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Induced Restricted Ramsey Theorems for Spaces

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The induced restricted versions of the vector space Ramsey theorem and of the Graham–Rothschild parameter set theorem are proved. © 1987 Academic Press, Inc.

INTRODUCTION

In recent years, numerous Ramsey-type results have been established for a variety of algebraic and combinatorial structures. These include, for example, subsets of finite sets, subspaces of finite-dimensional vector spaces (over finite fields), solution sets of systems of homogeneous linear equations, sublattices of finite lattices, partitions of finite sets, sub-parameter sets of parameter sets, and subcategories of various categories, to name a few. The essence of such a result is to assert for some collection X of "rank l" objects and any integers k and r, the existence of another collection Y of rank l objects so that no matter how the set of all rank k subobjects occurring in Y are partitioned into r classes, there is always a "copy" X' of X in Y which has all *its* rank k subobjects belonging to a single class. Such an X' is sometimes called *homogeneous*.

It turns out that in many cases it is possible to significantly strengthen results of this type by placing various restrictions both on the sought-after collection Y as well as on the homogeneous collection X'. For example, one might require that Y spans no rank s objects unless X also does (Y is

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restricted), or that the only rank k objects spanned by the points in X' are those which correspond to rank k objects in X (X' is an *induced* copy of X in Y).

Perhaps the most well-known such strengthening is the induced restricted Ramsey theorem for hypergraphs of Nešetřil and Rödl [7] (see also [8, 9] where stronger theorems are stated).

The purpose of this note is to give proofs of the corresponding strengthenings for two of the cornerstone theorems in Ramsey theory, namely, the vector space Ramsey theorem [3], and the n-parameter set Ramsey theorem [4]. As will be seen from the arguments, these techniques (based in part on ideas introduced in [7]) are in fact applicable to many other situations for which a basic Ramsey theorem already exists (cf. [14]).

Preliminaries

Let $A=\{a_1,..., a_q\}$ be a fixed finite set and let $B\subseteq A$ be nonempty. For non-negative integers $k\leqslant n$, we will define special subsets P_k, called *k-parameter sets*, of the cartesian product A^n, in the following way (cf. [4, 2]).

For disjoint, nonempty subsets $I_1,..., I_k$ of $[n]=\{1, 2,..., n\}$, define P_k to consist of all those $(x_1,..., x_n)\in A^n$ such that:

(i) If $u, v\in I_j$ for some j then $x_u=x_v$;

(ii) If $u\in [n]\backslash\bigcup_j I_j$ then $x_u=b_u$, a fixed element of B.

The elements of $\bigcup_j I_j$ are usually called the *moving* coordinates of P_k; the elements of the (possibly empty) subset $I_0=[n]\backslash\bigcup_j I_j$ are called the *constant* coordinates of P_k. In a certain sense, P_k is the combinatorial analogue of a k-dimensional affine space over a q-element field (at least, when q is a prime power). Observe that $|P_k|=q^k$ for $k\geqslant 0$. A set $X\subseteq A^n$ is said to be an i-parameter subset of P_k if X is an i-parameter set in A^n and $X\subseteq P_k$. A discussion of various properties of k-parameter sets can be found in [4].

When q is a prime power and $A=GF(q)$, more common substructures of A^n are those of either k-dimensional affine or k-dimensional vector spaces over $GF(q)$. Since we will be treating both k-parameter sets and k-dimensional spaces in A^n simultaneously, we will call them both k-spaces in A^n (although when we use the term we will always have one particular interpretation in mind). We will denote the set of k-spaces in A^n by $\binom{A^n}{k}$, and their number by $\left[{n \atop k}\right]$. X will be called a *subspace* of A^n if $X\in\binom{A^n}{k}$ for some k, in which case k is called the *dimension* of X, denoted by dim X.

The following statement expresses the basic Ramsey theorem for k-spaces.

