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Variation of mass in primordial nucleosynthesis as a test of induced matter brane gravity

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ABSTRACT

The variation of mass in induced matter theory using Ceroch–Stewart–Walter perturbations of submanifolds is redefined. It is shown that the deviation of primordial Helium production due to a variation on the difference between the “rest” mass of the nucleus is in agreement with induced matter brane gravity.

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1. Introduction

The aim of purely geometrical description of all physical interactions as well as that of a geometrical origin of matter as dreamed by Einstein [1] has attracted a lot of interest. A gravitational theory in which the matter is absorbed into the field itself, is called unified field theory. There exists various extensions of Einstein’s framework extracting matter from pure geometry. Much of the works trying to extend our knowledge of gravitational and other interactions has been concentrated on developing theories in more than four dimensions, like supergravity [2], superstrings [3] and various Kaluza–Klein (KK) theories [4]. In this theories the added extra dimensions are usually taken to be compact. To solve the problem of non-observability of the small “internal” space spanned by the extra dimensions, it is usually assumed that the size of the extra dimensions are of the order of Planck length, being itself a consequence of dynamical evolution of the higher-dimensional universe, as a result of the introducing the higher-dimensional stress–energy tensor. On the other hand, in [5], the authors show that the gravitational models with compact extra dimensions, linearly perturbed Einstein equations are in conflict with observation. There exist another extensions of Einstein’s theory in which our spacetime is a submanifold (brane) embedded in a higher-dimensional manifold (bulk). A revised KK approach in this direction in which the higher-dimensional stress–energy ten-

sor is taken to be identically zero is the Wesson induced matter theory (IMT) [6]. The starting points are the vacuum 5D Einstein gravitational field equations,

$$\mathcal{R}_{AB} = 0 \quad (A, B = 0, \dots, 4), \quad (1)$$

where \mathcal{R}_{AB} is the Ricci scalar of the bulk space. The induced field equations on the brane becomes [7]

$$G_{\mu\nu} = Q_{\mu\nu} - \varepsilon \mathcal{E}_{\mu\nu} \quad (\mu, \nu = 0, \dots, 3), \quad (2)$$

where $\mathcal{E}_{\mu\nu}$ is electric part of Weyl tensor of the bulk space and $Q_{\mu\nu}$ is defined as

$$Q_{\mu\nu} = \varepsilon \left[K_{\mu}^{\gamma} K_{\gamma\nu} - K K_{\mu\nu} - \frac{1}{2} (K_{\alpha\beta} K^{\alpha\beta} - K^2) g_{\mu\nu} \right], \quad (3)$$

where $g_{\mu\nu}$ is the induced metric on the brane, $\varepsilon = \pm 1$ denotes the signature of the extra dimension, $K_{\mu\nu}$ is the extrinsic curvature and K is its trace. The reason that this theory is called induced matter theory (IMT) is that the effective 4D matter is a consequence of the geometry of the bulk [7]

$$-8\pi G_N T_{\mu\nu} = Q_{\mu\nu} - \varepsilon \mathcal{E}_{\mu\nu}. \quad (4)$$

One of the outcomes of IMT is that the “rest mass” of particles varies from point to point in spacetime, in agreement with the ideas of Mach. To show the variation of mass, Wesson by using dimensional analysis [8] introduced the following relation between of fifth coordinate and the mass of the test particles

$$x^4 = \frac{G_N m}{c^2}, \quad (5)$$

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where m is the “rest” mass of a test particle and x^4 denotes the fifth dimension. Hence, according to the above equation if we consider that a variation of the rest mass of particles had occurred between the epoch of primordial nucleosynthesis and present, we can compute the deviation in the ^4He production from Hot Big Bang model production due to this fact. In [9] the authors show that if we use the relation (5) to obtain the variation of mass from primordial nucleosynthesis and our time, and compare with variation of mass obtained from nucleosynthesis bounds on the variation of the mass, the results are not in agreement with each other. They used the 5D metric with compact extra dimension. In this Letter we reobtain the variation of mass in IMT according to the recent developments in this theory and in a simple model it is showed that by correct defining of the induced mass, the variation of mass obtained from IMT is in agreement with mass variation bound obtained from Hot Big Bang.

2. Test particle dynamics and induced mass

In this section we wish to derive the 4D geodesic equations and induced mass of a test particle. To doing this, we start with the induced parallel displacement in 4D. According to the recent developments in IMT, the assumption is that our spacetime can be isometrically and locally embedded in a Ricci-flat 5D spacetime. In contrast to the Randall and Sundrum brane models where the matter field is confined to the fixed brane, in IMT there is no mechanism to confine induced matter field exactly on a specific brane. The authors of [10] and [11] show that to confine test particles on a brane it is necessary to exist either a non-gravitational centripetal confining force with an unknown source, or assume that our brane is totally geodesic in which case it is impossible to embed an arbitrary brane in the bulk space. In IMT however, if the induced matter field satisfies “Machian strong energy condition” then the test particles become stable around the fixed brane [12]. Finally, we can say that in IMT at the large scales we have matter field confined to a fixed brane, say $\bar{g}_{\mu\nu}$ that satisfies induced Einstein field equations and at small scales we find the matter fields having small fluctuations around this brane [13]. If we denote the metric of this brane by $g_{\mu\nu}$, then it becomes acceptable to assume that this new brane is a perturbation of the original one $\bar{g}_{\mu\nu}$ [14]. In the following we briefly review the relation of geometrical objects in these two branes, for more details see [15].

