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Theoretical Elasto-Plastic Solution for Piles Subject to Lateral Soil Movement

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Abstract

In this study, a nonlinear mechanical mechanism model for piles subject to lateral soil movement was proposed based on the Winkler elastic model. The plastic deformation of surrounding soils was taken into account using a simplified elastoplastic constitutive model. Then, the governing equations describing the interaction between passive piles and unstable slopes were deduced. By considering the continuity condition and the boundary conditions, the analytical solutions for the responses of pile embedded in linear-plastic soil were obtained. Moreover, a Matlab program was developed for the implementation of the solutions. Finally, a model test is simulated and it is shown that the proposed analytical method considering the soil nonlinear and plastic yielding is capable of representing the actual lateral performance of a pile more realistically than the existing elastic solutions.

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1. Introduction

The stability of soil mass, for example dams, foundation pit and landslide is a basic and important topic of geotechnical engineering. Stabilizing piles are widely used to improve the stability of slope because the installation of piles in unstable soils could keep informed the property of soil mass to adjust the design timely. The lateral performance of stabilizing piles embedded in slope is highly affected by the instability of soil mass, which brings larger lateral displacement because of its gravity. Generally,
stabilizing piles belong to the category of the "passive" pile. At present, several analytical methods have been developed to analyze the response of passive pile, including winkler elastic subgrade reaction[1] and the elastic continuum[2].

The methods mentioned above are especially suitable to capture pile deflection at smaller magnitude of soil movement because the soil condition is assumed to be elastic. If the elastoplastic behaviors or nonlinear properties are taken into account, the prediction of piles behaviors deviates from real condition. A number of laterally loading tests [3-4] showed that the property of soil is nonlinear, especially when the soil goes into plastic flow state. The interforce of pile-soil reaches its limit no longer increase with the increase of horizontal displacement. Thus, the method considering the soil nonlinear and plastic yielding is capable of representing the actual lateral performance of a pile more realistically than the existing elastic solutions which may lead to the big error. But there is not, at present, a validated analytical solution employed to solve the problem of stabilizing pile embedded in unstable elastoplastic layer.

In this paper, the theoretical solution for stabilizing pile subject to lateral soil movement in a uniform subgrade reaction modulus of winker-yielding soil is presented. Simplified linear-plastic soil behavior is employed to model soil-pile interaction. The general computational software Matlab is used to carry out the derived numerical calculations, making the method easy and efficient to use. To facilitate the use of the theoretical solution, methods for determining the modulus of subgrade reaction and yielding deflection are provided. Finally, comparisons between the theoretical solution and field measured results are also given.

2. Basic Equation

2.1. Basic assumption and differential equation

According to the static balance and material mechanics hypothesis[1], the beam flexure equation for the case of a pile subject to free-field lateral soil movement with no axial or externally applied lateral loads is

\[ EI \frac{d^4v}{dz^4} = k(s-v) \]  \(1\)

where, E=elastic modulus of concrete, kN/m^2; I=effective moment of inertia, m^4;k=khd, kh is horizontal subgrade reaction coefficient.

The pile-soil reaction can be evaluated as a function of the relative displacement between the soil and the piles [5]. In this analysis, the lateral soil reaction due to pile-soil interaction is obtained from the relative displacement between the pile and soil (s-v). Just as what Fig.1 (a) show, the term (s-v) is multiplied by soil stiffness (k) to give the load on the pile, as following

\[ F = k(s-v) \]  \(2\)

The following empirical equation developed by Vesic(1961)[6] who employed elastic subgrade reaction method to analysis the performance of long pile embedded in elastic soil based on infield lateral loading test data could be used to present a relation for k and E_s:

\[ k = \frac{0.65E_s E_t}{1-\nu_s^2} \sqrt{\frac{E_t d^4}{EI}} \]  \(3\)

where, \(\nu_s\)=Poisson’s ratio of soil; d=pile diameter, m; E_s=modulus of soil, kN/m^2, which represents a secant modulus for relatively low load levels.

If a nonlinear analysis is to be used, E_s is a key parameter required for lateral response analysis of stabilizing piles. For clays, E_s is usually related to the undrained shear strength, c_u, as follows: \(E_s = ac_u\),
where, \( a \) lies between 150 and 400 suggested by Poulos(1980)[7]. For sands, it is customary to assume that the modulus of soil, \( E_s \), is linearly with depth, \( z \), so that \( E_s = Nz \), where, \( N \) is a constant parameter and the typical values of it for saturated loose, medium, and dense sands are 1.5, 5.0 and 12.5 MPa/m, respectively[8].

