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Numerical procedure for the vibration analysis of arbitrarily constrained stiffened panels with openings

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ABSTRACT: *A simple and efficient vibration analysis procedure for stiffened panels with openings and arbitrary boundary conditions based on the assumed mode method is presented. Natural frequencies and modes are determined by solving an eigenvalue problem of a multi-degree-of-freedom system matrix equation derived by using Lagrange's equations of motion, where Mindlin theory is applied for plate and Timoshenko beam theory for stiffeners. The effect of stiffeners on vibration response is taken into account by adding their strain and kinetic energies to the corresponding plate energies whereas the strain and kinetic energies of openings are subtracted from the plate energies. Different stiffened panels with various opening shapes and dispositions for several combinations of boundary conditions are analyzed and the results show good agreement with those obtained by the finite element analysis. Hence, the proposed procedure is especially appropriate for use in the preliminary design stage of stiffened panels with openings.*

KEY WORDS: Stiffened panels; Openings; Vibration analysis; Energy approach; Arbitrary boundary conditions; Assumed mode method.

INTRODUCTION

Stiffened panels are primary design members in all fields of engineering: civil, mechanical, aerospace, naval, ocean, etc. Stiffening of a plate is done to increase its loading capacity and to prevent buckling (Sapountzakis and Mokos, 2008; Cho et al., 2014). Moreover, stiffened panels are often made with openings of different shapes and sizes to reduce structural weight and to provide passage ways. This is particularly emphasized in the case of ships and offshore structures where stiffened panels with oval openings can be found. At the same time, stiffening and plate openings significantly affect the dynamic properties of such structures.

The Finite Element Method (FEM) is nowadays an advanced and widespread numerical tool and gives a complete solution to the vibration analysis of stiffened panels with openings. However, the preparation of a model is a rather time-consuming task which makes its reasonable to be applied usually at the end of the detail design stage after all the structural members, their dimensions and boundary conditions have been completely defined. In this sense, at the preliminary design stage when the principal dimensions are being selected, it is useful to have some simplified method at hand.

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A short overview of methods for the vibration analysis of plates with openings is presented by Cho et al. (2013), where the advantages and drawbacks of different methods, for instance the finite difference method (Paramasivam, 1973; Aksu and Ali, 1976), the Rayleigh-Ritz (Kwak and Han, 2007) and the optimized Rayleigh-Ritz (Grossi et al., 1997) method, FEM (Monahan et al., 1970), etc., are discussed. Moreover, Kwak and Han (2007) presented an extensive literature survey on vibration analysis of rectangular plates with openings, with particular emphasis on the application of the Rayleigh-Ritz method. They proposed a Rayleigh-Ritz based method with different coordinate systems for a plate and opening with the particular aim to simplify the integration process in the determination of total energy. In the case of stiffened panels, there is extensive literature on the static analysis of stiffened panels using FEM, the element method or a combination of these methods, as discussed by Sapountzakis and Mokos (2008). However, to the authors' knowledge, there is a rather limited number of references to the dynamic analysis of stiffened panels. Generally, the most common methods applied to the vibration analysis of stiffened plates can be classified into closed-bound solutions, energy methods and other numerical methods (Samanta and Mukhopadhyay, 2004).

In the case of the vibration analysis of stiffened panels with openings, only a few references, based on the application of the finite element method, are available. Sivasubramonian et al. (1997) studied the effect of curvature and cut-outs on square panels with different boundary conditions applying the shell element having seven degrees of freedom per node. Sivasubramonian et al. (1999) also applied the same finite element to both stiffened and unstiffened plate with openings and presented comparative results. Recently, Srivastava (2012) applied the finite element method to the vibration analysis of stiffened panels with a single opening and different boundary conditions, under partial edge loading.

In order to provide a simple and efficient procedure, this paper presents the application of the assumed mode method to the problem of natural vibration analysis of stiffened plates with openings. Namely, the assumed mode method has already been successfully applied to the vibration analysis of solid plates (Chung et al., 1993; Kim et al., 2012), plates with openings (Cho et al., 2013) and stiffened panels (Cho et al., 2014). In that sense, its application to the dynamic analysis of stiffened panels with openings represents an extension, achieved by combining the former two approaches. The effect of stiffeners is taken into account by adding their strain and kinetic energy to the strain and kinetic energy of the plate, respectively. Similarly to this strain and kinetic energies of the openings are subtracted from the corresponding energies of plate, respectively. Illustrative numerical examples include an analysis of the natural vibrations of stiffened panels with different stiffening, opening types, sizes and various values of plate thickness. In addition, different combinations of boundary conditions are examined. A comparison of the results with those obtained by the FEM is also provided and commented on. The developed procedure based on the assumed mode method is shown to be very simple and fast and, from an engineering point of view, accurate enough. It is therefore recommended for practical use in the preliminary design phase of stiffened panel structures with openings.

OUTLINE OF MATHEMATICAL MODEL

A mathematical model for the vibration analysis of stiffened panels with openings actually represents a generalization of the procedures for the vibration analysis of thick plates with openings and stiffened panels, presented by Cho et al. (2013; 2014).

