THEORETICAL & APPLIED MECHANICS LETTERS **4**, 013013 (2014)

The present status and prospects in the research of orbital dynamics and control near small celestial bodies*

Pingyuan Cui,[1,2] Dong Qiao[1,2,a)]
[1)] *School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China*
[2)] *Key Laboratory of Dynamics and Control of Flight Vehicle, Ministry of Education, Beijing 100081, China*

(Received 26 December 2013; accepted 30 December 2013)

Abstract Small celestial body exploration is of great significance to deep space activities. The dynamics and control of orbits around small celestial bodies is of top priority in the exploration research. It includes the modeling of dynamics environment and the orbital dynamics mechanism. This paper introduced state-of-the-art researches, major challenges, and future trends in this field. Three topics are mainly discussed: the gravitational field modeling of irregular-shaped small celestial bodies, natural orbital dynamics and control, and controlled orbital dynamics. Finally, constructive suggestions are made for China's future space exploration missions.

 [doi:10.1063/2.1401313]

Keywords small celestial body exploration, dynamics and control, gravitational field of irregular-shaped body, natural orbital dynamics, controlled orbital dynamics

I. INTRODUCTION

As one of the major deep-space activities in the 21st century, small celestial body exploration plays an indispensable role in the course of human's knowledge of accumulation, for example, the recognizing of formation and evolution of the solar system, the understanding of the origins and evolution of life and the learning to defend against collision from other celestial bodies. Due to the abundant raw materials on small celestial body, it is known as "fossil" of the solar system. Therefore, the scientific data acquired through exploration provide significant clues for fundamental scientific issues, such as the study of the origin of the solar system, the evolution of planets, small celestial bodies' collision and defense and so on. In addition, the data can also help to effectively validate new technologies as well as improve the ability of deep-space exploration.

The close exploration of small celestial bodies began in 1990s. With the rapid development of space technology, the exploration has developed from flyby, rendezvous, landing to sample return. In recent years, small celestial body exploration has become increasingly important. Among the typical missions including Near Earth Asteroid Rendezvous (NEAR), Stardust, Deep Impact, Hayabusa, Rosetta Mission, and so on, NEAR firstly realized orbiting and landing on an asteroid, and Hayabusa was the earliest that accomplished the sample return mission.

NEAR probe is the first spacecraft succeeded in closing to the asteroid and orbiting it several times. On February 14, 2000, NEAR probe orbited the 433 Eros asteroid with a potato shape and

[a)]Corresponding author. Email: qiaodong@bit.edu.cn.
*Original article comes from P. Y. Cui, D. Qiao. Research progress and prospect of orbital dynamics and control near small bodies. *Advance in Mechanics* **43**, 526–539 (2013) (in Chinese).

the size of 34.4 km×11.2 km×11.2 km in dimension. NEAR forms circular orbits with radius ranging from 200 km, 50 km, to 35 km, and also proves the well stability of gravitational field model and the frozen orbits near the asteroid. Moreover, through changing the orbital inclination and radius, the probe comprehensively detected the surface of the Eros. On February 12, 2001, the spacecraft landed on the south of the saddle-shaped Himeros. From then on, famous institutions, such as Jet Propulsion Laboratory (JPL), Applied Physics Laboratory (APL), paid much attention to issues as precise modeling of irregular-shaped small body, motion mechanism of probe and dynamical mechanism in complex environment.

Hayabusa probe is so far the only one that accomplished rendezvous and sample return task. In November 2005, the probe arrived at 25143 Itokawa asteroid with an irregular elongation shape and the size of 0.535 km×0.294 km×0.209 km in dimension. Since the size and mass of Itokawa asteroid are relatively small, hover orbit is applied to help fulfill the exploring mission. Hovering is a complicated motion combining natural forces and active control together in weak gravitational environment. Later on, Hayabusa aroused great interests in the study of dynamic mechanism of complicated space environment among Japan Aerospace Exploration Agency (JAXA).

Rosetta space craft launched by European Space Agency (ESA) aimed at landing 67P/ Churyumov–Gerasimenko comet, which possesses a starfish shape and diameter of 4 km. The probe will be propelled into the orbit and have long-term observations and comprehensive exploration. At the same time, a lander (Philae) probe will be released on the surface of the comet. When the comet flies towards perihelion, its tail can extend and produce large amount of gas and dust. These products always have a crucial influence on flyby motion and landing orbits. The issues such as the evolution of comet tail, dynamics of gas/dust products and their influences on the stability of orbits stimulated intense interests in Advanced Concept Lab (ACT) of ESA, JPL, and so on.

II. THE GRAVITATIONAL MODELING OF IRREGULAR-SHAPED SMALL CELESTIAL BODIES

Gravitational field modeling is the foundation for analyzing orbital motion characteristics. A variety of shapes present great challenges. There are two commonly used methods for gravitational field modeling: series approximation and three-dimensional approximation. Series approximation method usually approximates the gravitational potential energy by infinite series of some special functions such as spherical harmonic function and ellipsoidal harmonic function. Three-dimensional approximation generally approximates through simplified geometrical bodies, such as triaxial ellipsoid model, polyhedral model, and particle swarm model.