Consider the background manifold \bar{V}_4 isometrically embedded in V_5 by a map $\mathcal{Y}: \bar{V}_4 \rightarrow V_5$ such that

$$\mathcal{G}_{AB}\mathcal{Y}^A{}_{,\mu}\mathcal{Y}^B{}_{,\nu} = \bar{g}_{\mu\nu}, \quad \mathcal{G}_{AB}\mathcal{Y}^A{}_{,\mu}\mathcal{N}^B = 0, \quad \mathcal{G}_{AB}\mathcal{N}^A\mathcal{N}^B = \varepsilon \quad (6)$$

where \mathcal{G}_{AB} ($\bar{g}_{\mu\nu}$) is the metric of the bulk (brane) space V_5 (\bar{V}_4) in an arbitrary coordinate with signature $(-, +, +, +, \varepsilon)$, $\{\mathcal{Y}^A\}$ ($\{x^\mu\}$) are the basis of the bulk (brane) and \mathcal{N}^A is a normal unit vector orthogonal to the brane. Perturbation of \bar{V}_4 in a sufficiently small neighborhood of the brane along an arbitrary transverse direction ζ [29]

$$\mathcal{Z}^A(x^\mu, x^4) = \mathcal{Y}^A + (\mathcal{L}_\zeta \mathcal{Y})^A, \quad (7)$$

where \mathcal{L} represents the Lie derivative and x^4 is a small parameter along \mathcal{N}^A parameterizing the extra noncompact dimension. By choosing ζ orthogonal to the brane we ensure gauge independency and have perturbations of the embedding along a single orthogonal extra direction $\bar{\mathcal{N}}$, giving the local coordinates of the perturbed brane as

$$\mathcal{Z}^A{}_{,\mu}(x^\nu, x^4) = \mathcal{Y}^A{}_{,\mu} + x^4 \bar{\mathcal{N}}^A{}_{,\mu}(x^\nu). \quad (8)$$

In a similar manner, one can find that since the vectors $\bar{\mathcal{N}}^A$ depend only on the local coordinates x^μ , they do not propagate along

the extra dimension. The above assumptions lead to the embedding equations of the perturbed geometry

$$\begin{aligned} \mathcal{G}_{\mu\nu} &= \mathcal{G}_{AB}\mathcal{Z}^A{}_{,\mu}\mathcal{Z}^B{}_{,\nu}, & \mathcal{G}_{\mu 4} &= \mathcal{G}_{AB}\mathcal{Z}^A{}_{,\mu}\mathcal{N}^B, \\ \mathcal{G}_{AB}\mathcal{N}^A\mathcal{N}^B &= \mathcal{G}_{44}. \end{aligned} \quad (9)$$

If we set $\mathcal{N}^A = \delta_4^A$, then the line element of the bulk space in the Gaussian frame (9) becomes

$$dS^2 = \mathcal{G}_{AB}d\mathcal{Z}^A d\mathcal{Z}^B = g_{\mu\nu}(x^\alpha, x^4)dx^\mu dx^\nu + \varepsilon(dx^4)^2, \quad (10)$$

where

$$g_{\mu\nu} = \bar{g}_{\mu\nu} - 2x^4 \bar{K}_{\mu\nu} + (x^4)^2 \bar{g}^{\alpha\beta} \bar{K}_{\mu\alpha} \bar{K}_{\nu\beta}, \quad (11)$$

is the metric of the perturbed brane, so that

$$\bar{K}_{\mu\nu} = -\mathcal{G}_{AB}\mathcal{Y}^A{}_{,\mu}\mathcal{N}^B{}_{,\nu}, \quad (12)$$

represents the extrinsic curvature of the original brane. Any fixed x^4 signifies a new perturbed brane, enabling us to define an extrinsic curvature similar to the original one by

$$K_{\mu\nu} = -\mathcal{G}_{AB}\mathcal{Z}^A{}_{,\mu}\mathcal{N}^B{}_{,\nu} = \bar{K}_{\mu\nu} - x^4 \bar{K}_{\mu\gamma} \bar{K}^\gamma{}_{\nu}. \quad (13)$$

