Modelling stock market crashes: the case of Bucharest Stock Exchange

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Abstract

In this paper we investigate the bubble behaviour of Bucharest Stock Exchange, using log periodic power laws models. Analysing the behaviour of the most speculative index from Bucharest Stock Exchange, BET-FI, we prove that LPPL models are a useful tool in recognizing the behaviour of a stock market bubble and they have predictive abilities for the critical point of a bubble.

Iterative calibration of the model for BET-FI regime led to a reasonable estimate of the stock market crash in January 2008. Using the same methodology, the anti-bubble regime from 2008 is fitted and we find an accurate prediction of the local point of phase transition from 27/10/2008.

\textit{Keywords: Log Periodic Power Law, Stock Market Bubble, Crash}

1. Introduction

At this moment, the researchers and practitioners involved in the study of economic phenomena are facing a critical time, when the very foundations of economics are questionable; the economy is a complex system and understanding

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the dynamics of such a system cannot be made without using a set of tools, methods and techniques combining mathematical rigor, empirical observation and methodological approach coming from physics.

This combined approach, also known as econophysics, has been proven to be extremely useful in understanding the dynamics of complex phenomena regarding the financial markets as a whole and especially the stock markets.

The history of stock markets could be regarded as a succession of stationary regimes, upwards and downwards trends and also severe crashes.

Large financial crashes like the Tulip Mania (17th century, The Netherlands) and the South Sea Bubble (18th century, The Great Britain) had an influential effect on the economic environment of their time.

The 20th century had plenty of such catastrophic events, from the Great Depression in the ’30s, to the Black Monday in 1987, the internet companies bubble in the late’90s in the US, to the stock markets crash in 2007.

In the context of the financial and economic crisis, it is essential to establish a general framework in order to identify the bubble behaviour of a stock price and to estimate the most probable time of a crash.

Detection of a speculative stock market bubble and estimation of the critical point, the moment when the transaction price drops dramatically, are topics extensively discussed by both researchers in the academia and practitioners.

In order to understand the origin and the evolution of stock market bubbles, which in most of the cases end in a severe crash, we need to address few fundamental questions, as the entire process of building a model for stock market crash depends on the responses to these questions.

Synthetically, these questions can be stated in the following manner (Sornette (2003)): What is a stock market crash? What is the mechanism of appearance and evolution of stock market crashes? What is the cause of stock market crashes? Can the moment of appearance, as well as its magnitude be predicted?

The answers to these questions can result after careful examination of the theoretical models of stock market crashes, as well as after applying some quantitative methods on the stock prices time series, as well as after testing some fundamental hypotheses related to the predictability regime of these data series.

2. Literature Review And Hypotheses

1.1. The origin and evolution of stock market bubbles

Building a theoretical model for stock market bubbles is proven to be extremely difficult, especially due to the lack of agreement among economists regarding a coherent definition of stock market bubbles.

From a theoretical perspective, the concept of stock market bubble could be embedded in one of the following frameworks: Efficient Market Hypothesis (EMH), Rational Bubble View (RBV) or LogPeriodic Power Laws (LPPL).

From the classical definition of Fama (1970), until more recent developments of Timmerman and Granger (2004), Efficient Market Hypothesis is inseparable from the concept of information and the mechanism of incorporation of a certain set of information in the trading price of a financial asset.

Under the EMH approach, all the relevant information is immediately reflected in the stock prices, so tomorrow's price change will reflect only the tomorrow’s information and will be independent of today’s price change (Malkiel, 2003). As the information is unpredictable, the price changes must be unpredictable and pure random.

In this context, the concept of bubble is questionable, since at any moment the prices reflect all available information. From this point of view, a crash is only the effect of negative information which is incorporated in the trading price for a short period of time and is virtually impossible to predict the magnitude or the time of a stock market crash.

The Rational Bubble View paradigm (Friedman and Abraham, 2007) assumes the existence of a fundamental value for a financial asset (intrinsic value), \( V \), which is unobservable, and a directly observable transaction price \( P \).

