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Research on Multi-Stage Inventory Model by Markov Decision Process

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Abstract

This paper researched multi-stage inventory system and established limited inventory Markov model, on the other hand it induced DP algorithm of limited inventory Markov model. The results proved that the reorder point of multi-stage inventory system can guarantee demand, and also allows the storage costs to a minimum level in accordance with the above model.

Keyword: Multi-stage inventory; Markov model;DP algorithm

1. Introduction

Multi-stage inventory control is a necessary strategy of handling uncertainties of demand in the production, supply and so on. Typically, most of probability distribution is used to treat uncertainty, but businesses often lack information to deal with this probability distribution, therefore resulting in high inventory costs.

Generally speaking, stocks change with the size of enterprises, decision on inventory levels of each product is based on the existing inventory, order point of products. but the manager also need not less than a certain value of the cash to meet the demand in order to pursue long-term profit, not just limited to a certain time. But if we know the transfer probability of demand between different period. Here decision on inventory can made based on Markov

2. Overview of MDP

Markov decision process (MDP) refers to decision-makers observe Markov random dynamic system periodically or continuously and make decision from the set of available actions to choose an action policy sequentially according to the state each time, the future system state is random, and the transition probability is a Markov process. Decision-makers make a new decision based on the new observed state and carry out repeatedly.

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3. Limited Stage Markov Inventory Model

As for a single product inventory problem, Markov decision process can be used to determine the optimal order point and the optimal order quantity, the time of decision-making inventory is weekly. the system state is the current inventory. In a given state, the action can be used to enable suppliers to provide products depends on state of transition probability ,the amount of order and the demand for the next week. a strategy is a decision of order quantity. which is a function of inventory, it can tell policy-makers how to determine order quantity to ensure customer satisfaction in the probability of not less than the given conditions at any time to minimum a long-term average cost if the system changes.

In this issue, the optimal strategy should have the property, easy to use and do not change over time. Considering company operation laws, if the company orders, which also includes some potential question, companies often need to predict future needs. But usually, this prediction is very difficult, often influenced by many factors. Taking into account many uncertainties, so this paper researches optimal strategies of the special structure by establishing MDP model.

In a discrete multi-stage inventory model, each decision-making time the state of inventory is $\Omega = \{0,1,...s_k, s_{k+1}, \cdots S_k\}$ and the action to choose is two: do not order (a = 0), or when the inventory bellow s ,the order quantity will reach $S\ (a=1)$ .As for each stock state $j \in \Omega$, the action is set $A(i) = A = \{0,1\}$

According to the foregoing (s, S) policy

$$u(x_k) = \begin{cases} S_k - x_k, x_k < s_k \\ 0, x_k \geq s_k \end{cases} \quad (1)$$

We can get the state transition equation

$$x_{k+1} = \begin{cases} x_k - w_k, x_k - w_k \geq s_k \\ S, x_k - w_k < s_k \end{cases} \quad (2)$$

Now assume that the demand between different periods satisfy the Markov property, that is, without aftereffect.

Asfor $j = s_k \cdots, S_k$,

and $i = 0,1,...j, j+1, \cdots, S_k$

1. $x_k > s_k$ In accordance with the definition of Markov transition probability

$$\begin{aligned} &P\{x_{k+1} = j | x_k = i, x_{k-1}, \cdots x_0\} \\ &= P(x_k - W_k = j | x_k = i, x_{k-1}, \cdots x_0) \\ &= P(i - W_k = j | x_k = i, x_{k-1}, \cdots x_0) \\ &= P(W_k = i - j) \end{aligned} \quad (3)$$

a corresponding transition probability is

$$P(j | i,1) = P(i - j) \quad (4)$$

2. $x_k \le s_k$,

$$\begin{cases} P\{x_{k+1} = j | x_k = i, x_{k-1}, \cdots x_0) = 1, j \in S \\ P\{x_{k+1} = j | x_k = i, x_{k-1}, \cdots x_0) = 0, j \notin S \end{cases} \quad (5)$$

Means when $x \le s_k$, the highest inventory is S_k。

transition probability is

$$P(j|i,0) = \begin{cases} 1, j = S \\ 0, j \notin S \end{cases} \quad (6)$$

For state j, the cost function is $c_k(j|i,a)$, according to the optimal equation , dynamic programming DP algorithm can be rewritten as

$$V_k(j) = \min_u \left[c_k(j|i,a) + E\{V_{k+1}(w_k)\} \right] \quad (7)$$

Accordingly, the probability distribution of demand in the past will be given to seek mathematical expectation based on non-parametric kernel estimation method.

State transition equation is

$$x_{k+1} = x_k + u_k - w_k$$
$$j = i + u_k - w_k \quad (8)$$

According to the optimal equation, the equation can be written as

$$V_\alpha^k(j) = \min\left\{ c_k(j|i,a) + \sum_{w_k} V_\alpha^{k+1}(i + u - w_k) P(w_k) \right\} = \min\{C_k(j|i,a) + \sum_{w_k > i - s_k} V_\alpha^{k+1}(s) P(j|i,a=0) + \sum_{w_k \le i - s_k} V_\alpha^{k+1}(i) P(j|i,a=1)\} \quad (9)$$

For limited stage Markov inventory model, the iterative method can be used to get the optimal (s, S) policy by Obtaining the corresponding minimum cost at all stages.

4. Applications

now take spare parts of a company store as example to seek the optimal storage strategy, known as the spare storage capacity is S = 9, subscription cost C = 10 yuan, storage cost Y = 20 yuan, shortage cost Z = 40 yuan. We assume that demand obey Poisson distribution ($\lambda = 2$)

$$P(x = k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (10)$$

k=1,2,3,…,9

According to Markov's no aftereffect, transition probability can be

$$p_{ij} = p(j|i) = \frac{p(k=j)}{p(k=i)} = \frac{\frac{\lambda^j e^{-\lambda}}{j!}}{\frac{\lambda^i e^{-\lambda}}{i!}} = \lambda^{j-i} \frac{i!}{j!} \quad (11)$$

i,j=1,2,3,…,9

Where K (Poisson intensity)is the average number of spare parts, called as the demand rate. input the above calculation condition to the computer and policy iteration will be got as follows:

Initial strategy：D1=[d1(0),d1(1),,d1(9)]=[0,0,0,0,0,0,0,0,0,0] result of the first iteration：

D2=[d2(0),d2(1),…,d2(9)]=[9,8,7,6,5,4,3,2,1,0]

result of the second iteration：

D3=[d3(0),d3(1),…,d3(9)]=[9,8,0,0,0,0,0,0,0,0]

result of the third iteration：

D4=[d4(0),d4(1),…,d4(9)]=[9,8,7,0,0,0,0,0,0,0]

result of the fourth iteration：

D5=[d5(0),d5(1),…,d5(9)]=[9,8,7,0,0,0,0,0,0,0]

Known by the above calculation after four iterations, spare parts storage strategy set D4 = D5, so D *= [9,8,7,0,0,0,0,0,0,0] is a optimal storage strategy of long-term storage, that is, when x is less than inventory limit (ie n = 2) when ordering spare parts for the supplement is 9,8,7, but, when x is greater than n , no order will be needed.

Replenishment of stocks x is lower than n (n> 2), the spare capacity savings will be N-x (7,6,5,4,3,2,1). Inventory reduction will significantly reduce the occupation of the capital reserve of spare parts, therefore inventory savings will be a considerable sum. order in accordance with the above strategy can not only guarantee the demand but also minimize the level of storage costs. More generally, different storage strategy will be got by inputting the continuous change of a particular variable under the condition of fixed capacity.

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