The above perturbation is needed in the reminding of the Letter. To obtain induced parallel displacement and the mass, consider an arbitrary vector in 5D bulk space X_A that has a 4D counterpart in a brane in which the vector X_μ is defined. These two vectors are related by the following inducing relation

$$X_\mu = \mathcal{G}_{AB}X^A \mathcal{Z}^B{}_{,\mu}. \quad (14)$$

Let us consider an infinitesimal parallel displacement of a vector in the bulk space

$$dX_A = -\bar{\Gamma}_{AC}^B X_B d\mathcal{Z}^C, \quad (15)$$

where $\bar{\Gamma}_{AC}^B$ denotes the Christoffel symbols of the bulk space. Now using Eqs. (14) and (15), the induced parallel displacement of X_μ is

$$dX_\mu = \mathcal{G}_{AM}\bar{\Gamma}_{BC}^M \mathcal{Z}^B{}_{,\mu} X^A d\mathcal{Z}^C + \mathcal{G}_{AB}X^A d\mathcal{Z}^B{}_{,\mu}. \quad (16)$$

As the bulk space may be mapped either by $\{\mathcal{Z}^A\}$ or by local coordinates of brane and extra dimension, one can write

$$d\mathcal{Z}^C = \mathcal{Z}^C{}_{,\alpha} dx^\alpha + \mathcal{N}^C dx^4. \quad (17)$$

Inserting decomposition (17) into the expression for the parallel displacement (16) we obtain

$$dX_\mu = \mathcal{G}_{AM}(\bar{\Gamma}_{BC}^M \mathcal{Z}^B{}_{,\mu} X^A + X^A \mathcal{Z}^M{}_{,\mu,C})\{\mathcal{Z}^C{}_{,\alpha} dx^\alpha + \mathcal{N}^C dx^4\}. \quad (18)$$

In the Gaussian frame (10) this may be rewritten as

$$dX_\mu = \Gamma_{\mu\alpha}^\beta X_\beta dx^\alpha + K_{\mu\alpha} X_4 dx^\alpha - K^\beta{}_{\mu} X_\beta dx^4, \quad (19)$$

where $\Gamma_{\mu\alpha}^\beta$ denotes the Christoffel symbols of the brane. In the particular case where the induced parallel displacement is discussed, we use 5-velocity vector

$$X^A = \frac{d\mathcal{Z}^A}{dS} = \mathcal{Z}^A{}_{,\alpha} \frac{dx^\alpha}{dS} + \mathcal{N}^A \frac{dx^4}{dS}, \quad (20)$$

where dS is line element in the bulk space. In this case X^A represents 5-velocity in the bulk. But it is not clear that the parameterization of path of test particles in the bulk and brane are proportional. Accordingly, we use in general the different parameterization on the brane. Hence, the corresponding induced component of X^A according to Eqs. (14) and (20) becomes

$$X_\mu = \mathcal{G}_{AB}X^A \mathcal{Z}^B{}_{,\mu} = e g_{\alpha\mu} u^\alpha, \quad (21)$$

where $e = \frac{d\lambda}{ds}$ and $u^\alpha = \frac{dx^\alpha}{d\lambda}$ is 4-velocity of test particle on the brane. Now using (19), the induced parallel displacement becomes

$$\frac{du^\mu}{d\lambda} + \Gamma_{\alpha\beta}^\mu u^\alpha u^\beta = -\frac{\dot{e}}{e} u^\mu + 2K_{\alpha}^{\mu} u^\alpha u^4, \quad (22)$$

where $u^4 = \frac{dx^4}{d\lambda}$ and overdot denotes derivative respect to λ . Repeating the above process with respect to induced normal component of 5-velocity, we obtain

$$dX_4 = \mathcal{G}_{AB} X^A (\mathcal{Z}_{,4}^M \bar{\Gamma}_{MC}^B + \mathcal{Z}_{,4,C}^B) \{ \mathcal{Z}_{,\alpha}^C dx^\alpha + \mathcal{N}^C dx^4 \}. \quad (23)$$

In the Gaussian frame the above equation takes the following form

$$dX_4 = -g_{\mu\nu} X^\mu K_\nu^4 dx^\alpha. \quad (24)$$

Hence the equation of motion of the test particle along the normal to the brane direction becomes

$$\frac{du^4}{d\lambda} = -K_{\mu\nu} u^\mu u^\nu - \frac{\dot{e}}{e} u^4. \quad (25)$$

In the continuum let us consider the square of the length $X^2 := g^{\mu\nu} X_\mu X_\nu$. Its change under parallel displacement as

$$dX^2 = g^{\mu\nu}{}_{,\gamma} X_\mu X_\nu dx^\gamma + g_{,4}^{\mu\nu} X_\mu X_\nu dx^4 + 2g^{\mu\nu} X_\mu dX_\nu. \quad (26)$$

