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Electron impact ionization of metastable 2P-state hydrogen atoms in the coplanar geometry

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ABSTRACT

Triple differential cross sections (TDCS) for the ionization of metastable 2P-state hydrogen atoms by electrons are calculated for various kinematic conditions in the asymmetric coplanar geometry. In this calculation, the final state is described by a multiple-scattering theory for ionization of hydrogen atoms by electrons. Results show qualitative agreement with the available experimental data and those of other theoretical computational results for ionization of hydrogen atoms from ground state, and our first Born results. There is no available other theoretical results and experimental data for ionization of hydrogen atoms from the 2P state. The present study offers a wide scope for the experimental study for ionization of hydrogen atoms from the metastable 2P state.

1. Introduction

The study of multiple ionization process by charged particle impact is of great interest in many branches of physics, such as astrophysics, plasma physics and also in life sciences, for understanding the various mechanisms leading to energy deposition by radiation of matter.

Ionization of the hydrogen atoms by electrons is a good check for the perturbation theory, because of the existence of experimental results, specially for the Triple differential cross sections (TDCS) [1–5]. Ionization by fast particles was first treated quantum mechanically by Bethe [6]. Electron impact of single and double ionization is one of the simplest and the most important fundamentals of such a process. The utilization of the multi-parameter detection technique, together with the progress in computational methods, have made it possible to perform a complete experiment in which kinematical parameters (like momentum and energies) of all acting particles are determined. In such calculations, the ejected electron is detected in coincidence with the scattered electrons and it is a well known experiment [4]. This kind of experiment, called (e, 2e) experiments, have been successfully used during the last four decades to investigate the fine details of the ionization process both in the ground state [7–16] and metastable [17–28] states of atomic Hydrogen.

Hafid et al. [20] have shown that the application to the H(2S) of the corrected double continuum wave function of Brauner et al. [29] gives results comparable to the second Born ones. In the BBK theory of Brauner et al. [29] focused attention was seen on the improvement of the final state wave function by including in it the effects of all the long range Coulomb interactions, including the electron–electron repulsion. This satisfies the correct boundary condition when the particle separations tend to infinity.

For the ionization of hydrogen atoms by electrons from the metastable 2S and 2P states, no such triple differential cross sections (TDCS) measurement is yet available in the literature, although the absolute total cross sections (TCS) were measured much earlier [30,31]. Recently, Dal et al. [25] has investigated in a greater detail the ionization of atomic hydrogen and helium atoms in the different metastable states by electrons and positrons. In this study we have investigated the ionization of metastable 2P state hydrogen atoms by electrons. To the best of our knowledge, the work reported here, is introducing the TDCS calculation for the ionization of metastable 2P-state hydrogen atoms by electrons for the first time.

A multiple scattering theory [10,11] has been followed in the present calculation of the triple differential cross sections (TDCS) in the metastable 2P-state hydrogen atom ionization by 250 eV electron energy. It is noted that the multiple scattering wave function [11] has been designed for two electrons moving in a coulomb field, which include higher order and correlation effects. Using this wave function, very interesting results for triple differential cross sections (TDCS) with various kinematic conditions have been obtained for electron hydrogen ionization collisions both in the ground state and metastable 2S state at non-relativistic energies [11,13,21,22] and many other calculations (the references of which

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are not given here) as well as for medium-heavy atoms ionization by electrons at relativistic energies [23,24,32,33]. So it will be interesting here to use this wave function in the present study for ionization of metastable 2P state hydrogen atoms by electrons.

2. Theory

2.1. Scattering mechanism

The most detailed information presently available is about the single ionization processes of the following type

$$e^- + H(2P) \rightarrow H^+ + 2e^- \quad (1)$$

where the symbol 2P denotes the metastable state of the hydrogen atoms, and has been obtained in the coplanar geometry by analyzing triple differential cross sections (TDCS) measured in (e, 2e) coincidence experiments. The TDCS is a measure of the probability that in an (e, 2e) reaction an incident electron of momentum $\bar{p}_i$ and energy E_i will produce on collision with the target two electrons having energies E_1 and E_2 and momenta $\bar{p}_1$ and $\bar{p}_2$, emitted respectively into the solid angles $d\Omega_1$ and $d\Omega_2$ centered about the directions (θ_1, ϕ_1) and (θ_2, ϕ_2).