The Mindlin thick plate theory which takes shear influence and rotary inertia into account (Mindlin et al., 1956) is adopted in this mathematical model. The Mindlin theory operates with three general displacements, i.e. plate deflection w, and angles of cross-section rotation about the x and y axes, ψ_x and ψ_y, respectively. From the equilibrium of sectional forces (bending moments, torsional moments and shear forces) and inertia forces, the equations of motions yield:

$$\frac{\rho h^3}{12}\frac{\partial^2 \psi_x}{\partial t^2} - D\left(\frac{\partial^2 \psi_x}{\partial x^2} + \frac{1}{2}(1-\nu)\frac{\partial^2 \psi_x}{\partial y^2} + \frac{1}{2}(1+\nu)\frac{\partial^2 \psi_y}{\partial x \partial y}\right) - kGh\left(\frac{\partial w}{\partial x} - \psi_x\right) = 0 \tag{1}$$

$$\frac{\rho h^3}{12}\frac{\partial^2 \psi_y}{\partial t^2} - D\left(\frac{\partial^2 \psi_y}{\partial y^2} + \frac{1}{2}(1-\nu)\frac{\partial^2 \psi_y}{\partial x^2} + \frac{1}{2}(1+\nu)\frac{\partial^2 \psi_x}{\partial x \partial y}\right) - kGh\left(\frac{\partial w}{\partial y} - \psi_y\right) = 0 \tag{2}$$

$$\frac{\rho}{kG}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y^2}+\frac{\partial \psi_x}{\partial x}+\frac{\partial \psi_y}{\partial y}=0 \tag{3}$$

where ρ is plate density, h is plate thickness, k is shear coefficient, while v is Poisson's ratio. Further, D represents plate flexural rigidity $D = Eh^3/\left(12\left(1-\nu^2\right)\right)$, while E and $G = E/\left(2\left(1+\nu\right)\right)$ are Young's and shear moduli, respectively.

Based on the above-described idea to include an opening and stiffener effect, one can write for the strain and kinetic energy of a stiffened panel with openings:

$$V = V_p + V_s - V_o, \tag{4}$$

$$T = T_p + T_s - T_o. \tag{5}$$

where V_p is the plate strain energy, V_s represents the strain energy of stiffeners and V_o is the strain energy of an opening. Similarly, T_p is the plate kinetic energy and T_s and T_o are the kinetic energies of the stiffeners and opening, respectively.

Energy of a rectangular plate

By introducing the non-dimensional parameters $\xi = x/a$, $\eta = y/b$, $\alpha = a/b$ and $S = kGh/D$ for a rectangular plate of length a and breadth b with arbitrary boundary conditions, one can write for the strain and kinetic energy of a rectangular plate without openings, V and T, respectively (Kim et al., 2012):

$$\begin{aligned} V_p = {} & \frac{D}{2\alpha}\int_0^1\int_0^1\Bigg[\left(\frac{\partial \psi_\xi}{\partial \xi}\right)^2+\alpha^2\left(\frac{\partial \psi_\eta}{\partial \eta}\right)^2+2\nu\alpha\frac{\partial \psi_\xi}{\partial \xi}\frac{\partial \psi_\eta}{\partial \eta}+\frac{1-\nu}{2}\left(\alpha\frac{\partial \psi_\xi}{\partial \eta}+\frac{\partial \psi_\eta}{\partial \xi}\right)^2 \\ & +S\left(\left(\frac{\partial w}{\partial \xi}-a\psi_\xi\right)^2+\alpha^2\left(\frac{\partial w}{\partial \eta}-b\psi_\eta\right)^2\right)\Bigg]\mathrm{d}\xi\mathrm{d}\eta \\ & +\int_0^1\left[K_{Rx1}\psi_\xi^2(0,\eta)+SK_{Tx1}w^2(0,\eta)\right]\mathrm{d}\eta+\alpha^2\int_0^1\left[K_{Ry1}\psi_\eta^2(\xi,0)+SK_{Ty1}w^2(\xi,0)\right]\mathrm{d}\xi \\ & +\int_0^1\left[K_{Rx2}\psi_\xi^2(1,\eta)+SK_{Tx2}w^2(1,\eta)\right]\mathrm{d}\eta+\alpha^2\int_0^1\left[K_{Ry2}\psi_\eta^2(\xi,1)+SK_{Ty2}w^2(\xi,1)\right]\mathrm{d}\xi, \end{aligned} \tag{6}$$

$$T_p = \frac{\rho ab}{2}\int_0^1\int_0^1\left[h\left(\frac{\partial w}{\partial t}\right)^2+\frac{h^3}{12}\left(\frac{\partial \psi_\xi}{\partial t}\right)^2+\frac{h^3}{12}\left(\frac{\partial \psi_\eta}{\partial t}\right)^2\right]\mathrm{d}\xi\mathrm{d}\eta, \tag{7}$$