Making use of $g^{\mu\nu}{}_{,\gamma} = -\Gamma_{\gamma\beta}^\mu g^{\nu\beta} - \Gamma_{\gamma\beta}^\nu g^{\mu\beta}$ and $g^{\mu\nu}{}_{,4} = 2K^{\mu\nu}$, we obtain from Eqs. (19) the change of the squared length of the 4-vector

$$dX^2 = 2X^\mu X^4 K_{\mu\alpha} dx^\alpha. \quad (27)$$

Thus, in general case, the brane possesses a non-integrable geometry [16,17], and only when the original 5D vectors do not have extra components, or when the extrinsic curvature vanishes one has a pseudo-Riemannian brane. In the non-integrable geometry, there is a well-known method to measure the ‘‘length curvature’’ $F := dA$, $A_\mu = K_{\mu\nu} X^\nu X^4$ by means of the so-called ‘‘second clock effect’’. Let us assume that, we have two standard clocks which are close to each other and synchronized in the beginning. Now if these two clocks are separated for a while and brought together again later, they will be out of synchronization in general. This is a well-known effect from general and special relativity and called ‘‘first clock effect’’ and often called the twin paradox. The second clock effect exists if, in addition, the units of the two clocks are different after their meeting again. In Lorentzian spacetime there is no second clock effect for standard clocks. Assuming that atomic clocks are standard clocks, then in general, after the above argument, they have different properties. To solve this problem Dirac [18] assumed that in practice we have two different intervals: ds_A and ds_E . The interval ds_A is referred to atomic units; it is not affected by A . The Einstein interval ds_E is associated with the field equations and the non-integrable geometry. Another solution to the problem was given by Wood and Papini [19]. In their approach, the atom appears as a bubble. Outside one has the non-integrable spacetime, and on the boundary surface and in the interior of the atom we have $A_i = 0$. The static spherical entity is filled with ‘‘Dirac matter’’ satisfying equation of state like cosmological constant. Finally the third method is discussed by Audretsch [20] and Flint [21]. In this approach, the above solutions are classified as non-quantum-mechanical ways and we can set second clock effect as a quantum effect.

To find the induced mass on the brane, we project 5-momenta P_A into the brane. This projection is done by vielbeins $\mathcal{Z}_{,\mu}^A$, then

$$p_\mu = \mathcal{G}_{AB} P^A \mathcal{Z}_{,\mu}^B. \quad (28)$$

For a 4D observer, the motion is described by 4-momenta (28) such that

$$g_{\mu\nu} p^\mu p^\nu = -m^2, \quad (29)$$

where m is 4D induced mass. On the other hand, we defined 4-velocity as $u^\mu = \frac{dx^\mu}{d\lambda}$. Hence we have

$$g_{\mu\nu} u^\mu u^\nu = \left(\frac{ds}{d\lambda} \right)^2 \equiv -l^2. \quad (30)$$

Now, comparing Eqs. (29) and (30) we obtain

$$p^\mu = \frac{m}{l} u^\mu. \quad (31)$$

Usually we assume that the length of 4-velocity is normalized to unity. But in this model, Eq. (21) implies if $X^\mu = eu^\mu$ then

$$d(-e^2 l^2) = 2K_{\mu\nu} u^\mu u^\nu u^4 d\lambda. \quad (32)$$

It is well known that in the non-integrable geometry the normal component of acceleration vanishes $u^\mu u^\nu u_{\mu;\nu} = 0$ [22]. Referring to the this fact, contracting Eq. (22) with 4-velocity of the test particle, the result is

$$\frac{\dot{e}}{e} = -\frac{2}{l^2} K_{\mu\nu} u^\mu u^\nu u^4. \quad (33)$$

Inserting this result in previous equation (32), we obtain

$$\frac{dl}{l} = \frac{1}{l^2} K_{\mu\nu} u^\mu u^\nu u^4 d\lambda. \quad (34)$$

Now we can compute the variation of the mass of test particle. Using (27) we have

$$d(g_{\mu\nu} p^\mu p^\nu) = 2K_{\alpha\beta} u^\alpha u^\beta p^4 d\lambda, \quad (35)$$

or using Eq. (29) and the corresponding definition of extra momenta $p^4 = \frac{m}{l} u^4$ we obtain

$$\frac{dm}{m} = -\frac{1}{l^2} K_{\mu\nu} u^\mu u^\nu u^4 d\lambda. \quad (36)$$