The TDCS is usually denoted by the symbol $d^3\sigma/d\Omega_1 d\Omega_2 dE_2$ for unpolarized incident electrons and targets, it is a function of the quantities E_i, E_1 or E_2, θ_1, θ_2 and $\phi = \phi_1 - \phi_2$. By integrating the TDCS over $d\Omega_1$, $d\Omega_2$ or dE_2 one can form various double and single differential cross sections. We will calculate the same in the near future. Finally, the total ionization cross section is obtained by integrating over all outgoing scattering angles and energies, and depends only in E_i, the incident electron energy. It is useful when studying (e, 2e) coincidence experiments to distinguish between several kinematical arrangements, since these have important implications for the theoretical analysis of the collision. A first distinction can be made between coplanar geometries-such that the momenta $\bar{p}_i$, $\bar{p}_1$ and $\bar{p}_2$ are in the same plane-and non-coplanar geometries such that the momentum $\bar{p}_2$ is out of the $(\bar{p}_i, \bar{p}_1)$ reference plane. There is another useful distinction between asymmetric and symmetric geometries. In asymmetric geometries, a fast electron of energy E_i is incident on the target atom, and a fast ("scattered") electron is detected in coincidence with a slow ("ejected") electron. This kind of experiment was first performed by Ehrhardt et al. [4]. On the other hand, symmetric geometries are defined by the requirement that $\theta_1 \cong \theta_2$ and $E_1 \cong E_2$.The first (e, 2e) symmetric coincidence experiments of Amaldi et al. [34] have been followed by a number of experiments of this type.

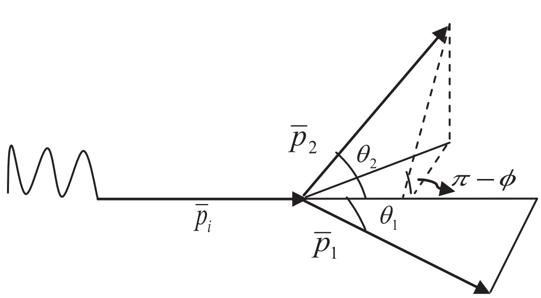

Fig. The kinematics of an (e, 2e) reaction. The incident electron momentum is $\bar{p}_i$ and the momenta of the outgoing electrons are $\bar{p}_1, \bar{p}_2$ respectively. Also the angles θ_1 and θ_2 are shown with respect to the incident direction and the angle $\pi - \phi$ is measuring the direction in a coplanar situation.

2.2. The T-matrix element

The direct T-matrix element for ionization of hydrogen atoms by electrons [11] may be written as

$$T_{fi} = \langle \Psi_f^{(-)}(\bar{r}_1, \bar{r}_2) | V_i(\bar{r}_1, \bar{r}_2) | \phi_i(\bar{r}_1, \bar{r}_2) \rangle \quad (2)$$

where the perturbation potential $V_i(\bar{r}_1, \bar{r}_2)$ is given by

$$V_i(\bar{r}_1, \bar{r}_2) = \frac{1}{r_{12}} - \frac{Z}{r_2}$$

For hydrogen atoms nuclear charge is $Z = 1$, r_1 and r_2 are the distances of the two electrons from the nucleus and r_{12} is the distance between the two electrons.

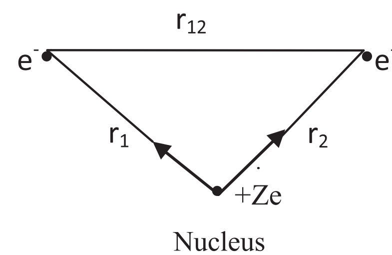

Fig. Interaction between two electrons and the nucleus.

The initial channel unperturbed wave function is given by

$$\phi_i(\bar{r}_1, \bar{r}_2) = \frac{e^{i\bar{p}_i \cdot \bar{r}_2}}{(2\pi)^{3/2}} \phi_{2p}(\bar{r}_1) = \frac{e^{i\bar{p}_i \cdot \bar{r}_2}}{8\sqrt{2}\pi^2} r_1 \cos\theta e^{-r_1 \lambda_1} \quad (3)$$

where

$$\phi_{2p}(\bar{r}_1) = \sqrt{\frac{1}{32\pi}} r_1 \cos\theta e^{-r_1/2} = \sqrt{\frac{1}{32\pi}} r_1 \cos\theta e^{-\lambda_1 r_1} \quad \left[\lambda_1 = \frac{1}{2}\right]$$

is the hydrogenic 2P state wave function, $\bar{p}_i$ is the incident electron momentum.