A. Spherical harmonic function models

At the end of the 19th century, the method by Fourier series to approximate gravitational potential function had been proposed. However, due to the complexity and slow convergence rate, this method did not win popularity. The series approximation by Legendre polynomial function was proposed in 1936. On account of its high rate of convergence, it was widely used in gravitational field modeling of Earth, Mars, and other planets. In 1966, Kaula et al.[1,2] established the spherical harmonic model theory. This model has positive advantages, such as simple form, small amount of calculation, explicit analytic expression of gravitational potential energy and readily derived coefficients. Based on in-orbit data, the coefficients of harmonic function can be achieved. Moreover, the analytical solution has been derived to describe perturbed orbital motion so that the influence of non-spherical gravitational field on probe orbit can be summarized. Based on the above advantages, the spherical harmonic function model is widely promoted in the research of celestial mechanics and orbital dynamics.

However, the spherical harmonic model also exposes disadvantages, which might result in error in the analysis of motion. These drawbacks can be summarized into two aspects: truncation error and convergence domain problem. The gravitational potential energy should be approximated by using the infinite series in theory. However, only finite terms can be obtained during the actual calculations. This will inevitably cause error between actual and ideal results. Rossi et al.[3] studied the truncation error and found that it tends to be more significant when the distance between sample points and small bodies are shortened. This phenomenon will lead to serious error during the gravitational field modeling. As for the convergence domain problem, based on Legendre polynomials, the spherical harmonics function could only take effect outside the Brillouin sphere. Therefore, for those planets, which are similar to sphere, the function has no remarkable influence on gravitational field modeling. However, as for the irregular-shaped small celestial bodies, most regions may be located in the Brillouin sphere, so the influence of convergence becomes obvious, as shown in Fig. 1.

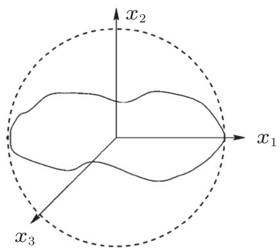


Fig. 1. Brillouin sphere of small body.[4]

When the probe is located in the Brillouin sphere, no matter how many orders of the spherical harmonic expansion are preserved, they can not accurately approximate the gravitational potential function. This will cause difficulties for all the following activities such as flying around the small body at close range, orbital dynamics analysis of hovering and landing, design and control of trajectory. In order to solve this problem, ellipsoidal harmonic model was introduced afterwards.

B. Ellipsoid harmonic function model

In the middle of 20th century, Hobson[5] proposed the ellipsoidal harmonic model by applying the Lamé polynomial approximation. This model can provide a description of the irregular shape of central body by ellipsoidal harmonic coefficients, and help to construct the gravitational field.[5,6] In 1973, Pick et al.[7] established the ellipsoidal harmonic function theory. Compared with the spherical harmonic function model, the ellipsoidal harmonic function extends convergence domain nearby small body surface. For the irregular-shaped small celestial bodies, their Brillouin ellipsoid can be shown in Fig. 2.

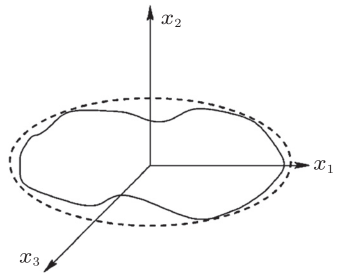


Fig. 2. The Brillouin ellipsoid of small body.[4]

As can be seen in Fig. 2, the region near small body surface can be approximated by the Brillouin ellipsoid. This method can effectively solve the convergence problem near small celestial body surface. In 2001, Garmier and Barriot[4] used the ellipsoidal harmonic model to design the landing orbits around Wirtanen comet, and implemented a series of simulation experiments. The analysis showed that the ellipsoid harmonic model can be employed in designing landing orbits. However, it still has disadvantages, such as complex algorithm, long CPU time, and the complicated derivation process of harmonic coefficients.[8] In order to simplify the algorithm, Dechambre and Scheeres[9] proposed a method to convert ellipsoidal harmonic coefficients to spherical harmonic coefficients. This method simplifies the solution of ellipsoidal harmonic coefficients, and laid solid foundation for the application and promotion of ellipsoidal harmonic model.

C. Triaxial ellipsoid model

The most typical and widely used method in three-dimensional approximation is triaxial ellipsoid model. This method relies on triaxial ellipsoid to approximate the shape of small body, and then obtains the gravitational potential function by integrating triaxial ellipsoid body. Length, width and height of a cuboid are regarded as the three major-axes of ellipsoid model, as shown in Fig. 3.

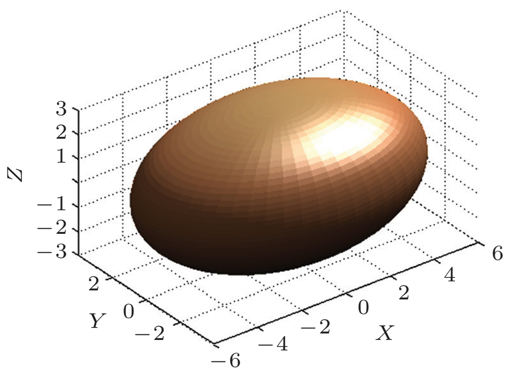


Fig. 3. Triaxial ellipsoid model.

Triaxial ellipsoid modeling has three obvious advantages: the small amount of calculation, easy acquisition of data by astronomical observation and comprehensive representation of gravitational field. However, due to variable shapes, this method also inevitably leads to error during the research.

D. Polyhedron model

In order to obtain precise gravitational field, the polyhedral model and particle swarm model are employed as two classical paradigms. In general, they approximate an irregular-shaped asteroid through new geometrical bodies. By line integral and area integral, the gravitational potential energy is derived to represent gravitational field. The polyhedral model is proposed by Werner,[10] and then developed into a relatively comprehensive method.[11,12] For example, 216 Kleopatra and 6489 Golevka asteroids shown in Figs. 4 and 5 are based on polyhedral model.