The author of [23] obtained the same result by using Hamilton–Jacobi formalism, and showed that this expression showing variation of mass is independent of the coordinates and any parameterization used along the motion. Now we are ready to discuss the physical meaning of the variation of mass and non-integrability. In general relativity we deal with large scales or at least up to scales of the order of millimeter. According to [13] the influence of matter fields on the bulk space is small and at large scales the matter ‘‘seems’’ to be on the original brane $\bar{g}_{\mu\nu}$. For this reason, we parameterize the path of a particle with an affine parameter in the original brane. According to (36) and fact that $u^\alpha = dx^\alpha/d\lambda = (dx^\alpha/d\tau)(d\tau/d\lambda)$ we have

$$\frac{dm}{m} = -K_{\mu\nu} \bar{u}^\mu \bar{u}^\nu \bar{u}^4 \left(\frac{d\tau}{ds} \right)^2 d\tau, \quad (37)$$

where $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is the line element of the perturbed brane, $d\tau^2 = -ds^2$ is the proper time defined on the original brane and \bar{u}^α is the 4-velocity of the test particle in the original non-perturbed brane. Now using Eqs. (10) and (11) we have

$$-\left(\frac{ds}{d\tau} \right)^2 = 1 + 2x^4 \bar{K}_{\mu\nu} \bar{u}^\mu \bar{u}^\nu + \mathcal{O}((x^4)^2), \quad (38)$$

and consequently inserting Eq. (38) into Eq. (37) and using Eq. (13) we obtain

$$\frac{dm}{m} = \left[\frac{1}{R} - \left(\frac{2}{R^2} + \bar{K}_{\mu\gamma} \bar{K}^\gamma{}_\nu \bar{u}^\mu \bar{u}^\nu \right) x^4 \right] \bar{u}^4 d\tau, \quad (39)$$

where

$$\frac{1}{R} = \bar{K}_{\mu\nu} \bar{u}^\mu \bar{u}^\nu \quad (40)$$

is the normal curvature [24]. In fact the normal curvature is nothing more than the higher-dimensional generalization of the familiar centripetal acceleration. Note that according to Eq. (3) the last term in (40) is related to the energy-momentum tensor of induced matter. Using (3) and (4) one can easily show that

$$\bar{K}_{\mu\gamma}\bar{K}^{\gamma\nu}\bar{u}^{\mu}\bar{u}^{\nu} = -8\pi G\varepsilon\left(\bar{T}_{\mu\nu}\bar{u}^{\mu}\bar{u}^{\nu} + \frac{1}{2}\bar{T}\right) + \frac{\bar{K}}{R}. \quad (41)$$

Hence the variation of mass is given by

$$\frac{dm}{m} = \left[\frac{1}{R} + \left\{-\frac{2}{R^2} - \frac{\bar{K}}{R} + 8\pi G\varepsilon\left(\bar{T}_{\mu\nu}\bar{u}^{\mu}\bar{u}^{\nu} + \frac{1}{2}\bar{T}\right)\right\}x^4\right]\bar{u}^4 d\tau. \quad (42)$$

One thing in above the equation for computing the variation of mass is to replace the normal component of velocity \bar{u}^4 . Using approximation (38), normal geodesic equation (25) up to first order x^4 , takes the following form

$$\frac{d^2x^4}{d\tau^2} + \left(\frac{2}{R^2} + \bar{K}_{\mu\gamma}\bar{K}^{\gamma\nu}\bar{u}^{\mu}\bar{u}^{\nu}\right)x^4 - \frac{1}{R} = 0, \quad (43)$$

or

$$\frac{d^2x^4}{d\tau^2} + \left[\frac{2}{R^2} + \frac{\bar{K}}{R} - 8\pi G\varepsilon\left(\bar{T}_{\mu\nu}\bar{u}^{\mu}\bar{u}^{\nu} + \frac{1}{2}\bar{T}\right)\right]x^4 - \frac{1}{R} = 0. \quad (44)$$

In general, there is not any general solution to the above equation. A useful method exists for determining an approximate solutions to the above differential equation. This is known in the literature as the Wentzel–Kramers–Brillouin (WKB) method. If we set

$$x^4 = A(\tau)e^{i\phi(\tau)} + B(\tau), \quad (45)$$

and substitute this solution into (44) we obtain

$$\begin{aligned} \left(\frac{d^2A}{d\tau^2} - A\left(\frac{d\phi}{d\tau}\right)^2 + PA\right)e^{i\phi} + i\left(2\frac{dA}{d\tau}\frac{d\phi}{d\tau} + A\frac{d^2\phi}{d\tau^2}\right)e^{i\phi} \\ + \frac{d^2B}{d\tau^2} + PB + Q = 0. \end{aligned} \quad (46)$$

Therefore we obtain

$$\begin{aligned} 2\frac{dA}{d\tau}\frac{d\phi}{d\tau} + A\frac{d^2\phi}{d\tau^2} = 0, \\ \frac{d^2A}{d\tau^2} - A\left(\frac{d\phi}{d\tau}\right)^2 + PA = 0, \\ \frac{d^2B}{d\tau^2} + PB + Q = 0, \end{aligned} \quad (47)$$

where

$$\begin{aligned} P &= \left[\frac{2}{R^2} + \frac{\bar{K}}{R} - 8\pi G\varepsilon\left(\bar{T}_{\mu\nu}\bar{u}^{\mu}\bar{u}^{\nu} + \frac{1}{2}\bar{T}\right)\right], \\ Q &= -\frac{1}{R}. \end{aligned} \quad (48)$$