And $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ is the final three-particle scattering state wave function with the electrons being in the continuum with momenta $\bar{p}_1$ and $\bar{p}_2$. Co-ordinates of the two electrons are taken to be $\bar{r}_1$ and $\bar{r}_2$. Here the approximate wave function $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ [11] is given by

$$\psi_f^{(-)}(\bar{r}_1, \bar{r}_2) = N(\bar{p}_1, \bar{p}_2)\left[\phi_{\bar{p}_1}^{(-)}(\bar{r}_1) e^{i\bar{p}_2 \cdot \bar{r}_2} + \phi_{\bar{p}_2}^{(-)}(\bar{r}_2) e^{i\bar{p}_1 \cdot \bar{r}_1} + \phi_{\bar{p}}^{(-)}(\bar{r}) e^{i\bar{P} \cdot \bar{R}} - 2e^{i\bar{p}_1 \cdot \bar{r}_1 + i\bar{p}_2 \cdot \bar{r}_2}\right] \Big/ (2\pi)^3 \quad (4)$$

where

$$\bar{r} = \frac{\bar{r}_1 - \bar{r}_2}{2}, \quad \bar{R} = \bar{r}_1 + \bar{r}_2, \quad \bar{p} = (\bar{p}_2 - \bar{p}_1), \quad \bar{P} = \bar{p}_2 + \bar{p}_1$$

The scattering amplitude [11] may be written as

$$f(\bar{p}_1, \bar{p}_2) = N(\bar{p}_1, \bar{p}_2)[f_{eT} + f_{PT} + f_{Pe} - 2f_{PWB}] \quad (5)$$

where f_{eT}, f_{PT}, f_{Pe} and f_{PWB} are the amplitudes corresponding to the four terms of Eq. (4) respectively.

The normalization constant $N(\bar{p}_1, \bar{p}_2)$ is given by

$$|N(\bar{p}_1, \bar{p}_2)|^{-2} = \left| 7 - 2[\lambda_1 + \lambda_2 + \lambda_3] - \left[\frac{2}{\lambda_1} + \frac{2}{\lambda_2} + \frac{2}{\lambda_3}\right] + \left[\frac{\lambda_1}{\lambda_2} + \frac{\lambda_1}{\lambda_3} + \frac{\lambda_2}{\lambda_1} + \frac{\lambda_2}{\lambda_3} + \frac{\lambda_3}{\lambda_1} + \frac{\lambda_3}{\lambda_2}\right] \right| \quad (6)$$

where

$$\lambda_1 = e^{\pi\alpha_1/2}\Gamma(1 - i\alpha_1), \quad \alpha_1 = 1/p_1,$$
$$\lambda_2 = e^{\pi\alpha_2/2}\Gamma(1 - i\alpha_2), \quad \alpha_2 = 1/p_2,$$
$$\lambda_3 = e^{\pi\alpha/2}\Gamma(1 - i\alpha) \qquad \alpha = -1/p.$$

Here $\phi_q^{(-)}(\bar{r})$ is the Coulomb wave function given by

$$\phi_q^{(-)}(\bar{r}) = e^{\pi\alpha/2}\Gamma(1+i\alpha)e^{iq.\bar{r}}{}_1F_1(-i\alpha, 1, -i[qr + \bar{q}.\bar{r}])$$

where

$\alpha_1 = \frac{1}{p_1}$ for $\bar{q} = \bar{p}_1$, $\alpha_2 = \frac{1}{p_2}$ for $\bar{q} = \bar{p}_2$ and $\alpha = -\frac{1}{p}$ for $\bar{q} = \bar{p}$.

Now applying Eqs. (3) and (4) on Eq. (2), it becomes

$$T_{fi} = T_B + T_{B'} + T_i - 2T_{PB} \tag{7}$$

where

$$T_{\mathbf{B}} = \langle \phi_{\bar{p}_1}^{(-)}(\bar{r}_1)e^{i\bar{p}_2.\bar{r}_2}|V_i|\phi_i(\bar{r}_1, \bar{r}_2)\rangle \tag{8}$$

$$T_{B'} = \langle \phi_{\bar{p}_2}^{(-)}(\bar{r}_2)e^{i\bar{p}_1.\bar{r}_1}|V_i|\phi_i(\bar{r}_1, \bar{r}_2)\rangle \tag{9}$$

$$T_i = \langle \phi_{\bar{p}}^{(-)}(\bar{r})e^{i\bar{P}.\bar{R}}|V_i|\phi_i(\bar{r}_1, \bar{r}_2)\rangle \tag{10}$$

$$T_{PB} = \langle e^{i\bar{p}_1.\bar{r}_1 + i\bar{p}_2.\bar{r}_2}|V_i|\phi_i(\bar{r}_1, \bar{r}_2)\rangle \tag{11}$$