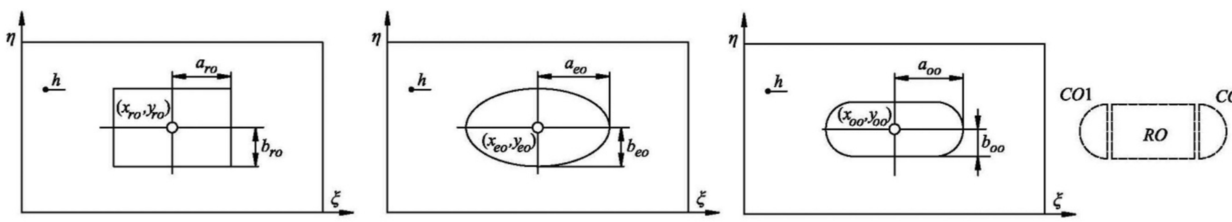
where $K_{Tx1}=\left(k_{Tx1}a/kGh\right)$, $K_{Tx2}=\left(k_{Tx2}a/kGh\right)$, $K_{Ty1}=\left(k_{Ty1}b/kGh\right)$ and $K_{Ty2}=\left(k_{Ty2}b/kGh\right)$ are non-dimensional stiffness at $x=0$, $x=a$, $y=0$ and $y=b$, respectively, and correspond to the translational spring constants per unit length k_{Tx1}, k_{Tx2}, k_{Ty1} and k_{Ty2} (Kim et al., 2012; Cho et al., 2013). In the same manner, $K_{Rx1}=\left(k_{Rx1}a/D\right)$, $K_{Rx2}=\left(k_{Rx2}a/D\right)$, $K_{Ry1}=\left(k_{Ry1}b/D\right)$ and $K_{Ry2}=\left(k_{Ry2}b/D\right)$ are for the rotational spring constants per unit length k_{Rx1}, k_{Rx2}, k_{Ry1} and k_{Ry2} at boundaries, respectively. The following quantities are related to the x direction: a, k_{Tx1}, k_{Tx2}, k_{Rx1} and k_{Rx2}. Other quantities (b, k_{Ty1}, k_{Ty2} k_{Ry1} and k_{Ry2}) are relevant for the y direction (Cho et al., 2013).

Energy of openings

In similar way, as for a plate without openings, one can write for the strain and kinetic energy of the rectangular opening shown in Fig. 1(a), respectively:

$$V_{ro}=\frac{D}{2\alpha}\int_{y_{ro}-b_{ro}}^{y_{ro}+b_{ro}}\int_{x_{ro}-a_{ro}}^{x_{ro}+a_{ro}}\left[\left(\frac{\partial\psi_{\xi}}{\partial\xi}\right)^{2}+\alpha^{2}\left(\frac{\partial\psi_{\eta}}{\partial\eta}\right)^{2}+2\nu\alpha\frac{\partial\psi_{\xi}}{\partial\xi}\frac{\partial\psi_{\eta}}{\partial\eta}+\frac{1-\nu}{2}\left(\alpha\frac{\partial\psi_{\xi}}{\partial\eta}+\frac{\partial\psi_{\eta}}{\partial\xi}\right)^{2}\right.$$
$$\left.+S\left(\left(\frac{\partial w}{\partial\xi}-a\psi_{\xi}\right)^{2}+\alpha^{2}\left(\frac{\partial w}{\partial\eta}-b\psi_{\eta}\right)^{2}\right)\right]\mathrm{d}\xi\mathrm{d}\eta \tag{8}$$

$$T_{ro}=\frac{\rho ab}{2}\int_{y_{ro}-b_{ro}}^{y_{ro}+b_{ro}}\int_{x_{ro}-a_{ro}}^{x_{ro}+a_{ro}}\left[h\left(\frac{\partial w}{\partial t}\right)^{2}+\frac{h^{3}}{12}\left(\frac{\partial\psi_{\xi}}{\partial t}\right)^{2}+\frac{h^{3}}{12}\left(\frac{\partial\psi_{\eta}}{\partial t}\right)^{2}\right]\mathrm{d}\xi\mathrm{d}\eta\,, \tag{9}$$



(a) (b) (c)

Fig. 1 Rectangular plates with different opening shapes; (a) Rectangular, (b) Elliptic, (c) Oval.

In Fig. 1(a), x_{ro} and y_{ro} denote the longitudinal and transverse coordinates of the rectangular opening centre of gravity, and a_{ro} and b_{ro} are one half of its length and breadth.

Analogously to the rectangular opening, the strain and kinetic energies of the elliptical opening shown in Fig. 1(b) yield:

$$V_{eo}=\frac{D}{2\alpha}\int_{y_{eo}-b_{eo}}^{y_{eo}+b_{eo}}\int_{x_{eo}-a_{eo}\sqrt{1-\left(\frac{\eta-y_{eo}}{b_{eo}}\right)^{2}}}^{x_{eo}+a_{eo}\sqrt{1-\left(\frac{\eta-y_{eo}}{b_{eo}}\right)^{2}}}\left[\left(\frac{\partial\psi_{\xi}}{\partial\xi}\right)^{2}+\alpha^{2}\left(\frac{\partial\psi_{\eta}}{\partial\eta}\right)^{2}+2\nu\alpha\frac{\partial\psi_{\xi}}{\partial\xi}\frac{\partial\psi_{\eta}}{\partial\eta}+\frac{1-\nu}{2}\left(\alpha\frac{\partial\psi_{\xi}}{\partial\eta}+\frac{\partial\psi_{\eta}}{\partial\xi}\right)^{2}\right.$$
$$\left.+S\left(\left(\frac{\partial w}{\partial\xi}-a\psi_{\xi}\right)^{2}+\alpha^{2}\left(\frac{\partial w}{\partial\eta}-b\psi_{\eta}\right)^{2}\right)\right]\mathrm{d}\xi\mathrm{d}\eta \tag{10}$$

$$T_{eo}=\frac{\rho ab}{2}\int_{y_{ro}-b_{ro}}^{y_{ro}+b_{ro}}\int_{x_{eo}-a_{eo}\sqrt{1-\left(\frac{\eta-y_{eo}}{b_{eo}}\right)^{2}}}^{x_{eo}+a_{eo}\sqrt{1-\left(\frac{\eta-y_{eo}}{b_{eo}}\right)^{2}}}\left[h\left(\frac{\partial w}{\partial t}\right)^{2}+\frac{h^{3}}{12}\left(\frac{\partial\psi_{\xi}}{\partial t}\right)^{2}+\frac{h^{3}}{12}\left(\frac{\partial\psi_{\eta}}{\partial t}\right)^{2}\right]\mathrm{d}\xi\mathrm{d}\eta\,, \tag{11}$$