Since P and Q are assumed to vary slowly, so are A and B , and thus we neglect the second derivatives of A and B . We thus obtain

$$x^4 = \frac{C}{P^{\frac{1}{4}}}\exp\left(\pm i\int\sqrt{P}d\tau\right) - \frac{P}{Q}, \quad (49)$$

where C is a constant of integration. This solution shows that the test particle becomes stable around the original non-perturbed brane, if P becomes greater than zero, i.e.,

$$-8\pi G\varepsilon\left(\bar{T}_{\mu\nu}\bar{u}^{\mu}\bar{u}^{\nu} + \frac{1}{2}\bar{T}\right) + \frac{2}{R^2} + \frac{\bar{K}}{R} > 0. \quad (50)$$

This, in turn means that the induced energy-momentum tensor satisfies some kind of energy condition. As a consequence, if the

energy-momentum tensor vanishes, so does the extrinsic curvature. This means that according to Eq. (44), the particle becomes totally unstable. Such a result seems to be in accordance with Mach's principal and for this reason we may call the above energy condition as Machian energy condition. Now, inserting Eq. (43) into Eq. (42) gives the following result

$$m = m_0 \exp\left(\frac{1}{2}(\bar{u}^4)^2\Big|_{\bar{u}_{in}^4}^{\bar{u}_{fin}^4}\right), \quad (51)$$

where m_0 is the initial mass, \bar{u}_{in}^4 and \bar{u}_{fin}^4 denote the initial and final velocity along extra dimension respectively. In the next section we will use this equation to obtain the variation of mass of nucleons from Big Bang nucleosynthesis up to now.

3. Variation of mass in FRW brane

Consider a FRW universe embedded (as a non-perturbed brane) in an 5D flat bulk space so that the extra dimension is spacelike. The FRW line element is written as

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \quad (52)$$

where κ takes the values ± 1 or 0 and $a(t)$ is the scale factor. Now, we proceed to analyze the variation of mass of the test particle. To do this, we first compute the extrinsic curvature through solving the Codazzi equations that gives [25]

$$\begin{aligned} \bar{K}_{00} &= -\frac{1}{a}\frac{d}{dt}\left(\frac{b}{a}\right), \\ \bar{K}_{ij} &= \frac{b}{a^2}g_{ij}, \quad i, j = 1, 2, 3. \end{aligned} \quad (53)$$

Here, b is an arbitrary functions of t . Consequently, the components of $\bar{Q}_{\mu\nu}$ using definition (3) become

$$\begin{aligned} \bar{Q}_{00} &= -\frac{3}{a^4}b^2, \\ \bar{Q}_{ij} &= \frac{1}{a^4}\left(2\frac{bb'}{H} - b^2\right)g_{ij}, \quad i, j = 1, 2, 3, \end{aligned} \quad (54)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and a dot denotes derivative with respect to the cosmological time t . Now the geodesic equation along the extra dimension (44) becomes

$$\frac{d^2x^4}{d\tau^2} + \frac{3}{R^2}x^4 + \frac{1}{R} = 0, \quad (55)$$

with approximate solution

$$x^4 \sim CR^{\frac{1}{2}}\sin\left(\sqrt{3}\int\frac{dt}{R} + \varphi\right) - \frac{R}{3}, \quad (56)$$

so that C and φ are integration constants and

$$\frac{1}{R} = \bar{K}_{\mu\nu}\bar{u}^{\mu}\bar{u}^{\nu} = \bar{K}_{00}. \quad (57)$$

Since within ordinary scales of energy we do not see the disappearance of particles, one may assume that the width of brane is very small. In braneworld models with large extra dimension, usually the width of brane should be in order or less than TeV^{-1} , i.e., $L \sim 10^{-17}$ cm. On the other hand, the Standard Model fields are usually confined to the brane within some localized width, i.e., the brane width [38–40]. Similarly, in induced matter theory, if the induced matter satisfies the restricted energy condition, the particles will be stabilized around the original brane [41]. The size of the fluctuations of the induced matter corresponds to the width of the brane. Since within the ordinary scales of energy we do not see the disappearance of particles, one may assume the fluctuations of the matter field exist only around the original brane. In

other words, if the brane width is d , it means that brane localized particles probe this length scale across the brane and therefore the observer cannot measure the distance on the brane to a better accuracy than d . On the other hand, the observational data constrains the brane width to be in the order of Planck length, see [26] and references therein. Hence, in this Letter according to [27] we assume that the size of the fluctuations of the brane (the width of the brane) is in order of Planck length which is much smaller than the effective size of the extra dimension L . This assumption may help us to investigate the correct quantum phenomenology in IMT. So we can neglect effect of this term in our calculation.