For First Born approximation, we calculated $T_{\mathbf{B}}$ matrix amplitude of Eq. (8) using (2)

$$\begin{aligned} T_{\mathbf{B}} &= \frac{1}{8\sqrt{2}\pi^2}\left\langle \phi_{\bar{p}_1}^{(-)}(\bar{r}_1)e^{i\bar{p}_2.\bar{r}_2}\left|\frac{1}{r_{12}} - \frac{1}{r_2}\right|e^{i\bar{p}_i.\bar{r}_2} r_1 \cos\theta e^{-r_1\lambda_1}\right\rangle \\ &= \frac{1}{8\sqrt{2}\pi^2}\int \phi_{\bar{p}_1}^{(-)*}(\bar{r}_1)e^{-i\bar{p}_2.\bar{r}_2}\left(\frac{1}{r_{12}} - \frac{1}{r_2}\right)e^{i\bar{p}_i.\bar{r}_2} r_1 \\ &\quad \times \cos\theta e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\ &= \frac{1}{8\sqrt{2}\pi^2}\int \phi_{\bar{p}_1}^{(-)*}(\bar{r}_1)e^{-i\bar{p}_2.\bar{r}_2}\frac{1}{r_{12}}e^{i\bar{p}_i.\bar{r}_2} r_1 \cos\theta e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \\ &\quad - \frac{1}{8\sqrt{2}\pi^2}\int \phi_{\bar{p}_1}^{(-)*}(\bar{r}_1)e^{-i\bar{p}_2.\bar{r}_2}\frac{r_1}{r_2}e^{i\bar{p}_i.\bar{r}_2}\cos\theta e^{-\lambda_1 r_1} d^3r_1 d^3r_2 \end{aligned} \tag{12}$$

After analytical calculation by using the Lewis integral [35], we evaluated the above expressions numerically using the Gaussian quadrature formula. The triple differential cross sections (TDCS) are finally given by

$$\frac{d\sigma}{d\Omega_1 d\Omega_2 dE_1} = \frac{p_1 p_2}{p_i}|T_{fi}|^2 \tag{13}$$

where E_1 is the energy of the incident electron. Therefore, in our present calculation we have computed the TDCS, given by the Eq. (13).

3. Results and discussion

In this section we have investigated the ionization of the metastable 2P state hydrogen atoms by electrons. Ionization of hydrogen atoms by electrons from the ground state theoretical results of Dal et al. [25], the BBK model of Brauner et al. [29] and the absolute data [4], are presented here for comparison. One of our earlier works on hydrogenic 2S-state ionization results [22] is also exhibited here for comparison. We have performed the triple differential cross section (TDCS) results of the present calculation and the first Born approximation for the process (1) at high incident energy E_i = 250 eV for some varied ejected angles (θ_1) and fixed scattering angles (θ_2). These results have been displayed in the following nine figures where we have plotted the electron impact TDCS varying against the angle of ejection (θ_1) of the ejected electron. In these figures, the region for $\theta_1(0° - 150°)$ and ϕ = 0°, refers to the recoil region, while $\theta_1(150° - 360°)$ and ϕ = 180°, refers to the binary region. Here θ varies from 0° to 360°.

In this study we have considered the triple differential cross sections (TDCS) for the ionization of metastable 2P state hydrogen atoms by electrons for the incident energy E_i = 250 eV, ejected electron energies E_1 = 5 eV and 50 eV eV corresponding to the different scattering angles θ_2 = 3° (Fig. 1), 15° (Fig. 2), 25° (Fig. 3), 5° (Fig. 4a), 7° (Fig. 4b), 9° (Fig. 4c), 11° (Fig. 4d), 15° (Fig. 4e) and 20° (Fig. 4f). The final state scattering wave function $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ is the continuum state of the atomic hydrogen. When the contribution of the final continuum state is considered in the ionization of 2P metastable state hydrogen atoms by electrons, it shows a fall of binary lobe amplitude and a rise of recoil lobe amplitude. In the present and first Born TDCS results, the amplitude is substantially large, in magnitude, compared to other amplitudes, such as first Born. This implies that near the peak, the projectile electron interactions are most important in the final channel.