If one introduces $x_{eo}=y_{eo}$ in the Eqs. (10) and (11), the strain and kinetic energies of the circular opening is obtained. As in previous case, in Fig. 1(b), x_{eo} and y_{eo} denote the longitudinal and transverse coordinates of the elliptic opening centre of gravity, while a_{eo} and b_{eo} are the major and minor semi-axes of the opening, respectively. An oval opening, which can often be found in ships and offshore structures, can be treated as a combination of rectangular and semi-circular openings, Fig. 1(c), where x_{oo} and y_{oo} are the longitudinal and transverse coordinates of the oval opening centre of gravity, while a_{oo} and b_{oo} are equal to one half of its length and breadth, respectively.

In this way, its strain and kinetic energies can be written in the following form:

$$T_{oo}=T_{co1}+T_{ro}+T_{co2}\,,\qquad V_{oo}=V_{co1}+V_{ro}+V_{co2} \tag{12}$$

and can be calculated with the above-listed formulae.

In addition, the procedure can be applied to multiple openings, but then the opening strain and kinetic energies have to be calculated separately, and subtracted from the corresponding energies of the plate without openings.

Energy of stiffeners

The strain and kinetic energy of stiffeners placed in longitudinal and transverse directions yield:

$$\begin{aligned} V_s = \frac{a}{2}\sum_{r=1}^{n_x}\left[\int_0^1 \left(\frac{EI_{x_r}}{a^2}\left(\frac{\partial \psi_\xi}{\partial \xi}\right)^2 + K_{x_r}A_{wx_r}G\left(\frac{\partial w}{a\partial \xi}-\psi_\xi\right)^2 + \frac{GJ_{x_r}}{a^2}\left(\frac{\partial \psi_\eta}{\partial \xi}\right)^2\right)_{\eta=\eta_r} \mathrm{d}\xi\right] \\ + \frac{b}{2}\sum_{r=1}^{n_y}\left[\int_0^1 \left(\frac{EI_{y_r}}{b^2}\left(\frac{\partial \psi_\eta}{\partial \eta}\right)^2 + K_{y_r}A_{wy_r}G\left(\frac{\partial w}{b\partial \eta}-\psi_\eta\right)^2 + \frac{GJ_{y_r}}{b^2}\left(\frac{\partial \psi_\xi}{\partial \eta}\right)^2\right)_{\xi=\xi_r} \mathrm{d}\eta\right] \end{aligned} \tag{13}$$

$$T_s = \frac{a}{2}\sum_{r=1}^{n_x}\left[\int_0^1 \left\{\rho I_{Rx_r}\left(\frac{\partial \psi_\xi}{\partial t}\right)^2 + \rho A_{wx_r}\left(\frac{\partial w}{\partial t}\right)^2\right\}_{\eta=\eta_r} \mathrm{d}\xi\right] + \frac{b}{2}\sum_{r=1}^{n_y}\left[\int_0^1 \left\{\rho I_{Ry_r}\left(\frac{\partial \psi_\eta}{\partial t}\right)^2 + \rho A_{wy_r}\left(\frac{\partial w}{\partial t}\right)^2\right\}_{\xi=\xi_r} \mathrm{d}\eta\right]. \tag{14}$$

where n_x and n_y represent the number of stiffeners in the x and y direction, respectively. K is the shear coefficient, and A and I_R are the cross-sectional area and the rotary moment of inertia of the stiffener cross-section, respectively. Further, GJ is torsional rigidity while EI is the bending rigidity of the stiffener, considering the plate flange contribution of effective width s as shown in Fig. 2.

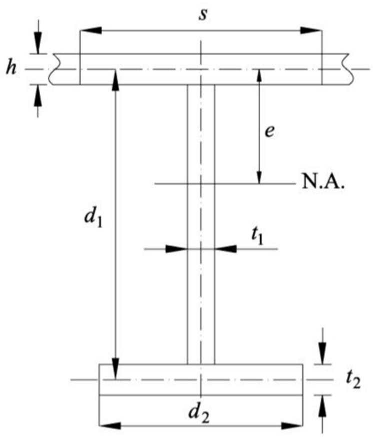


Fig. 2 Stiffener cross-section.

Outline of the assumed mode method

In the assumed mode method, lateral displacement and rotational angles are expressed by superposing the products of the orthogonal polynomials:

$$w(\xi,\eta,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}a_{mn}(t)X_m(\xi)Y_n(\eta),\quad \psi_\xi(\xi,\eta,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}b_{mn}(t)\Theta_m(\xi)Y_n(\eta),\quad \psi_\eta(\xi,\eta,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}c_{mn}(t)X_m(\xi)\Phi_n(\eta) \tag{15}$$

where $X_m(\xi)$, $Y_n(\eta)$, $\Theta_m(\xi)$ and $\Phi_n(\eta)$ are the orthogonal polynomials satisfying the specified elastic edge constraints with respect to ξ and η. Furthermore, $a_{mn}(t)$, $b_{mn}(t)$ and $c_{mn}(t)$ are the influence coefficients of orthogonal polynomials. Also, M and N are the number of orthogonal polynomials used for an approximate solution in the ξ and η directions, respectively. Eq. (15) can be alternatively written in matrix form:

$$\{z(\xi,\eta,t)\} = [H(\xi,\eta)]\{q(t)\} \tag{16}$$

where

$$\{z(\xi,\eta,t)\} = \{w(\xi,\eta,t),\ \psi_\xi(\xi,\eta,t),\ \psi_\eta(\xi,\eta,t)\}^T, \tag{17}$$

$$[H(\xi,\eta)] = \begin{bmatrix} X_1Y_1 \ldots X_MY_N & 0 \ldots 0 & 0 \ldots 0 \\ 0 \ldots 0 & \Theta_1Y_1 \ldots \Theta_MY_N & 0 \ldots 0 \\ 0 \ldots 0 & 0 \ldots 0 & X_1\Phi_1 \ldots X_M\Phi_N \end{bmatrix}, \tag{18}$$

$$\{q(t)\} = \{a_{11} \ldots a_{MN}\ b_{11} \ldots b_{MN}\ c_{11} \ldots c_{MN}\ \}^T. \tag{19}$$

By substituting Eqs. (4) and (5) into Lagrange's equation of motion below

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = 0, \tag{20}$$

the discrete matrix equation with 3xMxN degrees of freedom can be obtained as the following equation:

$$[M]\left\{\frac{\partial^2 q(t)}{\partial t^2}\right\} + [K]\{q(t)\} = 0, \tag{21}$$

where [M] and [K] are the mass and the stiffness matrix, respectively. Their structure using characteristic orthogonal polynomials is described in detail by Kim et al. (2012). The procedure for the formulation of the stiffness and mass matrices of the attached stiffeners using the same polynomials can also be found in Cho et al. (2014).

If harmonic vibrations are assumed, i.e.

$$\{q(t)\} = \{Q\}e^{j\omega t}, \quad w(\xi,\eta,t) = W(\xi,\eta)e^{j\omega t}, \quad \psi_\xi(\xi,\eta,t) = \Psi_\xi(\xi,\eta)e^{j\omega t}, \quad \psi_\eta(\xi,\eta,t) = \Psi_\eta(\xi,\eta)e^{j\omega t} \tag{22}$$

Eq. (21) leads to an eigenvalue problem which gives the natural frequencies and eigenvectors of the system, where j is the imaginary unit and ω represents the angular frequency. The mode shape corresponding to each natural frequency is obtained from the following equation:

$$\{W(\xi,\eta),\ \Psi_\xi(\xi,\eta),\ \Psi_\eta(\xi,\eta),\}_l^T = [H(\xi,\eta)]\{Q\}_l \tag{23}$$

where l represents the order of the mode. It should be noted here that when using the assumed mode method, the orthogonal polynomials corresponding to the property of the target model should be applied to achieve accurate analysis. For this purpose, the characteristic orthogonal polynomials having the property of Timoshenko beam functions which satisfies the specified edge constraints are used. Their complete derivation is presented by Kim et al. (2012).

In addition, it should be mentioned that developed procedure is not fully physically consistent, since generally in the Rayleigh-Ritz method all of the assumed shape function should satisfy at least the geometric boundary conditions (Szilard, 2004), which is not unconditionally the case in the presented procedure at the opening boundaries. Namely, it is necessary to say that a difference exists in the real boundary conditions at the free opening edge and remains after excluding that part. In the former case, the boundary forces are zero, while in the latter case they are equal to the sectional forces (Cho et al., 2013). Although the developed procedure is primarily aimed for stiffened panels with small openings, but at the same time it is quite good approximation in the case of panel with relatively large openings, as shown in the following section of the paper. The inclusion of stiffeners in the mathematical model is fully consistent.

NUMERICAL RESULTS

The developed procedure is verified with several numerical examples, where different stiffening, plate thicknesses, openings of different shapes (rectangular, oval, circular) and sizes are included. In the numerical examples, different boundary conditions are also applied. In all the calculations, the values of Young's modulus, material density and Poisson's ratio are set to 2.1x10^{11} N/m^2, 7,850 kg/m^3 and 0.3, respectively. Furthermore, the value of the shear factor k is adjusted to 5/6 and the number of polynomials has been set to 13, for both ξ and η directions, respectively. It should also be mentioned that in all the calculations the stiffeners are modelled as beam elements in the FE analysis, since a comparison of the results in that case is the most realistic. Namely, in the proposed procedure, the beam theory is applied in order to take into account the effect of the stiffeners.

The natural vibration analysis of a stiffened panel with 3 rectangular openings and 4 equidistant transverse stiffeners (unidirectionally stiffened panel) shown in Fig. 3 is performed first. The panel length and breadth is 4.0 *m* and 3.0 *m*, respectively, and the stiffeners are I profiles with a height of 150 *mm* and a web thickness of 10 *mm*. Two values of plate thickness are taken into account to verify the applicability of the method to the panels with both thin and thick plating. Table 1 shows the results of the natural vibration analysis of the simply supported (SSSS) stiffened panel, the clamped (CCCC) stiffened panel and the stiffened panel with mixed boundary conditions (FCSC), i.e. with one free longitudinal edge and clamped transverse edges. The results obtained by the proposed procedure are denoted with PS (Present Solution). The natural frequencies calculated using NASTRAN (MSC, 2010) are also included in Table 1, where quite good agreement can be noticed. This is especially true for the lowest natural frequencies that are actually most relevant for structural design from the viewpoint of vibration. The corresponding FE model in Table 1 consists of 1,476 plate and 120 beam finite elements, respectively, and the number of degrees of freedom varies depending on the selection of boundary conditions. The natural modes obtained by the proposed method and NASTRAN are almost the same, as shown in Fig. 4.