To proceed with geometrical interpretation of the energy-momentum tensor, let us consider an analogy between $\bar{Q}_{\mu\nu}$ and a simple example of matter consisting of free radiation field plus dust, that is

$$\begin{aligned}\bar{Q}_{\mu\nu} &= -8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \\ &= -8\pi G [(p + \rho)u_\mu u_\nu + p g_{\mu\nu}] + \Lambda g_{\mu\nu},\end{aligned}\quad (58)$$

with equation of state

$$p = \omega\rho. \quad (59)$$

Using Eqs. (54) and (58) the energy density and pressure takes the following forms

$$\begin{aligned}\rho &= -\frac{3}{8\pi G a^4} b^2 - \frac{1}{8\pi G} \Lambda, \\ p &= \frac{1}{8\pi G a^4} \left(\frac{2b\dot{b}}{H} + b^2 \right) - \frac{1}{8\pi G} \Lambda.\end{aligned}\quad (60)$$

Using the above two equations and equation of state of the matter we obtain

$$\begin{aligned}\Lambda &= \frac{b^2}{(\omega + 1)a^4} \left(\frac{2h}{H} + 3\omega - 1 \right), \\ \rho &= -\frac{b^2}{4\pi G(\omega + 1)a^4} \left(\frac{h}{H} - 2 \right),\end{aligned}\quad (61)$$

where $h = \dot{b}/b$. On the other hand, the conservation of energy-momentum tensor on the original brane gives

$$\rho = \rho_0 a^{-3(\omega+1)}, \quad (62)$$

where ρ_0 is energy density in the corresponding epoch. Hence, using Eqs. (61) and (62) we obtain

$$\frac{h}{H} = 2 - \frac{4\pi G \rho_0 (\omega + 1)}{b^2 a^{3\omega-1}}. \quad (63)$$

Consequently, using (61) and (63) we are left with

$$b^2 = \frac{\Lambda}{3} a^4 + \frac{8\pi G \rho_0}{3} a^{1-3\omega}. \quad (64)$$

Also we have

$$\frac{1}{R} = \bar{K}_{\mu\nu} \bar{u}^\mu \bar{u}^\nu = \left(1 - \frac{h}{H} \right) \frac{b}{a^2}. \quad (65)$$

Hence according to (61), (64) and (65) we obtain

$$\frac{1}{R} = \frac{4\pi G(3\omega + 1)a^{-3(1+\omega)} - \Lambda}{\sqrt{3}(8\pi G \rho_0 a^{-3(1+\omega)} + \Lambda)^{\frac{1}{2}}}. \quad (66)$$

Note that the existence of cosmological constant in the open universe models ($k = 0, -1$) is necessary to stabilize the test particles. Inserting Eq. (66) into (56) shows that in open universe in the absence of cosmological constant normal curvature in late time universe tends to the zero and consequently test particles become

unstable. According to the recent observations, we live in a flat universe $k = 0$. Hence the induced Freedman equation in the radiation dominated universe is

$$\dot{a}^2 = \frac{\Lambda}{3} a^2 + \frac{8\pi G \rho_0}{3a^2}, \quad (67)$$

with solution

$$a^2 = \sqrt{\frac{8\pi G \rho_0 \gamma}{\Lambda}} \sinh\left(2\sqrt{\frac{\Lambda}{3}} t\right). \quad (68)$$

Consequently the normal curvature in the radiation dominated epoch becomes

$$\frac{1}{R} = \sqrt{\frac{\Lambda}{3}} \frac{(1 - \sinh^2(2\sqrt{\frac{\Lambda}{3}} t))}{\sinh(2\sqrt{\frac{\Lambda}{3}} t) \cosh(2\sqrt{\frac{\Lambda}{3}} t)}. \quad (69)$$

This equation in the nucleosynthesis epoch ($2\sqrt{\Lambda/3}t \ll 1$) take the form

$$\frac{1}{R} = \frac{1}{2t}. \quad (70)$$

On the other hand, in the dust dominated universe we obtain from Freedman equation induced on the original brane the following solution

$$a = \left(\frac{8\pi G \rho_{0m}}{\Lambda} \right)^{\frac{1}{3}} \sinh^{\frac{2}{3}}\left(\frac{\sqrt{3\Lambda}}{2} t\right). \quad (71)$$

In this case the normal curvature becomes

$$\frac{1}{R} = 2\sqrt{\frac{\Lambda}{3}} \frac{1 - \sinh^2(\frac{\sqrt{3\Lambda}}{2} t)}{\sinh(\sqrt{3\Lambda} t)}. \quad (72)$$

Now Eq. (55) in the nucleosynthesis epoch using approximation takes the form

$$\frac{d^2 x^4}{dt^2} + \frac{3}{4} t^{-2} x^4 + \frac{1}{2} t^{-1} = 0, \quad (73)$$

which have the following exact solution

$$x_\gamma^4 = \xi_{0\gamma} \sqrt{\frac{t}{t_1}} \sin\left(\frac{\sqrt{2}}{2} \ln \frac{t}{t_2}\right) - \frac{2}{3} t. \quad (74)$$