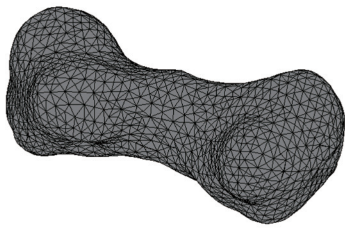


Fig. 4. Polyhedral model of 216 kleopatral asteroid.

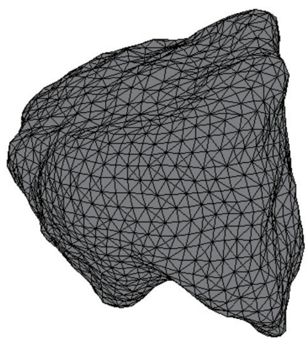


Fig. 5. Polyhedral model of 6489 Golevka asteroid.

Particle swarm method usually applies microspheres or cubes with different densities to approximate irregular-shaped small bodies. Therefore, it is possible to obtain a more precise model.

With the foregoing analysis, we can see that two typical methods (series approximation and three-dimensional approximation) both have merits and shortcomings. On one hand, spherical harmonic or ellipsoidal harmonic function can provide relevant analytical expressions and analytical description of motion behavior to facilitate qualitative and quantitative theoretical analysis but fails to construct a highly precise gravitational field. On the other hand, triaxial ellipsoid and polyhedron models derive gravitational field with high precision, yet fail to provide analytical expression of potential energy function. In summary, it presents to be a major issue to combine the two typical methods together to establish both accurate and easily analyzed gravitational field.

III. NATURAL ORBITAL DYNAMICS AROUND SMALL CELESTIAL BODIES

There are various motional behaviors around small celestial bodies. Their motional characteristics are highly influenced by shape, spin, and other physical parameters. According to controlled condition of motion around small body, the orbit around small body can be classified into two categories, namely, natural motion and controlled motion. The natural motion is an orbital motion which is only affected by gravitational force, non-spherical perturbation force, and other natural forces. The controlled motion is the orbital motion which is formed under combined effects of natural forces and controlled forces.

The natural motional orbit can be distinguished into orbit around the center and orbit around the equilibrium point. Some details about these two types of orbit are introduced as follows.

A. Orbital dynamics of small celestial bodies

For small celestial bodies with relatively large mass, there may exist surrounding orbits like that of a planet, but the motional behavior is very different indeed. Due to the non-spherical perturbation and spin effect of a small body, the great changes of orbital energy and angular momentum of probe maybe occur in a relative short period of time. These changes would make the probe collide with or escape from the small bodies. So the existence and stability of the orbit are the major issues of orbital dynamics. According to gravitational field model, these researches can be classified into three categories: (1) the orbital dynamics on the basis of spherical harmonic or elliptical harmonic models; (2) the orbital dynamics on the basis of triaxial ellipsoid models; (3) the orbital dynamics on the basis of polyhedron models.

1. Orbital dynamics on the basis of spherical harmonic or elliptical harmonic model of small bodies

On the basis of spherical harmonics and ellipsoid harmonics research, the second degree and order gravity field is considered as a typical model of gravitational field. A great amount of researches have been carried out by Scheeres and Hu.[13–15] Firstly, motional types and qualitative analysis have been presented in the second degree and order gravity field with no rotation. After that, some results showed that motional plane of particles will be fixed or precess along the axis of the maximum or minimum moment of inertia of the gravitational body. Some analytical descriptions were also provided.[13,14] Then the motional problem of particle in the slow-rotation gravitational field was investigated. The relational average Lagrange planetary equations were derived. The motional description was given by combining Lagrange planetary equations and Jacobi integral. Some studies found that slow-rotation leads to 1:1 resonance of orbital plane, and the resonance has a critical value. When the spin speed is greater than the critical value, the resonance motion disappears.[15]

Based on the assumption of homogeneous spin, criterion of the second degree and order gravity field was proposed by Hu and Scheeres.[16,17] With this criterion, the stability of near circular equatorial orbits was evaluated. The stable and unstable regions were presented, the resonance relationships between mean motions and self-spin rates are found.[16] In 2008, the periodic orbits in second degree and order gravity field were investigated. The four equilibrium points and five basic orbital families were brought forward. The existence and stability of these orbital families were discussed under different self-spin rates.[17]

These researches provided a relatively clear and comprehensive understanding of the motional characteristics in the second degree and order of gravity fields. In 2009, Hu et al.[18] made a preliminary summary motion around small body and analyzed the influence of dynamical environment. Taking the second degree and order gravity field and Castalia asteroid as an example, the research obtained a summary of characteristics of dynamics and control around small celestial bodies.