The speculative bubble can be defined in this context as an abnormal deviation of the transaction price from the
intrinsic value \( (B_t = P_t - V_t, >> 0) \) and consequently, the stock market crash can be seen as the event consisting of the sudden evolution of \( B_t = P_t - V_t \), from a large, positive value, to zero, or even a negative value. The major challenge arising from this approach is how to define and estimate the intrinsic value of a financial asset.

Using the LPPL approach, Sornette and Johansen (1999) have analysed the stock market bubbles and crashes at macroeconomic and microeconomic levels. From a macroeconomic point of view, the model assumes that we are dealing with rational markets which have incomplete information. In such a market, the trade price not only reflects the fundamental value, but also the future expectations related to profitability and risk.

From a microeconomic point of view, the Sornette – Johansen model assumes that investors (rational investors and noisy traders) are connected locally through certain networks that govern their anticipations regarding market earnings. Also, along with this imitative behaviour manifested on a horizontal level, each investor receives information at a vertical level, from other public or private sources.

Moreover, trading decisions depend on the decisions of other members of the network, but may also include external influences.

Following these interactions, investors develop a self-similar behaviour, pushing the market into a speculative bubble regime, which may end in a severe crash or may exhibit a smooth evolution around a descending local trend. In this approach, a stock market bubble is a market regime where trading prices exhibit a super-exponential behaviour, i.e. the price changes have an exponential evolution.

1.2. The Johansen – Ledoit – Sornette model

At a microscopic level, JLS (2000) and Sornette (2003) assume that the noise traders are locally connected through a network and every investor could be described by two trading positions: \( s_i = 1 \) (buy) or \( s_i = -1 \) (sell).

If, in addition, for every investor \( i \) there are \( N(i) \) investors in the local network, then the state of the investor \( i \) is a result of the following Markov process: \( s_i = \text{sign}(K \sum_{k \leq N(i)} s_k + \sigma_i) \) where \( K > 0 \) controls for the herding behaviour, \( \sigma > 0 \) controls for the idiosyncratic behaviour and \( \varepsilon_i \) is a Gaussian white noise.

According to this model, at the origin of stock crashes is not a chaotic behaviour, but an ordered one, resulting from herding effect among investors. Imitative behaviour of irrational agents leads to the development of a speculative bubble. When this trend is growing up near a critical point, most investors will take a short position, leading to a dramatic decrease in the transaction price. A crash is not yet a certain event, but is characterized by a certain probability distribution: therefore, to invest in the context of a speculative bubble is a rational choice, because the risk of a crash is compensated by the expected large returns, since the probability that a speculative bubble will collapse suddenly in a crash is negligible.

At a macroscopic level, according to the mean field theory, an imitative process could be described through the hazard rate \( h(t) \), as a solution of a differential equation:

\[
\frac{dh}{dt} = Ch^\delta
\]

where \( C > 0 \) and \( \delta + 1 > 1 \) is the average number of interactions among investors. The hazard rate \( h(t) \) is the probability per time-unit of having a crash, conditioned by the fact that the crash has not happened yet till the time \( t \).
The solution of equation (1) is \( h(t) = \left( \frac{h_0}{t_c - t} \right)^\alpha \) with, \( \alpha = \frac{1}{\delta - 1} \), \( t_c \) being the critical time of the crash.

There are several restrictions in this formula:
- \( \alpha > 0 (\delta > 1) \), meaning that the hazard rate increases before the critical time;
- \( \alpha < 1 (\delta > 2) \), so every investor is connected via a local network with at least two other investors.

The behaviour of the hazard rate before the critical time could be expressed using a periodical-power law, following the Ising model originated from physics:

\[
h(t) \approx A + B(t_c - t)^\beta + C(t_c - t)\cos[\omega \ln(t_c - t) + \phi].
\]  

(2)

1.3. LPPL fit for stock market bubbles

Under risk neutrality and rational expectations hypothesis, JLS (2000) have deduced the price dynamics before the crash:

\[
\ln \frac{p(t)}{p(0)} = k \int_{t_0}^{t} b(u)du.
\]

The reason behind this expression is that the crash probability should be compensated by larger price changes, prior to the stock market crash (Blanchard, 1979).

Sornette (2000) compares seismic activity to the evolution of speculative bubbles, and deduces the evolution law for stock prices before and during the crash, which is seen as a critical time.