2.2. Linear-plastic profile of pile-soil reaction system

The yielding is likely to occur if relative displacement between pile and soil is slightly larger than the yielding displacement, \( v^* \). It is necessary to incorporate the effects of the yielding of soil on the performance of the pile. The Winkler-yielding soil model composed of spring and sliding block is shown in Fig. 1(b). In the linear range where pile deflection is in keeping with that of soil, the soil is considered to be like a linear-plastic spring. When the yielding displacement has reached, the interforces between pile and soil becomes a constant, \( p_u = kv^* \), and the sliding block may be trigged and the deflection continuously increase.

Matlock (1970) [9] were given the experience expression of yielding displacement for cohesive soil. Reese (1974) [10] suggested that typical values of yielding displacement for sand would be 3d/80 (d, the diameter of pile).

To simplify analysis, Bowles (1977) [11] had idealized the nonlinear behavior to linear-plastic behavior as shown in Fig. 1(b) to analyze the behavior of beam. Poulos (1989) [12] proved that simplifying the constitutive relation could provide enough of the calculation precision, and obtain higher computational efficiency.

3. Theoretical solution of pile subject to soil movement

3.1. Development of Control Equation

Practically, the stabilizing pile is defined as infinite beam embedded in elastic foundations subject to lateral soil movement, \( s(x) \). It assumed that two segments of pile are above and below the critical failure surface respectively (Fig.1 a). The deflections of the two pile segments are governed by

\[
EI \frac{d^4v}{dz^4} = \begin{cases} 
  k(s - v) & h < z \leq 0 \\
  -kv & H < z \leq h
\end{cases}
\]  

where, \( h \)=the length of pile above the critical surface; \( H \)=the total length of pile.
(1) For the pile segment within the stable layer \((H<z \leq h)\) shown by Fig.1a, its deflection is given by
\[
v_1 = e^{k_d z} \left( C_{11} \cos \lambda_d z + C_{13} \sin \lambda_d z \right) + e^{-k_d z} \left( C_{13} \cos \lambda_d z + C_{14} \sin \lambda_d z \right)
\]
where, \(\lambda_d = \sqrt{k_d / 4EI} \); \(k_d\) = the subgrade reaction modulus of stable layer.

(2) For the pile segment within the sliding layer \((h<z \leq 0)\) subject to lateral soil movement, its deflection is given by
\[
v_2 = e^{k_u z} \left( C_{21} \cos \lambda_u z + C_{23} \sin \lambda_u z \right) + e^{-k_u z} \left( C_{23} \cos \lambda_u z + C_{24} \sin \lambda_u z \right) + v_n^*
\]
where, \(\lambda_u = \sqrt{k_u / 4EI} \); \(k_u\) = the subgrade reaction modulus of sliding layer; \(v_n^* = \sum_{n=0}^{\infty} v_n^*\) is particular solution of eq.(4.1) respect to \(k_s\).

The analytical solution of passive pile considering interaction of pile-soil is obtained by Zhang (2011) [13] on the assumtion that the soil is on the elastic state. This method may be presented in the next section to evaluate the efficient of the linear-plastic analytical solution by comparing with the method proposed by this paper and infield monitoring results.

(3) If the relative displacement between pile and soil increases bigger than the yielding displacement, the soil may be yielding and gradually downwards from ground surface. Herein, the interforces between pile and soil, \(k(s - v)\), becomes constant \(k v^*\) assuming that subgrade reaction is approximately constant with depth. Eq (4.1) can be rewritten as:
\[
EI \frac{d^4 v}{dz^4} = p_u = k v^*
\]
where, \(p_u\) = ultimate soil resistance.

The solution of eq (7) can be obtained as following
\[
v_3 = P z^3 + C_{32} z^2 + C_{33} z + C_{34}
\]
where, \(P = k_v v^* / 24EI\).

The main work of this study is to derive the analytical solution of pile subject to lateral soil movement in linear-plastic soil.

3.2. Boundary conditions and formula derivation

There are totally eight constant parameters for eqs.(5) and (8), \(C_{11}, C_{12}, C_{13}, C_{14}, C_{31}, C_{32}, C_{33}, C_{34}\).

By the boundary conditions and the plastic critical state, we obtain \(C_{11} = C_{12} = C_{32} = 0\) and \(C_{34} = v^*\).