2. Orbital dynamics around small bodies on the basis of triaxial ellipsoid models

Triaxial ellipsoid modeling is the commonly used method in astronomical observation. Though there are few researches based on this model, they are representative enough. In 1994, Scheeres[19,20] pioneered the orbital dynamics with spinning in the triaxial homogeneous ellipsoid gravity. According to the spin rate and shape, the triaxial ellipsoid body can be classified into two categories. Researchers found that synchronous orbits of both types are unstable; the stability of quasi-synchronous orbit depends on the types of triaxial ellipsoid bodies. Taking the Vesta and Eros asteroids as an example, the research finally validated parts of conclusions.[19,20] In 2004, Cui et al.[21] studied the motion around "asteroids" by triaxial ellipsoid bodies. The research presented zero-velocity curve, analyzed the possible motion area for the probe, and proposed boundary conditions of collision between asteroid and probe. After that, the research also addressed the effects of asteroid's oblateness and ellipticity on the probe orbital perturbation especially in the case of slow-spin asteroids. Several frozen orbital families and their stability conditions were finally presented. Based on the above research, Cui et al.[21] took Ivar asteroid as an example. (Ivar was regarded as the first target asteroid for Chinese asteroid exploration activity[22]). The zero-velocity curve and collision boundary conditions were presented. The effects of Ivar's oblateness and ellipticity on the orbit were also analyzed (as shown in Fig 6).[23] In 2007, on the basis of triaxial ellipsoid models, the gravitational field of comet nuclear was established by Byram,[24] and then the orbital dynamics of probe near comet was also investigated. At the same time, it proposed a coma conical jet model and analyzed the damping effect of comet shape and gas on the orbit of the comet probe.

It can be seen from the above discussions that the research framework of orbital dynamics of triaxial ellipsoid model has been basically established. It includes the classification of triaxial ellipsoid models, the relationship between models and the stability of specific type of orbits, and motion near small celestial bodies and comets. Meanwhile, further studies on motion characteristics of orbits of triaxial ellipsoid model are in urgent need in the near future. The existence and stability of periodic and quasi-periodic orbits are the focus in the future study.

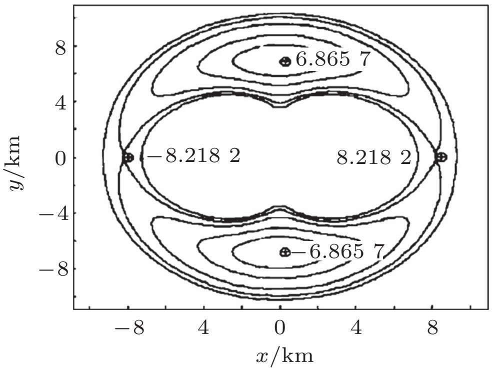


Fig. 6. Zero-velocity curve of Ivar asteroid.[23]

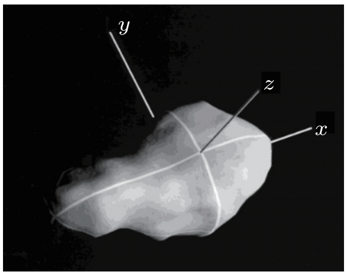
3. *Orbital dynamics of small celestial body on the basis of precise polyhedron models*

The polyhedral model and particle swarm model are the most accurate models of small celestial bodies until now. With the development and improvement, researches have become more accurate and mature. It is worth mentioning that Scheeres et al.[25] did a lot of work related to this field.

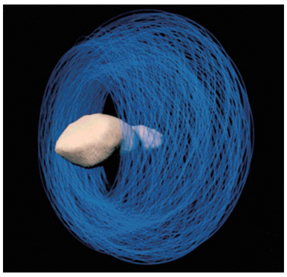
In 1996, Scheeres et al.[25] studied the orbital dynamics based on the polyhedral model of 4769 Castalia asteroid. The research found that there are periodic orbits near Castalia asteroid. The further analysis showed that all of synchronous orbits and direct orbits in the region with triple average diameter of the asteroid are unstable and prone to collide or escape. The conditions and regions of escape and capture were given by numerical analysis.[25] This research revealed the basic laws of motion near the 4769 Castalia asteroid. It laid firm foundation for future close exploration.

4179 Toutatis asteroid, as the target of Chang'E-2 (CE-2) probe, also received wide attention.[26–29] In 1998, Scheeres et al.[30] presented the polyhedral model of 4179 Toutatis asteroid as shown in Fig. 7(a). The orbital dynamics around the non-spindle self-spin asteroid was studied on the basis of the model. The study showed that this system belongs to Hamilton system and possesses all its properties. However, there is no Jacobi constant and the zero-velocity surfaces can not be used for the analysis of motion behavior. The quasi-periodic "frozen orbits" were found by the Lagrange equations of planetary motion (shown in Fig. 7(b)). Some of these orbits are stable, especially the retrograde orbit family in "frozen orbits".[30] Toutatis asteroid is a typical non-spindle self-spina steroid. However, it still remains as a question whether all of non-spindle spin asteroids have these properties.

In terms of orbital dynamics, the periodic orbit and its stability have received special attention. Recently, Yu and Baoyin[31,32] took the polyhedral model of 216 Kleopatra asteroid as an example and studied the dynamic behavior and stability of three-dimensional periodic orbits. 29 basic periodic orbits families were found. The motional characteristics and stability evolution were revealed.[31] What is more, the resonance orbit families near asteroid were also studied.[32] Through numerical analysis, Lara and Scheeres[33] studied the boundary conditions of stability of three-dimensional orbits around a small celestial body. The study also analyzed orbital stability of three-dimensional periodic orbits, and discussed the resonance orbits, and so on.[33] In addition,



(a)



(b)

Fig. 7. Polyhedral model and frozen orbit of Toutatis asteroid.[30]

some researchers have proposed the terminator orbits,[34] which show strong robustness under all kinds of perturbation forces, such as weak gravity force, non-spherical perturbation force, solar radiation pressure force, and so on.[35–38] This kind of orbit most satisfies the requirements for long time in-orbit work. In a word, the study proposed a novel idea for application of perturbation forces around small body.