Here t_1 and t_2 are two constants. If we assume $t_2 = t_n$ so that t_n is the nucleosynthesis epoch then we have

$$u_\gamma^4 = \frac{dx^4}{dt} \Big|_{t=t_n} = \frac{\sqrt{3}\xi_{0\gamma}}{2\sqrt{t}t_1} \sin\left(\frac{\sqrt{2}}{2} \ln\left(\frac{t}{t_n}\right) + \theta\right) - \frac{2}{3}, \quad (75)$$

where $\tan\theta = \sqrt{2}$. In the above relation $\xi_{0\gamma}$ denotes the width of the brane in the nucleosynthesis duration. Hence

$$u_\gamma^4(t_n) \sim -\frac{2}{3}. \quad (76)$$

Also the corresponding solution of Eq. (55) in the present epoch becomes

$$u_m^4(t_0) \sim -\frac{1}{3} \frac{dR}{dt} \Big|_{t=t_0} = -\frac{1}{2} \frac{\sinh^2(\frac{\sqrt{3\Lambda}}{2} t)}{[1 - \sinh^2(\frac{\sqrt{3\Lambda}}{2} t)]^2} \Big|_{t=t_0}. \quad (77)$$

The age t_0 of the universe can be found by the condition $a(t_0) = 1$. Using the identity $\tanh^{-1} x = \sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ we get the expression

$$t_0 = \frac{2}{\sqrt{3\Lambda}} \tanh^{-1} \sqrt{\Omega_\Lambda}. \quad (78)$$

Inserting the values $t_0 = 13.7 \times 10^9$ years and $\Omega_\Lambda = 0.7$ found from the WMAP measurements of the temperature fluctuations in the

cosmic microwave background radiation, and from the determination of the luminosity-redshift relationship of supernova of type Ia, we get $\frac{8\pi G_N}{3}\rho_0 = \frac{1-\Omega_\Lambda}{\Omega_\Lambda} = 0.43$ and $\Lambda = 1.1 \times 10^{-20}$ years $^{-2}$. Consequently the velocity of particles in the present epoch along the extra dimension becomes

$$u_m^4(t_0) \sim -0.662. \quad (79)$$

We can now estimate the variation of mass from nucleosynthesis up to the present epoch. Defining the quotient $\frac{\Delta m}{m}$ as

$$\frac{\Delta m}{m_0} = \frac{m(t_0) - m(t_n)}{m(t_0)}, \quad (80)$$

where t_0 and t_n denote the age of universe and time of nucleosynthesis respectively. Now using Eqs. (51), (76) and (79) we have

$$\frac{\Delta m}{m_0} \sim 1 - e^{0.004} \sim -0.004. \quad (81)$$

If we consider that a variation of the rest mass of particles had occurred between the epoch of primordial nucleosynthesis and present, then one can calculate the deviation in the ${}^4\text{He}$ production from the Hot Big Bang model prediction with this fact. According to [9] if we the masses are changed then the upper bound of the deviation of primordial Helium production due to a variation on the difference between the rest mass of the nucleons between the present and nucleosynthesis epochs is given by

$$\delta(M_n - M_p) \leq 0.129 \text{ MeV}, \quad (82)$$

where M_n and M_p are neutron and proton masses respectively. If we define $\Delta Q = M_n - M_p$, then the above limit gives [9]

$$\left| \frac{\delta(\Delta Q)}{\Delta Q_0} \right| \leq 10\%, \quad (83)$$

with

$$\Delta Q_0 \simeq 294 \text{ MeV}. \quad (84)$$

The authors of [9] used the 5D induced matter brane cosmological model with compact extra dimension in the radiation dominated universe. Then using the original definition of induced mass (5) we find the following quotient for the mass variation of nucleons

$$\left| \frac{\delta(\Delta Q)}{\Delta Q_0} \right| \approx 100\%, \quad (85)$$

which is in disagreement with the previous bound (83). Note that, if we use Eq. (42) or equivalently (51) as variation of mass, then in compact models of IMT [28] the mass of particles remain unchanged. On the other hand, in noncompact IMT, Eq. (81) gives a better outcome

$$\left| \frac{\delta(\Delta Q)}{\Delta Q_0} \right| \approx 0.4\%, \quad (86)$$

which is in agreement with Hot Big Bang result (83).

4. Conclusions

In this Letter we have analyzed the variation of nucleon masses in flat FRW cosmological model, embedded in a 5D Ricci flat bulk space, using IMT ideas. We have showed that the mass variation is a consequence of non-integrability of the 4D embedded space-time. From the point of view of a 4D observer, according to (42), the variation of the mass of particles is a direct result of the distribution of matter in 4D universe. This relation can be regarded as an explanation of Mach's principle, that inertial forces should be generated by the motion of a body relative to the bulk of induced matter in the universe. In the theory outline in this Letter, the variation of mass obtained from IMT is in agreement with mass variation bound obtained from Hot Big Bang.

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