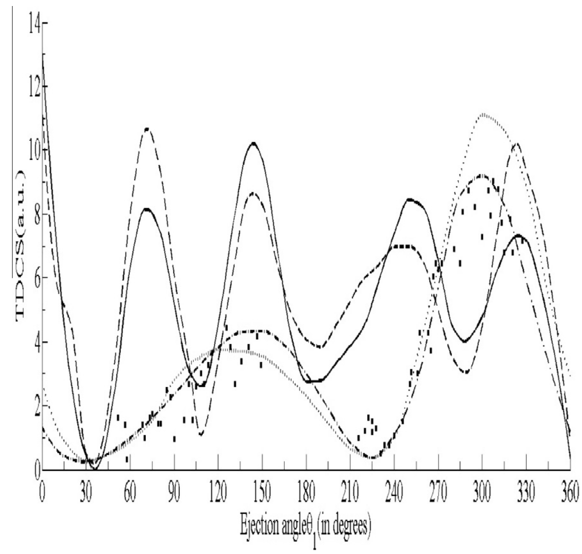

Fig. 1. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for θ_2 = 3° vary against the ejected electron θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 5 eV. Theory: full curve: present results; dash curve: present first Born result; short dash curve: hydrogenic ground state 2nd Born results [25]; dash-dotted curve: hydrogenic ground state BBK model [29] and square: hydrogenic ground state experiments [4] (multiplied by 0.88).

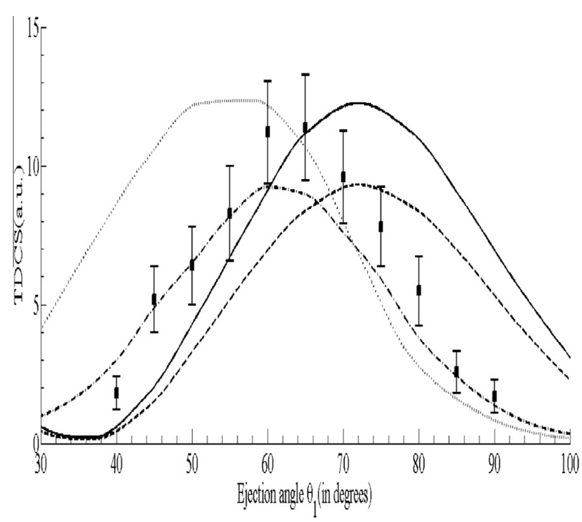

Fig. 2. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for θ_2 = 15° vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is E_1 = 50 eV. Theory: full curve: present results; dash curve: present first Born result; short dash curve: hydrogenic ground state 2nd Born results [25]; dash-dotted curve: hydrogenic ground state BBK model [29] and square: hydrogenic ground state experiments [4] (multiplied by 0.00224).

From this point of view it can be said that the present results play a vital role in the ionization of atomic hydrogen for these

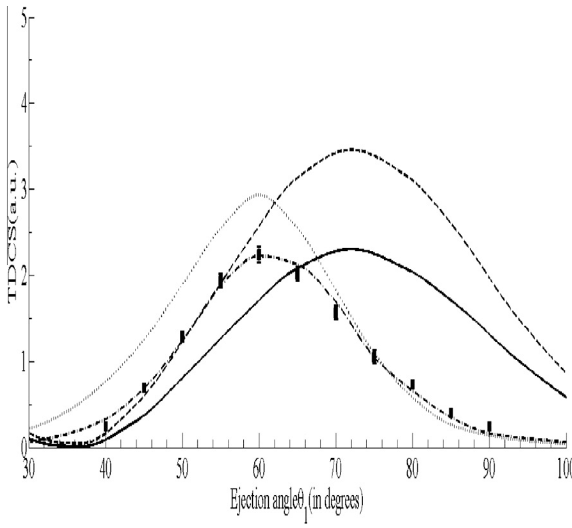

Fig. 3. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 25°$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is $E_1 = 50$ eV. Theory: full curve: present results; dash curve: present first Born result; short dash curve: hydrogenic ground state 2nd Born results [25]; dash-dotted curve: hydrogenic ground state BBK model [29] and square: hydrogenic ground state experiments [4] (multiplied by 0.00224).