Thus, the trading price before the crash follows a log-periodic power law:

\[
\ln p(t) \approx A + B(t_c - t)^\beta \{ 1 + C \cos[\omega \ln(t_c - t) + \phi]\}
\]

(3)

where \( p(t) \) is the price at moment \( t \), \( t_c \) is the critical time (the most probable moment of the crash), and \( \beta, B_0, B_1, \omega, \phi \) are the parameters of the model which give its log-periodic feature.

In order to have a proper specification of the model, there are several constrains applied to the parameters:
- \( A > 0 \) - usually this the price at the critical time \( t_c \);
- \( B < 0 \);
- \( C \neq 0 \), \( |C| < 1 \) – this parameter controls the magnitude of oscillations around the exponential trend;
- \( 0 < \beta < 1 \) – controls the growth rate of the magnitude and is the most important feature capturing the imminence of a regime switching, as his value is close to zero;
- \( \omega \in (0, \infty) \) - controls for the amplitude of oscillations;
- \( \phi \in [0,2\pi] \) – a phase parameter.

Altough the above equation is written in terms of the logprice, there is a lot of papers in which the raw trading price is used in order to estimate the critical time and the litterature is quite inconclusive whether the logprice or the raw price shoud be used.
Johansen, Ledoit and Sornette (2000) have applied these models to successfully predict famous crashes like the one in October 1987 and for the Brazilian market, Cajueiro, Tabak and Werneck (2009) have applied these models to predict the catastrophic behaviour of the price series of 21 stocks.

Financial Crisis Observatory (ETH – Zurich) has released during the past few years predictions about the bubble behaviour of different assets and they have succeeded to predict two famous events of this type: Oil Bubble – 2008 and Chinese Index Bubble – 2009.

Fantazzini and Geraskin (2011) provide an extensive review of theoretical background behind the LPPL models, estimation methods and various applications, pointing out that although the literature on this subject is heterogeneous, LPPL fit for asset bubbles could be a useful tool in predicting the catastrophic behaviour of capital markets as a whole.

Moreover, even using such a model, the prediction of critical time is not very accurate, Kurz-Kim (2012) shows that LPPL models could be used as a early warning mechanism of regime switching in case of a stock market.

Although these modes have been applied on a large variety of markets and assets, to the best of our knowledge, this is the first study regarding the fitting of a bubble regime for the Romanian stock market.

In the light of the literature, we propose the hypotheses as following:

\textbf{H1: The evolution of BET-FI index of Bucharest Stock Exchange had a bubble signature for the period 2001-2007.}

\textbf{H2: LPPL models have predictive ability for the critical time of stock market bubble in case of BET-FI index.}

\section{Methodology}

\subsection{Research Goal}

In this paper we aim to identify the bubble behavior of the BET-FI index for the Romanian stock market and to estimate the critical times using log-periodic power laws.

Using daily data from the Bucharest Stock Exchange, we apply Sornette’s methodology and we predict the moment of regime switching from January 2008 and also, applying LPPL for the anti-bubble developed during the year 2008, we obtain an accurate estimation of the critical point from 27 October 2008, which is a local minimum for BET-FI index.

\subsection{Data}

We used daily data for BET-FI index of Bucharest Stock Exchange, for the period 3.01.2001 – 23.12.2008(1978 daily observations). Although Bucharest Stock Exchange reports the values of two other major indexes (BET, the index of the most liquid companies and BET-C, a composite index for the entire market), we choose the BET-FI index due to its speculative potential, as a index of financial investment companies.
Fig.1. Closing price of BET-FI Index

The Romanian stock market, like all the emergent markets in the Eastern and Central Europe, did not react immediately to the critical event from 15\textsuperscript{th} September 2007, when the collapse of Lehman Brothers was announced, triggering a severe financial crisis for markets all around the world.

Actually, from 2001 to 2007, the BET-FI index exhibits a near exponential behaviour, reaching its historical maximum on 25\textsuperscript{th} July 2007 and for the rest of the year 2007 the evolution was quite stable.