"Frozen orbit", "terminator orbit", periodic orbits, and stability issues are the hot topics in orbital dynamics research. Due to variable shape, spin and other physical parameters, these topics proposed great challenges.

B. Abstract model its equilibrium points and their periodic orbits of irregular-shaped small celestial bodies

Equilibrium points near small celestial body usually include Lagrange points generated by the third-body gravity and libration points generated by the irregular shape of small celestial body. This paper focuses on the latter one. These equilibrium points are a kind of unique phenomenon caused by irregular-shape and self-spin of small celestial bodies. There are a plenty of periodic and quasi-periodic orbits families near the equilibrium points. Due to the variety in shapes, small celestial bodies' equilibrium points are distributed in different places with different degrees of stability. All these factors exert influence on the orbit family. In order to explore and discover the universal laws, some researchers observed the abstract or ideal model. They found some possible universal laws of motion, and then extended the approach to the motion around general small bodies.

From the perspective of ideal model, Liu et al.[39] extended the previous studies on orbital dynamics near ideal model such as strips, rings, and so on, and presented homogeneous cube as well. The positions of equilibrium points and their stability were studied, periodic orbits and the heteroclinic orbits were analyzed as shown in Fig. 8. After that, Liu et al.[40] further investigated periodic orbits around the fixed homogeneous cube by the Poincare sections and homotopy methods. More widely applied periodic orbits were presented in Ref. 40. These studies facilitate the research of orbital dynamics near irregular-shaped small bodies, which can be approximated by a number of cubes.

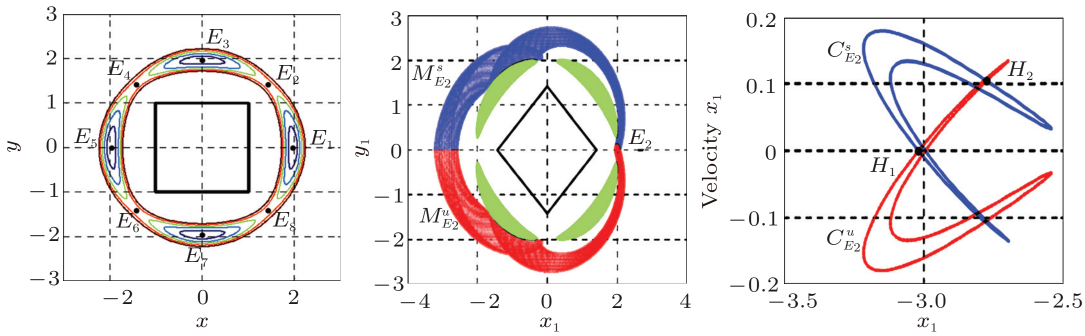


Fig. 8. Equilibrium points and heteroclinic orbits near the homogeneous rotating cube.[39]

In 2012, Yu and Baoyin[41] studied the equilibrium points and periodic orbits in the vicinity of 216 Kleopatra asteroid from the perspective of polyhedral model. Four equilibrium points were found through zero-velocity surface. Analysis showed that these equilibrium points are non-linear and unstable. The six major periodic orbits family were found at the same time also showed instability.[41]

According to the previous studies, Li et al.[42] put forward dumbbell-shaped model which is abstracted from 216 Kleopatra asteroid. They further analyzed this model from the perspective of equilibrium points, the degree of stability, periodic orbits, and some related motion characteristics. The dumbbell-shaped body is further evolved to double spherical cap through increasing the diameter as shown in Fig. 9. The changes of equilibrium points and stability were also studied.[42] In a word, this research extended the motional analysis from single body to combined bodies and promoted the orbital dynamics research in the vicinity of irregular-shaped small celestial bodies.

The above analysis shows that equilibrium points and periodic or quasi-periodic orbits have unique influence on small celestial body exploration. For research in this area, the major difficulty comes from the variety in shape of small celestial body. Therefore, it is almost impossible to obtain the universal laws on distribution of equilibrium points and the periodic orbits.

IV. CONTROLLED ORBITAL DYNAMICS AROUND SMALL BODIES

The probe can perform special motion behaviors under the effect of combined forces such as two-body gravity, irregular-shaped perturbation, environmental disturbance, and active control force. The weak gravitational field of small bodies resulted in unstable natural orbit. However, this property also improves the active control ability. Hover motion is such an example, and the hover orbit near the Eros asteroid is shown in Fig. 10. In terms of the orbital dynamics, this kind of motion belongs to a motion under continuous control and is different from orbit corrections and sustaining. Some research progresses and difficulties in this area are introduced as follows.

In this aspect, Sawai et al.[44,45] studied the hovering orbit control and its stability of uniform spin or non-uniform spin asteroid. For the small bodies with different spin rate, the stable and unstable hovering points were presented, and the possibility of translational motion was discussed. At the same time, the resonance phenomenon between spin of an asteroid and motion of a probe