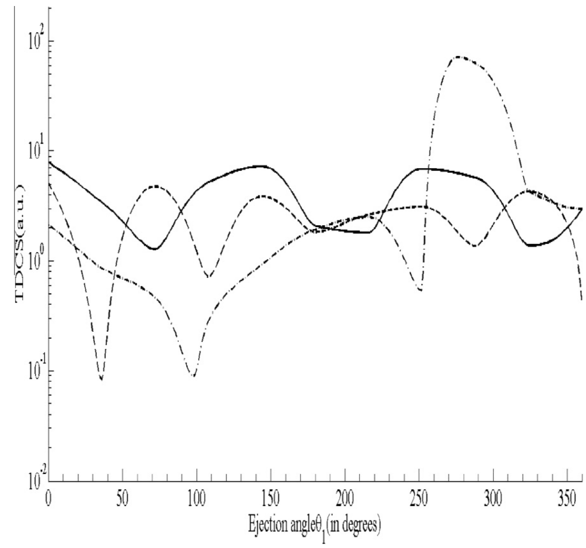

Fig. 4b. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 7°$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is $E_1 = 5$ eV. Theory: full curve: present results; dash curve: present first Born result; dash-dotted curve: hydrogenic 2S-state results [22].

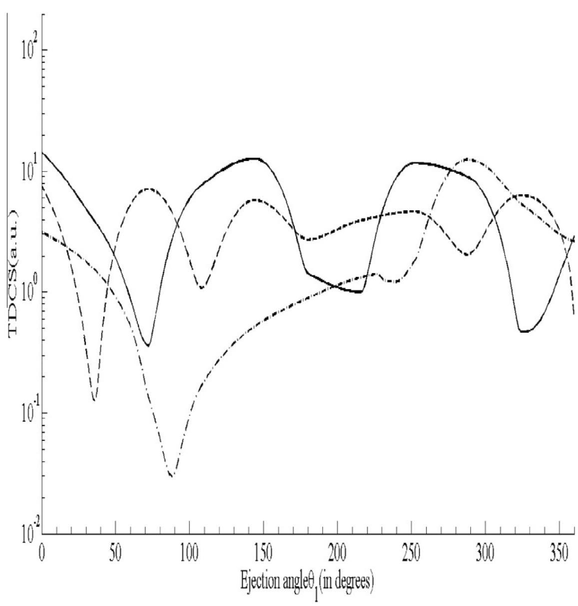

Fig. 4a. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 5°$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is $E_1 = 5$ eV. Theory: full curve: present results; dash curve: present first Born result; Dash-dotted curve: hydrogenic 2S- state results [22].

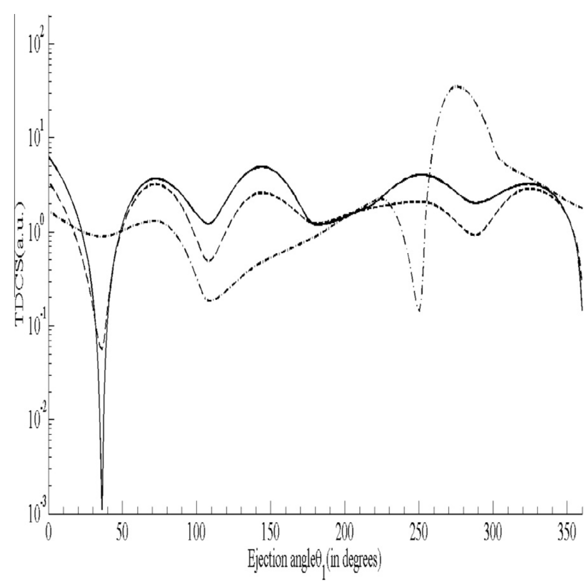

Fig. 4c. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 9°$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is $E_1 = 5$ eV. Theory: full curve: present results; dash curve: present first Born result; dash-dotted curve: hydrogenic 2S-state results [22].

particular kinematical conditions (large incident energy compared to the energy of the ejected electron).

Here we have used the approval of Jones and Madison [14,15] and multiplied the available hydrogenic ground state absolute measurements by a scaling factor of 0.88 (for $\theta_2 = 3°$) and 0.00224 (for $\theta_2 = 15°, 25°$). They noted that this scaling factor of 0.88 was used to change the relative uncertainty up to 10%. This indicates that the experiment should be within ±10% of the theoretical curve for quantitative evaluation.