Table 1 Natural frequency f (Hz) of a transversally stiffened panel with rectangular openings.

Mode no.	h (*mm*)	Boundary conditions								
		SSSS			CCCC			FCSC		
		PS	FEM	Diff., %	PS	FEM	Diff., %	PS	FEM	Diff., %
1	25	23.04	22.97	0.30	49.91	48.37	3.18	9.08	8.97	1.23
	50	25.71	25.43	1.10	51.48	50.61	1.72	18.42	18.12	1.66
2	25	29.62	29.42	0.68	55.85	54.46	2.55	24.34	23.93	1.71
	50	47.27	45.20	4.58	74.21	72.37	2.54	42.50	42.04	1.09
3	25	44.98	43.46	3.50	71.46	68.73	3.97	36.51	36.31	0.55
	50	85.05	78.94	7.74	114.68	110.78	3.52	49.95	48.46	3.07

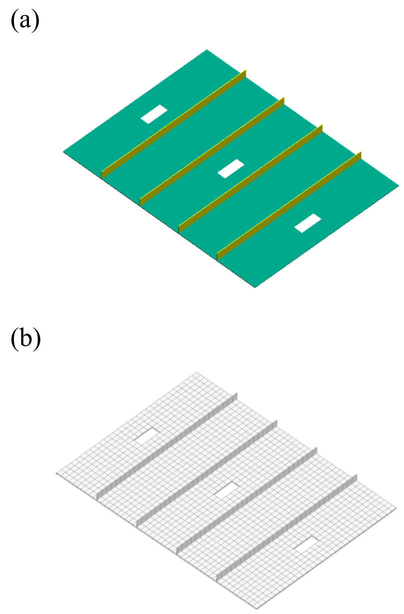


Fig. 3 Model of a transversally stiffened panel with rectangular openings; (a) PS, (b) FEM.

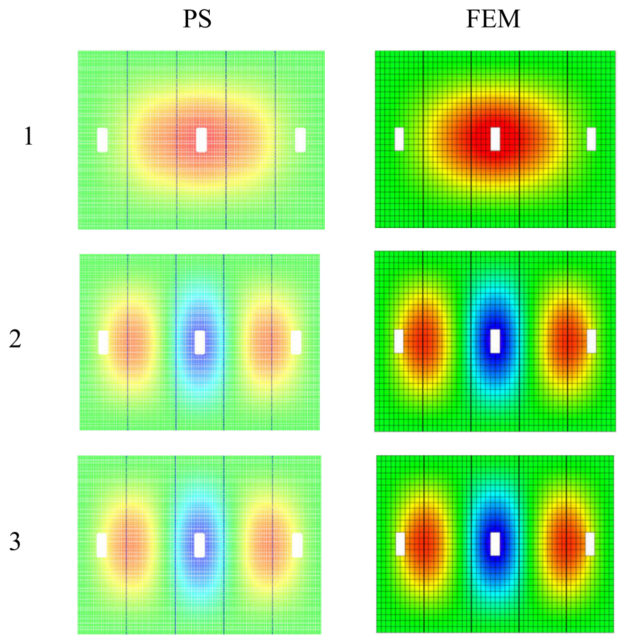


Fig. 4 Mode shapes of transversally stiffened panel with rectangular openings, h=50 *mm*, CCCC.

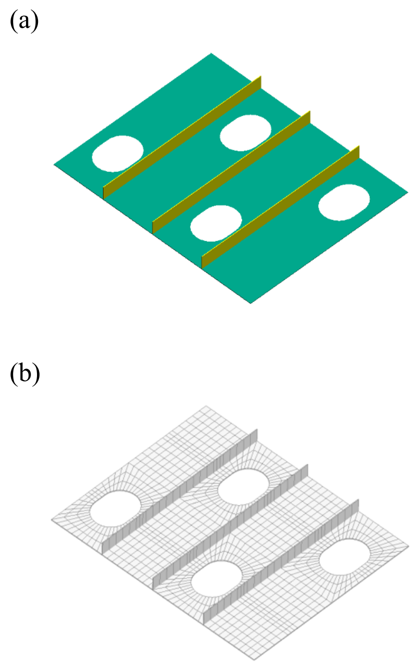


Fig. 5 Model of a transversally stiffened panel with oval openings; (a) PS, (b) FEM.

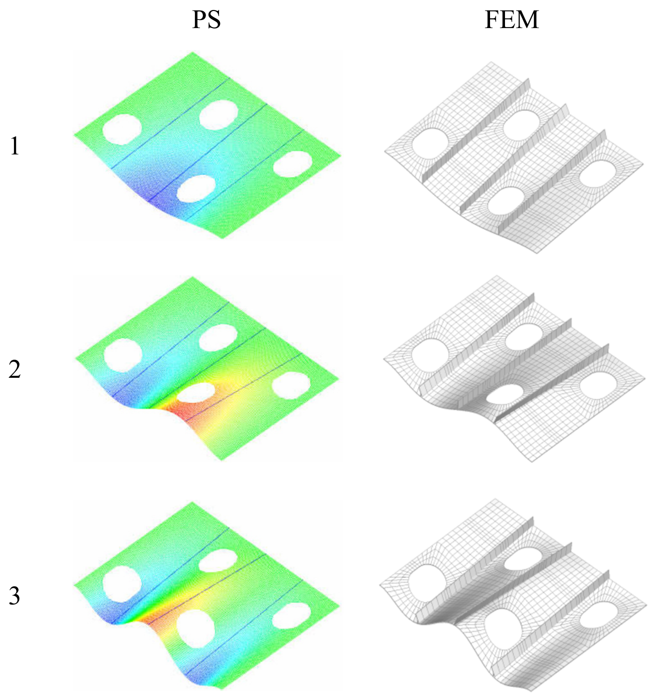


Fig. 6 Mode shapes of a transversally stiffened panel with oval openings, FCSC.