Accordingly, we can get the solution of other integration constants as what done by Zhang (2011) [13]
\[
C_{34} = A_{34} V_{44} =
\begin{bmatrix}
NE_i^z (h) & NE_i^z (h) & -h^3 & -h
\end{bmatrix}
\]
\[
A =
\begin{bmatrix}
-\lambda_i [NE_i^z (h) + NE_i^z (h)] & \lambda_i [NE_i^z (h) - NE_i^z (h)] & -3h^2 & -1
2 \lambda_i^2 \left[ NE_i^z (h) \right] & -2 \lambda_i \left[ NE_i^z (h) \right] & -6h & 0
2 \lambda_i \left[ NE_i^z (h) \right] & 2 \lambda_i \left[ NE_i^z (h) \right] & -6 & 0
\end{bmatrix}
\]
\[
C =
\begin{bmatrix}
C_{31} & C_{32} & C_{33}
\end{bmatrix}
\]
\[
V =
\begin{bmatrix}
Ph^4 + v^* & 4Ph^3 & 12Ph^2 & 24Ph
\end{bmatrix}
\]

Introducing into eq.(9) three symbols, \(P, NE_i^z (z)\) and \(NE_i^z (z)\), where \(P = k_v v^* / 24EI\)
\[
NE_i^z (z) = e^{i \lambda_i z} \sin (\lambda_i z) \quad NE_i^z (z) = e^{i \lambda_i z} \cos (\lambda_i z) \quad i = d
\]
The integration constants $C_{13}, C_{14}, C_{31}, C_{32}$ can be determined by eq. (9). Substituting all integration constants in eqs. (5) and (8) above, we have the whole deflection curve of the pile $v(z)$, the angle of deflection $\theta(z) = v(z)'$, the bending moment $M(z) = -EIv(z)''$ and shearing force $Q(z) = -EIv(z)'''$. Thus, the behaviour of stabilizing pile subject to soil movement is drawn.

4. Verification

The above calculation procedure has been coded into Matlab program. The accuracy of the present solution is verified with model test reported by Guo (2010) \[14\] based on a series of lateral load tests. The test was performed on free head single tube piles, made by alloy aluminum, with a diameter of 32 mm, wall thickness of 1.5 mm, bending stiffness $EI = 1.7 \text{kN} \cdot \text{m}^2$. A pile of these, named TS32-70, had a total embedment length of 700 mm and the length of pile segment within stable layer was 500 mm. The shaft was embedded in homogeneous dry sand with bulk density of 16.27 $\text{kN/m}^3$, poisson ratio $\nu_s = 0.35$, relative density of 89%. The elastic modulus of dry sand, $E_s$, the soil stiffness, $k$, and the yielding displacement for the dry sand, $v^*$, is determined by equations suggested by Poulos (1980) \[7\], Vesic (1961) \[6\] and Decourt (1991) \[8\] respectively.

The distribution profile of dry sand movement is triangle. That is, dry sand movement at sand surface or 200mm distance to the surface is 70mm or 0mm, respectively. The aluminum tube pile has been installed 7 strain gauges and 2 inclinometer to determine the displacements and stress of piles or soils. The constraint condition at the pile head and tip are free.

The pile deflections and moments measured by Guo (2010) \[14\], predicted by Zhang (2011) \[13\] and the proposed solution for plastic soil-pile interaction are presented in Fig. 2. It can be seen that a very good match on the values of maximum bending moment is achieved by the proposed solution, other than its position is slightly high. Moreover, the pile head deflection values were 9.65 mm and 12.57 mm predicted by the two methods respectively. Obviously, the difference is very small. It is indicated that on the condition of soil-pile large displacement, such as model experiment in the literature \[14\], when the soil surrounding pile would be yielding, employing the proposed solution is reliable and may obtain more precision results than the existing elastic solutions.

5. Conclusion

The study reported in this paper regarding lateral soil movement in a linear-plastic soil may be
summarized as follows:

(1) Based on analyzing the mechanics mechanism of pile-soil reaction associated with slope instability, the governing differential equation for piles subject to lateral soil movement are presented to describe the behavior of stabilizing pile embedded in linear-plastic soil and its analytical solution is derived. Generally, it is applied to analyze nonlinear mechanics mechanism for passive pile in a uniform subgrade reaction modulus considering the yielding of soil.

(2) The procedure is programmed with Matlab code. Furthermore, comparisons between the plastic theoretical solution and model test data showed good agreement, which means that the suggested analytical solutions are reliable. The approach can provide theoretical help for calculating internal forces and displacements of stabilizing pile design.

(3) The proposed method is well validated against the existing available elastic solutions for certain simplified soil layer conditions. It implies that through calculation, when plastic flow of soil appears, soil-pile interforces would be reach limit, and no longer increasing with lateral soil movement. Thus, the proposed method considering the soil nonlinear and plastic yield is capable of representing the actual lateral performance of a pile more realistically than the existing elastic solutions. While ever, the use of elastic method may lead to the big error because of soil mass yielding.

References