Fig. 1 shows a comparison among the present results with the hydrogenic ground state results of the BBK model [29], the second Born approximation [25] and the experimental data [4] and our first Born result. The binary peak values of the present results and the first Born result show qualitative agreement with those of the hydrogenic ground state results of the BBK model [29], the second Born approximation [25] and the experimental data [4] in magnitude for the higher ejection angle (θ_1). Here the recoil peak values of the present and first Born results are about double those of the hydrogenic ground state second Born calculation [25], BBK model [29] and the experimental data [4]. In this case, the binary peak values of the present and first Born results are the lowest among all theoretical hydrogenic ground state calculations

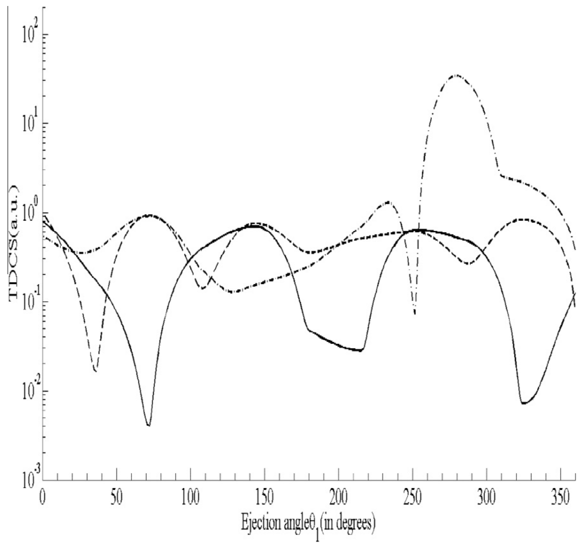

Fig. 4d. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 11°$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is $E_1 = 5$ eV. Theory: full curve: present results; dash curve: present first Born result; dash-dotted curve: hydrogenic 2S-state results [22].

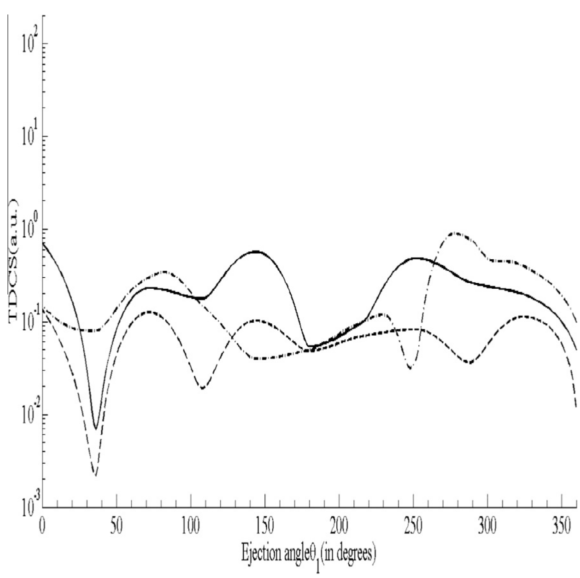

Fig. 4e. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 15°$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is $E_1 = 5$ eV. Theory: full curve: present results; dash curve: present first Born result; dash-dotted curve: hydrogenic 2S-state results [22].

[25,29] and show good qualitative agreement with the absolute data[4] for the ionization of hydrogen atoms by electrons from the ground state. In all cases the peak position of the present first Born approximation is almost similar to the peak position of the present results.

Here Figs. 2 and 3 are presented for $E_i = 250$ eV, $E_1 = 50$ eV, $\theta_2 = 15°$ and $E_i = 250$ eV, $E_1 = 50$ eV, $\theta_2 = 25°$ respectively. We note that in Fig. 2, our present results generally exist in between the hydrogenic ground state second Born result [25] and the present first Born result. For the same case, we also note that the present results are almost close to those of the hydrogenic ground state BBK model [29] and the experimental results [4]. Again, for the ejected electron energy $E_1 = 50$ eV, our first Born result for the 2P

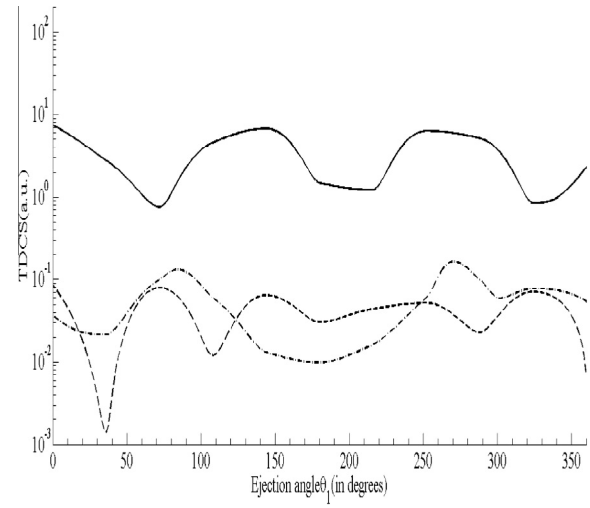

Fig. 4f. Triple-differential cross sections (TDCS) for ionization of atomic hydrogen by 250 eV electron impact for $\theta_2 = 20°$ vary against the ejected electron angle θ_1 relative to the incident electron direction. The ejected electron energy is $E_1 = 5$ eV. Theory: full curve: present results; dash curve: present first Born result; dash-dotted curve: hydrogenic 2S-state results [22].

metastable state represents exactly similar behavior as the hydrogenic ground state experimental results [4].

