative copula is linked with truth):

"To be" and "is" mean that something is true, and "not to be" that it is not true but false ... For example, in "Socrates is musical", the "is" means that it is true that Socrates is musical, and in "Socrates is not-white", that this is true; but in "the diagonal is not commensurate with the side" the "is not" means that it is false that the diagonal is commensurate with the side.

Within Aristotle's simple correspondence theory of truth, truth and falsity are interrelated as the two terms of a contradictory opposition. But contradictory negation does not reduce to falsity, since negation and falsity are about different things and operate on different levels: "A falsity is a statement of that which is that it is not, or of that which is not that it

is; and a truth is a statement of that which is that it is, or of that which is not that it is not" (Met. 1011b25-27; cf. De Int. 18b2-4).

The equation of negation (often specifically "logical" negation) and falsity is a frequent maneuver among the Idealists of the late 19th and early 20th century, often going hand-in-hand with a view of negation as a second-order comment on a first-order affirmation, and/or as a more subjective act than simple affirmation:

> To say "A is not B" is merely the same as to deny that "A is B", or to assert that "A is B" is false.   (Bradley [10, p. 118])

> "A is not B" means "it is false, it must not be believed that A is B"... Immediately and directly, the negation is a judgment concerning a positive judgment that has been essayed or passed.   (Sigwart [90, p. 122])

> a is not b = that a is b is false.   (Baldwin [5, p. 147])

> The pure negative judgment 'A is not B' is equivalent in every case to 'it is false that A is B'... 'Snow is not black' is a shorthand statement for 'snow is black is an erroneous judgment'. (Wood [104, p. 421])

For Russell [86, p. 81], too, every negation is a shorthand for some assertion of falsity: "It is unnecessary to have the two words "false" and "not", for, if *p* is a proposition, "*p* is false" and "not-*p*" are strictly synonymous".

Within the modern logical (and linguistic) tradition, the temptation to identify negation and falsity stems directly from the Fregean line that all negation is propositional and reducible to a suitably placed *'it is not true that...'*. In multi-valued logics, there is one form of negation (internal, strong, choice) which does not display the logic of contradictory opposition, being governed by the Law of Non-Contradiction but not the Law of Excluded Middle (see Horn [52]). Within such approaches, at least some negations cannot be "reduced to" assertions of falsity. Similarly, there may be illocutionary distinctions between the negation of a proposition and the statement that proposition is false, as in Heinemann's differentiation [37, p. 143] of not-*p* ('*p* is valid') from '*p* is not valid'. But even within classical two-valued logic itself, there are sufficient grounds for rejecting the identification of negation and falsity. Philosophers as diverse as Frege [21], Austin [2], Quine [82], and Geach [25] have observed that the identification of *not* and *false* results from a confusion of language and metalanguage.

Here is Austin's response to the view (represented by Ayer) that 'is true' and 'is false' are logically superfluous:

> An important point about this view is that it confuses falsity with negation: for according to it, it is the same thing to say 'He is not at home' as to say 'It is false that he is at home'... Too many philosophers maintain, when anxious to explain away negation, that a negation is just a second order affirmation (to the effect that a certain first order affirmation is false), yet, when anxious to explain away falsity, maintain that to assert that a statement is false is just to assert its negation (contradictory)... Affirmation and negation are exactly on a level, in this sense, that no language can exist which does not contain conventions for both and that both refer to the world equally directly, not to statements about the world.
>
> (Austin [2, pp. 128–129])

Quine [82, pp. 27–28]) is also at pains to distinguish the predicates 'is false' and 'is true', which are used to speak about statements, from the connective "∼", which is used to make statements. *'Jones is ill' is false* is a statement about the statement *Jones is ill*, while ∼(*Jones is ill*), read 'Jones is not ill', is a statement about Jones. Quine lays the mistaken identification of '∼' with falsehood at Whitehead and Russell's door, but the underlying mistake both antedates and survives the *Principia* – as does its rectification. The Stoics were careful to make the same distinction as Quine, that 'between the negation of a proposition and a (metalinguistic) statement that the proposition is false'; these two operations played different roles in the Stoics' account of syllogistic reasoning [67, pp. 64–65].

In the same vein as Austin and Quine, Geach [25, p. 76] inveighs against the 'widespread mistake' of assuming that 'the negation of a statement is a statement that statement is false, and thus is a statement *about* the original statement and logically secondary to it'. The error of this approach emerges clearly when we look at non-declaratives: "'Do not open the door!" is a command on the same level as "Open the door!" and does not mean (say) "Let the statement that you open the door be false!"'