Furthermore, the developed procedure is applied to a transversally stiffened panel with oval openings as shown in Fig. 5. Oval openings are actually used very often in ships and offshore structures, and here the dimensions of the openings are 800× 600 *mm*. The panel length and breadth is 4.0 *m* and 3.2 *m*, respectively. Three equidistant stiffeners are I profiles with a height of 250 *mm* and a width of 10 *mm*, while the thickness of the panel plate is 15 *mm*. In this case the corresponding FE model consists of 1,150 elements (1,072 plates and 78 beams). The natural frequencies and mode shapes for the selected representative boundary conditions are shown in Table 2 and Fig. 6, respectively, which are within acceptable tolerances.

Table 2 Natural frequency f (Hz) of a transversally stiffened panel with oval openings.

Mode no.	Boundary conditions					
	FCSC			FFCC		
	PS	FEM	Diff., %	PS	FEM	Diff., %
1	5.05	4.95	2.02	7.15	6.92	3.32
2	13.66	13.05	4.67	15.83	15.48	2.26
3	25.79	24.22	6.48	17.61	17.23	2.21

In the next numerical example, a stiffened panel with a length of 14.0 *m* and breadth of 10.0 *m*, longitudinal framing (I300×15) and six circular openings is considered, Fig. 7. The thickness of the plate is 50 *mm* and the opening diameter is equal to 1,000 *mm*. The FE model of the longitudinally stiffened panel consists of 4,100 plate and 280 beam finite elements. Two combinations of boundary conditions are considered, i.e. a clamped stiffened panel (CCCC) and a stiffened panel with a free transverse edge and the other edges clamped (CFCC). The natural frequencies of a stiffened panel with circular openings and longitudinal stiffeners are presented in Table 3, and the mode shapes are shown in Fig. 8. The agreement of the results is also good, as is the case in the previous numerical examples.

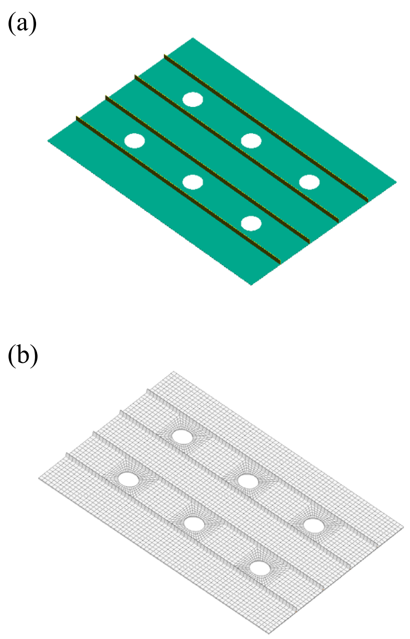


Fig. 7 Model of a longitudinally stiffened panel with circular openings; (a) PS, (b) FEM.

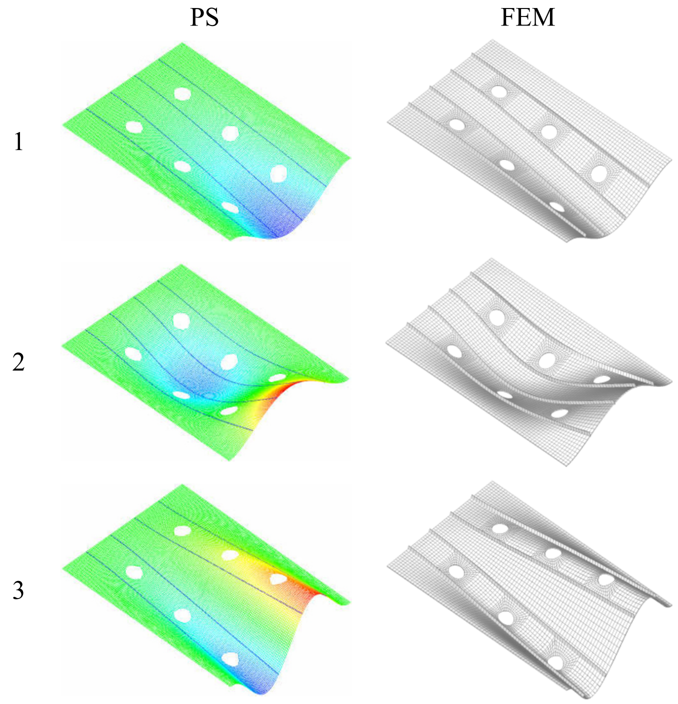


Fig. 8 Mode shapes of a longitudinally stiffened panel with circular openings, CFCC.

Table 3 Natural frequency f (Hz) of a longitudinally stiffened panel with circular openings.