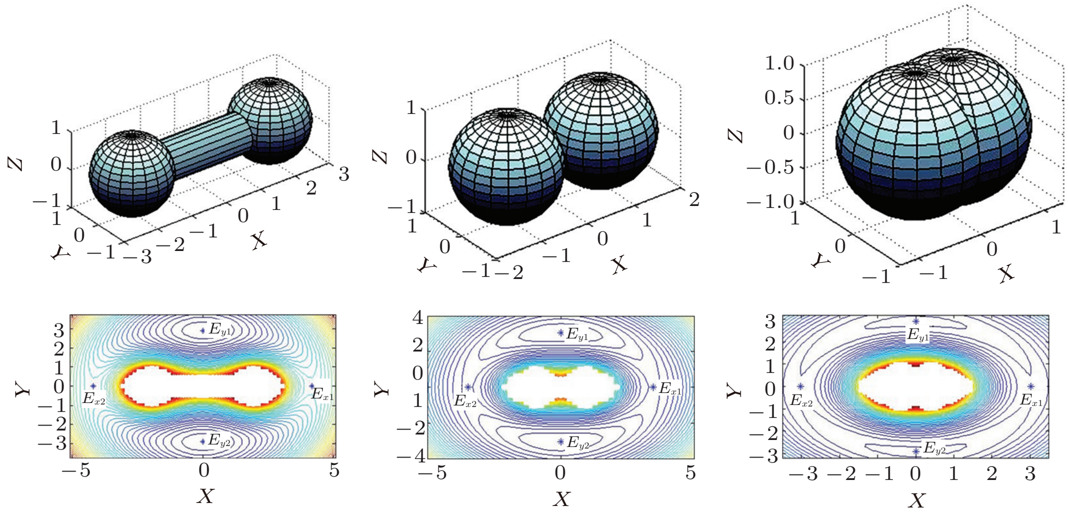


Fig. 9. Dumbbell-shaped body and equilibrium points of different length-diameter ratios.[42]

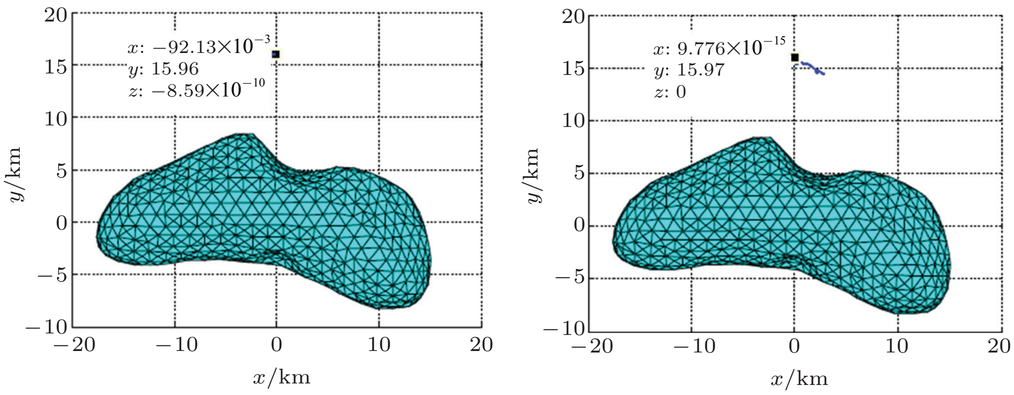


Fig. 10. The hovering orbit around Eros asteroid on basis of different controlled law.[43]

was found. This phenomenon may lead to unstable hover motion in the vicinity of asteroids.[44] For uniform-spin small bodies, the closed-loop control strategy of hovering orbit was analyzed. The controlled method of stable hovering orbit is presented based on the altitude information. And this method was applied to the spherical and ellipsoidal uniform-spin small bodies, as well as the control of hovering orbits.[45]

The stability of hovering orbit is crucial in orbital dynamics and control. Broschart and Scheeres[46] presented a numerical method to analyze the stability of hovering orbits. The stability and stable region of the hovering motion in the body-fixed frame and inertial frame were examined by this method. The research found that the hovering motion is stable in the region of resonance radius in the body-fixed frame. On the contrary, it is unstable outside of resonance radius region in the inertial frame. Later, Broschart and Scheeres[47] verified the stability of hovering controlled law and reliability of thrust-solution in the body-fixed frame and inertial frame. Taking the Itokawa asteroid as an example, the study analyzed the stability of hovering region.

Solar sail propulsion system exhibits a promising prospect in the deep space exploration. This propulsion system generates power or controlled force only from the solar radiation pressure. Thereby it saves certain amount of energy on satellite. Morrow et al.[48] proposed the concept of hovering in the vicinity of an asteroid by the controlled force generated from solar sail, and searched for the hovering points and orbits in different sizes of asteroids for solar sail probes. Meanwhile, the study also found that, as for the asteroids with small mass, the controlled force generated by solar sail is greater than required. The stable hovering orbit can still be found by restricting the magnitude of control force.

Hovering is a special type of motion in small celestial bodies' exploration. The hovering orbit problem is associated with both the dynamics environment and control force. These factors resulted in the complexity and diversity of this problem. It is a major issue to find an optimal control strategy of hovering orbit, which could not only meet the mission's requirements but also keep the orbit stable.

V. PROSPECTION TO THE DEVELOPMENT OF ORBITAL DYNAMICS AND CONTROL FOR SMALL BODY EXPLORATION

After successful implementation of the lunar exploration project, related researches on asteroid exploration have been carried out in China. Orbital dynamics and control, which is the key problem in the asteroid exploration mission has received wide attentions. The future development of orbital dynamics and control in China may confront the following challenges:

(1) Modeling and evaluation of dynamics environmental around small celestial bodies

Small celestial body exploration will concentrate on orbiting, landing, and sample return. For these tasks, accurate and reliable dynamic environment is indispensable. How to comprehensively evaluate major physical parameters (such as mass, shape, gravity field and spin state, and so on) and construct an accurate model are essential issues for future research.