For symmetricalists like Austin, Geach, and Ayer, conclusions as to the secondary status of negative statements with respect to affirmatives often betoken a confusion of meaning with 'use'. This is how Ayer – and Grice – diagnose Strawson's account of negation. In Ayer's words,

> From the fact that someone asserts that it is not raining one is not entitled to infer that he has ever supposed, or that anyone has ever suggested, that it is, any more than from the fact that someone asserts that it is raining one is entitled to infer that he has ever supposed, or that anyone has ever suggested, that it is not. No doubt negative forms of expression are very frequently used to deny some previous suggestion; it may even be that this is their most common use. But whatever the interest of this fact it cannot be the ground of any viable distinction between different types of statement.
>
> (Ayer [4, p. 39])

Ayer goes on to challenge the oft-maintained epistemological worth-less-ness (if not worthlessness) of negative statements: "Why should it not be allowed that the statement that the Atlantic Ocean is not blue is as much a description of the Atlantic as the statement that the Mediterranean Sea is blue is a description of the Mediterranean?" While the negative might well be less informative than its affirmative counterpart, "to say that a description is relatively uninformative is not to say that it is not a description at all" (Ayer [4, p. 47]).

"Perhaps", Ayer rhetorically wonders, "there are psychological grounds for sequestering negations as a special class of statements used only for rebuttals or denials? But any statement can be so used" [4, p. 38]. Along the same lines, Grice [32], while providing "The man at the next table is not lighting his cigarette with a $20 bill" as an instance of a negation that sounds odd if the corresponding affirmative has not been entertained, also brings up cases like "I went to the meeting of my own free will", "I remember my own name", and "Your wife is faithful", where it is the affirmative that is inappropriate in the absence of a specially marked context. When a positive sentence is less informative than the corresponding negation, it is the positive that is odder or presuppositionally richer. (See Horn [47, §3.3.1] for a neo-Gricean derivation of this asymmetry.) Thus, while Strawson may be correct in claiming that "the standard and primary use" of negatives is "to correct and contradict", this cannot be a definitional criterion of the property of negation; use is not meaning.

21. Envoi: A Gricean program for negation and implicature

While absent from otherwise complex systems of animal communication, negation is a sine qua non of every human language. Indeed, if our species can be dubbed *homo linguisticus*, it is negation that makes us fully human, providing us with the capacity to deny, to contradict, to misrepresent, to lie, and to convey irony. The apparent simplicity of logical negation as a one-place operator that toggles truth and falsity belies the intricate complexity of the expression of negation in natural language. For these reasons, the form and function of negation has engaged the interest and often the passion of scholars for thousands of years. But it is arguably the contributions of Paul Grice that have enabled us to sort out the contributions of general principles of communication and rational interchange to the specific habits of negating and denying that show up in language.

Grice departs from his logician predecessors and from the ordinary language philosophers in the Oxford Play-Group in subsuming his view of linguistic cooperation in the conversational enterprise within a general theory of rationality (see Kasher [62]). But, as he also reminds us, "It is irrational to bite off more than you can chew whether the object of your pursuit is hamburgers or the Truth" [32, p. 369]. Ever true to the spirit of the Quantity maxim, Grice was always rational enough to bite off neither more nor less than his appetite permitted. But no man lives by meat alone, much less a philosopher of language large enough to bestride the warring camps of Russell's Modernists and Strawson's Neo-Traditionalists; bread is important as well. So it is meet that a healthy portion of the Gricean oeuvre consists not of solutions but of problems, questions, and menus. For, as Grice reminds us elsewhere in offering a defense of absolute value admittedly "bristling with unsolved or incompletely solved problems" [30, p. 106], "If philosophy generated no new problems it would be dead... Those who still look to philosophy for their bread-and-butter should pray that the supply of new problems never dries up". As the history of work on negation eloquently demonstrates, there is no danger of that dread eventuality coming to pass any time soon.

Acknowledgements

Readers interested in current work on negation in philosophy and (especially) linguistics are urged to peruse the papers and extensive bibliography in Horn [55]. For more background on the history of negation in logic and philosophy, see Horn [47] (and the 2001 reissue with updated introductory and bibliographic material) and Brann [11]. The interaction of negation, contradiction, and the Square of Opposition are surveyed in the Stanford Philosophy of Logic entries by Horn [52] and Parsons [76].

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