Mode no.	Boundary conditions					
	CCCC			CFCC		
	PS	FEM	Diff., %	PS	FEM	Diff., %
1	4.78	4.79	-0.21	2.90	2.89	0.35
2	9.09	8.73	4.12	4.97	4.99	-0.40
3	10.94	10.63	2.92	7.89	7.53	4.78

The developed method can also be applied to the vibration analysis of stiffened panels with relatively large openings, as shown in the case of a cross-stiffened panel with a large rectangular opening. The dimensions of the panel and the rectangular opening are shown in Fig. 9, and, for the sake of simplicity, only the SSSS boundary condition is considered. As shown in Fig. 10, the natural frequencies and the mode shapes are in acceptable agreement with the FE solutions.

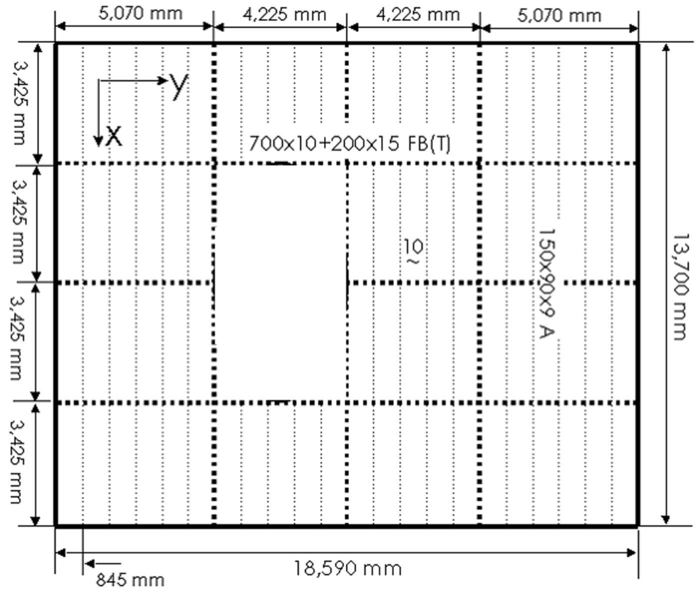


Fig. 9 Cross-stiffened panel with a large rectangular opening.

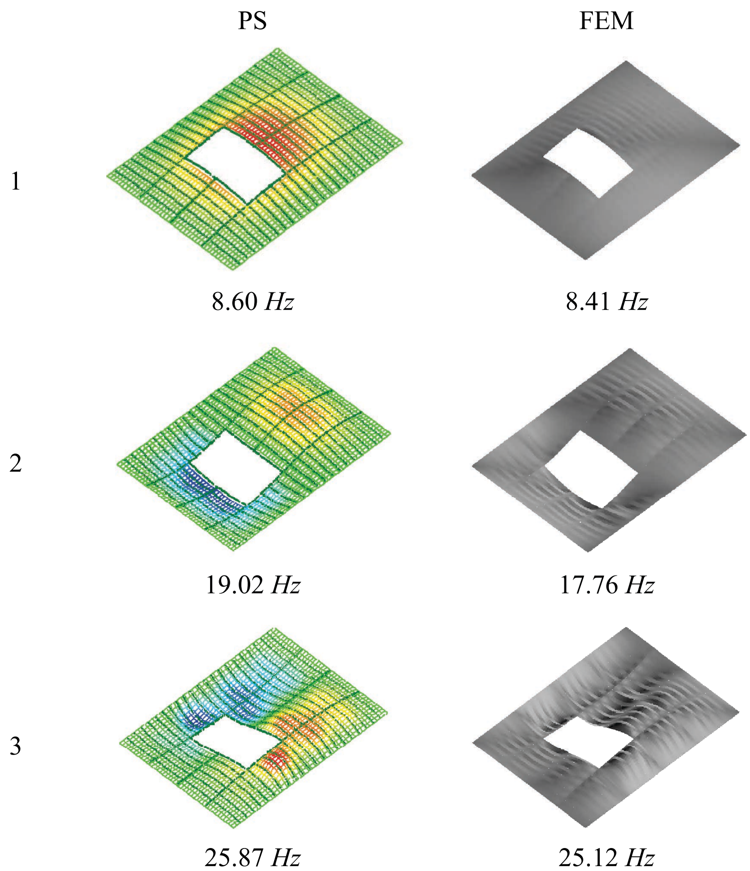


Fig. 10 Mode shapes of a cross-stiffened panel with a large rectangular opening, SSSS.

CONCLUSION

A simple and efficient procedure for the vibration analysis of stiffened panels with openings of different shapes and arbitrary edge constraints is presented. The assumed mode method (energy method) is applied and the effect of the stiffeners and openings is taken into account in a very simple way, i.e. by adding and subtracting their strain and kinetic energies to the corresponding plate energies, respectively. The procedure is validated through comparisons with finite element results obtained with general purpose FE software. Various combinations of stiffener spacing, stiffened number and orientation together with various opening shapes and dimensions are considered. It is shown that a good agreement of natural frequencies and mode shapes is achieved for all combinations of boundary conditions. Despite its simplicity, there are no limitations in the applicability of the presented method concerning plate thickness bounds or stiffener dimensions and types. It can also be applied to plates with carlings or partial stiffeners which are frequently arranged near the opening or below heavy equipment in ship structures. Thus, the proposed procedure can be used as a reliable alternative to the widely used FEM, especially in the initial design stage where the principal structural dimensions are being defined. This makes the procedure advantageous when the vibration performances of stiffened panels with different topologies are investigated. Future investigations will be oriented to the extension of the developed procedure to the vibration analysis of orthotropic plates assuming different materials, and to more complex structural parts of ships and offshore structures, such as, for instance, stiffened panels with additional attachments such as lumped inertia and stiffness elements, and stiffened panels partially or fully immersed in the water, etc.

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