(2) Dynamic mechanism under the effects of various disturbing forces

The effect of disturbance force (such as solar radiation pressure, solar tide, etc.) becomes more prominent in weak gravitational environment. Making full use of combined effects to design long-term and stable periodic orbits is a significant direction.

(3) Orbital dynamics and control issues for accompanying fly and hovering motion in week gravitational environment

Compared with the planet, the mass of small bodies is much smaller. But it still covers a range of magnitude. For the asteroid with small mass, it is difficult to form an orbit only by the gravity force. To achieve the hovering or accompanying fly, the dynamics relationship among the Sun, asteroid, disturbing forces should be further studied. What is more, the conditions and criteria of forming special orbits should also be given enough attention.

(4) Orbital dynamics and control issues of binary bodies

According to astronomical observations, almost 10% of near-Earth asteroids belong to the binary body type. For the binary-body system, the basic motion behavior will be an important issue in the future. The orbital dynamics of binary-body system is different from two-body and three-body models. Since the mass of two bodies and the probe can not be ignored, the mutual influence becomes significant.

VI. CONCLUSION

Orbital dynamics and control is of great significance in achieving orbiting and landing exploration activities. This paper firstly reviewed the development of foreign exploring missions. Secondly, it discussed the following three aspects: modeling around irregular-shaped small celestial bodies, natural orbital dynamics, and the controlled orbital dynamics. Then it analyzed the orbiting dynamics and control from the perspective of periodic orbits, quasi-periodic orbits, and the hovering orbits. Finally, constructive suggestions are made for China's future space exportation.

At present, orbital families with high practical value have been put forward and applied in some of the researches. With the development of small celestial body exploration, periodic, quasi-periodic, and hovering orbits are the main choices for exploration activities. China has fulfilled the first flyby mission, and is planning to accomplish orbiting, landing, and sample return task in the future. Due to the numerous of impact factors such as large amount, shape, complex dynamical environment, etc., the researches and analysis of orbital motion are still confronting serious challenges. In conclusion, it is proved absolutely necessary to further promote systematical and thorough research in small celestial body exploration.

1. W. M. Kaula. Theory of Satellite Geodesy: Applications of Satellites to Geodesy. Dover Publications, Mineola (2000).
2. A. K. Sharma, M. L. Narasaiah. Dynamics of Rigid Bodies. Discovery Publishing House, New Delhi (2007).
3. A. Rossi, F. Marzari, P. Farinella. Orbital evolution around irregular bodies. *Earth Planets Space* **51**, 1173–1180 (1999).
4. R. Garmier, J. P. Barriot. Ellipsoidal harmonic expansions of the gravitational potential: theory and application. *Celestial Mechanics and Dynamical Astronomy* **79**, 235–275 (2001).
5. E. Hobson. The Theory of Spherical and Ellipsoidal Harmonics. Chelsea Publishing Company, New York (1955).
6. H. G. Walter. Dynamics of Satellites. Springer Heidelberg, Berlin (1969).
7. M. Pick, J. Picha, V. Vyskocil. Theory of the Earth's Gravity Field. Elsevier Scientific Publishing Company, New York (1973).
8. B. William. Comment on a formula for the gravitational harmonic coefficients of a triaxial ellipsoid. *Celestial Mechanics and Dynamical Astronomy* **67**, 107–110 (1997).
9. D. Dechambre, D. J. Scheeres. Transformation of spherical harmonic coefficients to ellipsoidal harmonic coefficients. *Astronomy & Astrophysics* **387**, 1114–1122 (2002).
10. R. A. Werner. The gravitational potential of a homogeneous polyhedron. *Celestial Mechanics and Dynamical Astronomy* **59**, 253–278 (1994).
11. R. A. Werner. On the gravity field of irregularly shaped celestial bodies, [Ph. D. Thesis]. University of Texas, Austin (1996).
12. R. A. Werner, D. J. Scheeres. Exterior gravitation of a polyhedron derived and compared with harmonic and Mascon gravitation representations of asteroid 4769 Castalia. *Celestial Mechanics and Dynamical Astronomy* **65**, 313–344 (1997).
13. D. J. Scheers. The effect of C22 on orbit energy and angular momentum. *Celestial Mechanics and Dynamical Astronomy* **73**, 339–348 (1999).
14. D. J. Scheeres, W. D. Hu. Secular motion in a 2nd degree and order-gravity field with no rotation. *Celestial Mechanics & Dynamical Astronomy* **79**, 183–200 (2001).
15. W. D. Hu, D. J. Scheeres. Spacecraft motion about slowly rotating asteroids. *Journal of Guidance Control and Dynamics* **25**, 765–775 (2002).
16. W. D. Hu, D. J. Scheeres. Numerical determination of stability regions for orbital motion in uniformly rotating second degree and order gravity fields. *Planetary and Space science* **52**, 685–692 (2004).
17. W. D. Hu, D. J. Scheeres. Periodic orbits in rotating second degree and order gravity fields. *Chinese Journal of Astronomy and Astrophysics* **8**, 108–118 (2008).
18. W. D. Hu, D. J. Scheeres, K. H. Xiang. The characteristics of near asteroid orbital dynamics and its implication to mission analysis. *Progress in Astronomy* **27**, 152–166 (2009).
19. D. J. Scheeres. Dynamics about uniformly rotating tri-axial ellipsoids applications to asteroids. *Icarus* **110**, 225–238 (1994).