As it can be gleaned from Table 1, the moment of regime switching for BET-FI index was the beginning of 2008, when during January the cumulated daily returns reached around -26%.

<table>
<thead>
<tr>
<th>Month</th>
<th>Cumulated Returns</th>
<th>Month</th>
<th>Cumulated Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jun-07</td>
<td>19.46%</td>
<td>Mar-08</td>
<td>-13.85%</td>
</tr>
<tr>
<td>Jul-07</td>
<td>-1.24%</td>
<td>Apr-08</td>
<td>-0.42%</td>
</tr>
<tr>
<td>Aug-07</td>
<td>-2.16%</td>
<td>May-08</td>
<td>6.48%</td>
</tr>
<tr>
<td>Sep-07</td>
<td>-8.17%</td>
<td>Jun-08</td>
<td>-19.10%</td>
</tr>
<tr>
<td>Oct-07</td>
<td>2.95%</td>
<td>Jul-08</td>
<td>-29.12%</td>
</tr>
<tr>
<td>Nov-07</td>
<td>-12.32%</td>
<td>Aug-08</td>
<td>-9.21%</td>
</tr>
<tr>
<td>Dec-07</td>
<td>8.83%</td>
<td>Sep-08</td>
<td>-21.68%</td>
</tr>
<tr>
<td><strong>Jan-08</strong></td>
<td><strong>-25.40%</strong></td>
<td>Oct-08</td>
<td>-83.12%</td>
</tr>
<tr>
<td>Feb-08</td>
<td>-0.79%</td>
<td>Nov-08</td>
<td>28.13%</td>
</tr>
</tbody>
</table>

After this point, the evolution of the index followed a descending trend, until the turbulent period from October 2008, when the daily return was lower than -10% for several days and the local minimum value of the index was reached on 27.10.2008.

<table>
<thead>
<tr>
<th>Date</th>
<th>BET-FI Index</th>
<th>Daily return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Large negative returns from October 2008
### 3.3. LPPL fit for BET-FI Index

The initial sample for fitting LPPL model in the case of BET-FI index for predicting the phase transition from January 2008 was 03.01.2001 – 31.06.2007 (1603 daily observations); starting from the last observation in the initial sample, we extended the sample using a rolling window with fixed lower limit, so we estimated at every step the LPPL model for \( t \in [I, T+k], k=1 \ldots 100 \):

\[
p(t) = A_k + B_k (t_c - t)^{\beta_k} \{1 + C_k \cos[k \ln(t_c - t) + \phi_k] \}.
\]

As initial parameters we used the values validated in the literature (see for example Kurz-Kim(2012)): \( A(0) = p(\tau), \ B(0) = -|p(\tau) - p(\tau - 1)|, \ \tau = 1 \ldots k, \ C^{(0)} = 0, \ \beta^{(0)} = 0.33, \ \omega^{(0)} = 6.36, \ \phi^{(0)} = \pi. \)

Based on each iterative estimation, we computed the RMSE and the best model was chosen as the one that minimizes RMSE.

Also, in order to detect the local minima from 27.10.2008, we noticed that during the year 2008, the evolution of BET-FI index could be described as an “anti-bubble”, meaning a super-exponential evolution of the price inverse.

In this case, the sample for fitting LPPL model was 03.01.2008 – 13.10.2008 (129 daily observations); starting from the last observation in the initial sample, we have extended the sample using a rolling window with fixed lower limit, so we have estimated at every step the LPPL model for, \( t \in [I, T+k], k=1 \ldots 30 \):

\[
\frac{1}{\ln p(t)} = A_k + B_k (t_c - t)^{\beta_k} \{1 + C_k \cos[k \ln(t_c - t) + \phi_k] \}.
\]

Our initial values for the parameters were the following: \( A^{(0)} = 1/\ln p(\tau), \ B^{(0)} = -|1/\ln p(\tau) - 1/\ln p(\tau - 1)|, \ \tau = 1 \ldots k, \ C^{(0)} = 0, \ \beta^{(0)} = 0.33, \ \omega^{(0)} = 6.36, \ \phi^{(0)} = \pi \) and the best model was chosen as the one that minimizes RMSE.

In both cases, the model was estimated using a a dual quasi-Newton nonlinear fitting algorithm, implemented in SAS 9.2.