Figs. 2 and 3 show that the first Born result is increased simultaneously with the increase of the scattering angles (θ_2), whereas the peak values of the present results remain almost same in magnitude as the hydrogenic ground state experimental data [4]. However, these peaks shifted slightly to the higher ejected angle (θ_1).

Figs. 4a and b show that the present TDCS curves exhibit a distinct three peaked structures where both the binary and recoil regions consist of one prominent peak. But here the present first Born result shows less prominent peaks. However, the hydrogenic 2S-state results provide binary peaks and one deep peaked structure in the recoil regions.

In the Fig. 4c, the present and the first Born results exhibit two nice distinct peaks with the same position both in recoil and binary regions. The recoil peaks of our present results also represent the same pattern as the hydrogenic 2S-state results, whereas a peak with large magnitude appears in the recoil region.

For higher scattering angles (Figs. 4d–f), the peak values of the present and the first Born results and the hydrogenic 2S-state results, show almost similar position with different magnitude in the recoil region. But in the binary region, the present peak values are slightly smaller than the corresponding compared hydrogenic 2S-state results [22].

The physical origins of the findings are presented here. Four different scattering amplitudes corresponding to different terms of the scattering state wave function $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ from the Eq. (4) is considered.

In the Eq. (5), for f_{eT}, $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2) = \phi_{\bar{p}_1}^{(-)}(\bar{r}_1) e^{i\bar{p}_2 \cdot \bar{r}_2}$ is the first Born amplitude in which the scattered electrons are described by a plane wave while the ejected electrons are described by a Coulomb wave. The amplitude f_{PT}, $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2) = \phi_{\bar{p}_2}^{(-)}(\bar{r}_2) e^{i\bar{p}_1 \cdot \bar{r}_1}$ is similar to the first Born amplitude f_{eT} except for the fact that the role of the electron and projectile is interchanged. For f_{Pe}, $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ is given by $\phi_{\bar{p}}^{(-)}(\bar{r}) e^{i\bar{P} \cdot \bar{R}}$, where $\bar{r} = \frac{\bar{r}_1 - \bar{r}_2}{2}$, $\bar{R} = \bar{r}_1 + \bar{r}_2$, $\bar{p} = (\bar{p}_2 - \bar{p}_1)$, $\bar{P} = \bar{p}_2 + \bar{p}_1$. In this term the projectile electron interaction is exactly treated in the final channel, where the center of mass moves as a plane wave. For f_{PWB}, $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ is given by $e^{i\bar{p}_1 \cdot \bar{r}_1 + i\bar{p}_2 \cdot \bar{r}_2}$ corresponding to two plane waves for the ejected electron and scattered particle. Here $N(\bar{p}_1, \bar{p}_2)$ is the normalization constant [11]. It has been calculated numerically using the Eq. (6) for the electron impact and the approximate value of N is nearly 1. The physical origins of the peaks in the triple differential cross sections curves for the ionization of metastable 2P state collisions at 250 eV electron energy with all the desired qualitative features have also been investigated. This indicates that these peaks arise from second order scatterings, scattering first by atomic nucleus or atomic electron and then a second time by the atomic electron.

4. Conclusions

In this work we have calculated the triple differential cross sections (TDCS) for ionization of metastable 2P-state hydrogen atoms by 250 eV electron impact. We have noted that when the full wave function $\psi_f^{(-)}$ is used, then the present results represent qualitative agreement with the available hydrogenic ground state experimental data [4] and those of hydrogenic ground state theoretical models [25,29] and the present first Born results. It is also noted that Figs. 2 and 3 show similar peak patterns with the compared hydrogenic ground state results [25,29] but the peaks slightly shifted in position. The present calculation using the multiple scattering theory of Das and Seal [11] provides a significant contribution in the field of metastable 2P-state ionization problems. Due to the absence of any experimental data for the TDCS results of the hydrogenic metastable 2P-state ionization process, it is not possible to compare the present computational results with the experimental findings. Thus for judgment of this work, experimental study in the relevant field is needed. Therefore, experimental results for ionization of metastable 2P state hydrogen atoms by electrons will be valuable and will add a new dimension to the significant study of this field of research.

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