20. D. J. Scheeres. Satellite dynamics about asteroids. *Advances in the Astronautical Sciences Series Spaceflight Mechanics* **87**, 275–292 (1994).
21. H. T. Cui, X. Y. Shi, P. Y. Cui, et al. Spacecraft motion analysis about slowly rotating small body. *Acta Aeronauticaet Astronautica Sinica* **25**, 16–20 (2004).
22. P. Y. Cui, D. Qiao, H. T. Cui, et al. Target selection and transfer trajectories design for exploring asteroid mission. *Science China Technological Sciences* **53**, 1150–1158 (2010).
23. H. T. Cui, X. Y. Shi, P. Y. Cui, et al. Motion analysis of spacecraft around the asteroid Ivar. *Journal of Astronautics* **25**, 251–255 (2004).
24. S. M. Byram, D. J. Scheeres, R. C. Michael. Models for the comet dynamical environment. *Journal of Guidance, Control, and Dynamics* **30**, 1445–1454 (2007).
25. D. J. Scheeres, S. J. Ostro, R. S. Hudson, et al. Orbits close to asteroid 4769 Castalia. *Icarus* **121**, 67–87 (1996).
26. J. C. Huang, X. L. Wang, L. Z. Meng, et al. Approaching strategy and imaging technique for CE-2 flyby 4179 asteroid mission. *Science China Technological Sciences* **43**, 478–486 (2013) (in Chinese).
27. S. C. Hu, J. H. Ji, Y. H. Zhao, et al. Orbit determination and precision analysis of Toutatis asteroid for CE-2 flyby asteroid mission. *Science China Technological Sciences* **43**, 506–511 (2013) (in Chinese).
28. D. Qiao, J. C. Huang, P. Y. Cui, et al. Selection target for CE-2 flyby asteroid mission. *Science China Technological Sciences* **43**, 602–608 (2013) (in Chinese).
29. D. Qiao, J. C. Huang, P. Y. Cui, et al. Trajectory design of CE-2 flyby Toutatis asteroid mission. *Science China Technological Sciences* **43**, 487–492 (2013) (in Chinese).
30. D. J. Scheeres, S. J. Ostro, R. S. Hudson, et al. Dynamics of orbits close to asteroid 4179 Toutatis. *Icarus* **132**, 53–79 (1998).
31. Y. Yu, H. X. Baoyin. Generating families of 3D periodic orbits about asteroids. *Monthly Notices of the Royal Astronomical Society* **427**, 872–881 (2012).
32. Y. Yu, H. X. Baoyin. Resonant orbits in the vicinity of asteroid 216 Kleopatra. *Astrophysics Space Science* **343**, 75–82 (2013).
33. M. Lara, D. J. Scheeres. Stability bounds for three dimensional motion close to asteroids. *Journal of the Astronautical Sciences* **50**, 389–409 (2002).
34. S. B. Broschart, D. J. Scheeres, B. F. Villac. New families of multi-revolution terminator orbits near small orbits. *Advances in the Astronautical Sciences* **135**, 1685–1702 (2010).
35. D. J. Scheeres. Satellite dynamics about small bodies: Averaged solar radiation pressure effects. *Journal of the Astronautical Sciences* **47**, 25–46 (1999).
36. J. Bellerose, H. Yano, D. J. Scheeres. Solar radiation pressure perturbations at binary asteroid systems. *Advances in the Astronautical Sciences* **135**, 747–764 (2010).
37. D. J. Scheeres. Orbit mechanics about asteroids and comets. *Journal of Guidance Control and Dynamics* **35**, 987–997 (2012).
38. D. J. Scheeres. Orbital mechanics about small bodies. *Acta Astronautica* **72**, 1–14 (2012).
39. X. D. Liu, H. X. Baoyin, X. R. Ma. Equilibria, periodic orbits around equilibria, and heteroclinicconnections in the gravity field of a rotating homogeneous cube. *Astrophysics Space Science* **333**, 409–418 (2011).
40. X. D. Liu, H. X. Baoyin, X. R. Ma. Periodic orbits in the gravity field of a fixed homogeneous cube. *Astrophysics Space Science* **334**, 357–364 (2011).
41. Y. Yu, H. X. Baoyin. Orbital dynamics in the vicinity of asteroid 216 Kleopatra. *The Astronomical Journal* **143**, 62–71 (2012).
42. X. Y. Li, D. Qiao, P. Y. Cui. The equilibria and periodic orbits around a dumbbell-shaped body. *Astrophysics and Space Science* **348**, 417–426 (2013).
43. T. M. Winkler. Fuel-efficient feedback control of orbital motion around irregular-shaped asteroids, [M. S. Thesis]. Iowa State University, Iowa (2013).
44. S. Sawai, D. J. Scheeres. Hovering and translational motions over small bodies. *Advance in the Astronautical Sciences* **108**, 781–796 (2001).
45. S. Sawai, D. J. Scheeres, S. B. Broschart. Control of hovering spacecraft using altimetry. *Journal of Guidance, Control and Dynamics* **25**, 786–795 (2002).
46. S. B. Broschart, D. J. Scheeres. Numerical solution to the small-body hovering problem. *Advances in the Astronautical Sciences* **114**, 875–894 (2003).
47. S. B. Broschart, D. J. Scheeres. Control of hovering spacecraft near small bodies: Application to Asteroid 25143 Itokawa. *Journal of Guidance Control and Dynamics* **28**, 343–354 (2005).
48. E. Morrow, D. J. Scheeres, D. Lubin. Solar sail orbit operations at asteroids. *Journal of Spacecraft and Rockets* **38**, 279–286 (2001).