### 3.4. Results

<table>
<thead>
<tr>
<th>Date</th>
<th>Value</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/6/2008</td>
<td>24625.029</td>
<td>-11.09%</td>
</tr>
<tr>
<td>10/8/2008</td>
<td>19284.430</td>
<td>-15.83%</td>
</tr>
<tr>
<td>10/10/2008</td>
<td>15200.570</td>
<td>-16.08%</td>
</tr>
<tr>
<td>10/22/2008</td>
<td>13638.860</td>
<td>-14.53%</td>
</tr>
<tr>
<td>10/24/2008</td>
<td>11100.850</td>
<td>-13.72%</td>
</tr>
<tr>
<td>10/27/2008</td>
<td>10012.260</td>
<td>-10.32%</td>
</tr>
</tbody>
</table>
3.4.1. The bubble regime

The best fit for the model trying to predict the regime switching from January 2008 was estimated for the sample with the last observation 6.07.2007 (5 months in advance).

Table 3. The best fit for model (4)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>(t_c)</th>
<th>(\beta)</th>
<th>(\omega)</th>
<th>(\phi)</th>
<th>(t)</th>
<th>RMSE</th>
<th>AdjRSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>1078545</td>
<td>-855721</td>
<td>-0.003</td>
<td>0.031</td>
<td>6.909</td>
<td>6.28</td>
<td>04.01.2008</td>
<td>1730.603</td>
<td>06.07.2007</td>
<td>3808</td>
</tr>
</tbody>
</table>

The critical time estimated for the model with the minimum RMSE is the first trading day of 2008, 04.01.2008, which corresponds to the fact that January was the first month with a severe decline in BET-FI index.

Fig.2. Log periodic power law and BET-FI index

Fig.3. LPPL fit with critical time 24.07.2007

The second best fitted model, based on the RMSE criterion, gave us as 24.07.2007 as critical point, this trading day corresponding to the historical maximum of BET-FI index.

Based on this estimates, we can conclude that log periodic models could predict the bubble signature of the BET-FI index and the critical time corresponds to the real evolution of the index.

This conclusion is strengthened even further by the evolution of the parameter \(\beta\), the one controlling for magnitude of oscillations around the super-exponential trend before the critical time. Thus, small values of \(\beta\) and a stationary behaviour around the critical time are a warning signal of an imminent phase transition.
Plotting the values of parameter $\beta$ for each value of estimated critical time, we have noticed that starting from the beginning of January 2008, these values have a stationary dynamic, with values close to zero, a clear evidence of a regime switching in the BET-FI index.

3.4.2. The “anti-bubble” regime

The best fit for the model trying to predict the local minimum from 27 October 2008 was estimated for the sample with the last observation 15.10.2008(12 days in advance).

Table 4. The best fit for model (5)

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$t_c$</th>
<th>$\beta$</th>
<th>$\omega$</th>
<th>$\phi$</th>
<th>$t$</th>
<th>RMSE</th>
<th>AdjRSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-8.996</td>
<td>0.007</td>
<td>208.06</td>
<td>0.039</td>
<td>6.875</td>
<td>0</td>
<td>201</td>
<td>0.049</td>
<td>0.93</td>
</tr>
</tbody>
</table>

The critical time estimated for the model with the minimum RMSE is 27.10.2008, which is the day of the local minimum for the BET-FI index.
4. Conclusions

Analysing the behaviour of the most speculative index from Bucharest Stock Exchange, BET-FI, we have proved that LPPL models can be a useful tool in recognizing the behaviour of a stock market bubble. Iterative calibration of the model for BET-FI regime led to a reasonable estimate of the stock market crash in January 2008. Using the same approach, but in a reverse manner, we have estimated the critical time for the anti-bubble regime from 2008 and we have obtained an accurate prediction of the local point of phase transition from 27/10/2008.

Yet, the research in this direction needs to be improved, as this type of models have several weaknesses, like overparametrisation or the absence of a standardized method to recognize a developing bubble. Another serious constraint of the LPPL model is the restriction that during a bubble the trading price cannot decrease, this assumption being quite unrealistic.

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