THEOREM [4, 3]. *For all integers k, l, r with $0 \leqslant k \leqslant l$, there exists $N_0(k, l, r, A)$ such that if $N \geqslant N_0(k, l, r, A)$ and $\binom{A^N}{k} = \mathscr{C}_1 \cup \cdots \cup \mathscr{C}_r$ is an arbitrary partition of the k-spaces of A^N into r classes, then there is always an l-space $X \in \binom{A^N}{l}$ with $\binom{X}{k} \subseteq \mathscr{C}_i$ for some i.*

We should remark that the case of this theorem for k-parameter sets with $l = 1$, $k = 0$, and $B = A$ is known as the Hales–Jewett theorem [6], and will be needed in the proof of Proposition 2.

ADVANCED PRELIMINARIES

Before proceeding to our main result, we first need several additional results. For a finite set X, A^X will denote the set of all $|X|$-tuples $(a_x : x \in X)$. A map $f: A^X \to A^Y$ is called *linear* if $f(V)$ is a subspace of A^Y (written $f(V) < A^Y$) for each subspace V of A^X. For $Y \subseteq X$, the *projection map* P_Y is defined by setting

$$p_Y((a_x : x \in X)) = (a_y : y \in Y).$$

It is easy to see that p_Y is linear.

For $Y \subseteq X$, a subspace $V < A^X$ is called *Y-transverse* if

$$\dim V = \dim p_Y(V).$$

Note that in this case the projection map p_Y is $1-1$ on V.

For sets $X^{(1)}, \ldots, X^{(m)}$, we define the *amalgamated direct product* $\bigoplus_i A^{X^{(i)}}$ to be the set of all tuples $(a_x : x \in \bigcup_{i=1}^m X^{(i)})$.

PROPOSITION 1. *Suppose for sets $X^{(1)}, \ldots, X^{(m)}$ we have $X^{(i)} \cap X^{(j)} = Y \neq \varnothing$ for $1 \leqslant i < j \leqslant m$, and $F^{(i)}$ is a Y-transverse k-space in $A^{X^{(i)}}$ with all $P_Y(F^{(i)}) = E$, $1 \leqslant i \leqslant m$. Then there is a unique Y-transverse k-space $F = F^{(1)}F^{(2)} \cdots F^{(m)} \subseteq \bigoplus_i A^{X^{(i)}}$ satisfying $p_Y(F) = E$ and $p_{X^{(i)}}(F) = F^{(i)}$ for $1 \leqslant i \leqslant m$.*

Proof. Since each $F^{(i)}$ is Y-transverse then for each $\bar{a} \in p_Y(F^{(i)}) = E$, there is a unique point $\bar{a}^{(i)} \in F^{(i)}$ which has the same Y-coordinates as $\bar{a}$. That is, knowledge of the "Y-part" $\bar{a} = p_Y(\bar{a}^{(i)})$ is sufficient to reconstruct the remaining part of $\bar{a}^{(i)}$. Thus, for any $\bar{a} \in E$ we can uniquely extend it to an element $\hat{a} \in \bigoplus_i A^{X^{(i)}}$ (no contradictions can arise since $X^{(i)} \cap X^{(j)} = Y$ for $i \neq j$). In this way we obtain a Y-transverse set F satisfying $p_Y(F) = E$ which is easily verified to be a subspace of $\bigoplus_i A^{X^{(i)}}$. ∎

A central notion used in the paper is the following. Suppose $X \supseteq Y$, $W \supseteq Y$ and $f: A^W \to A^X$. Then f is called *Y-linear* if f is linear and for all

$\bar{a} \in A^W$, $f(\bar{a})$ has the same Y-part as $\bar{a}$. Further, for Y-transverse subspaces $U < A^W$, $V < A^X$ and families $\mathscr{F} \subseteq \binom{U}{k}$, $\mathscr{G} \subseteq \binom{V}{k}$, we say that $\mathscr{F}$ and $\mathscr{G}$ are Y-*isomorphic* if there exists a Y-linear map $f\colon A^W \to A^X$ which is $1-1$ and which satisfies

$$f(\mathscr{F}) = \{f(F)\colon F \in \mathscr{F}\} = \mathscr{G}.$$

A subspace $H \in \binom{U}{k}$ is called *l-complete* or simply *complete* if $\binom{H}{l} \subseteq \mathscr{F}$ holds. Let $\binom{\mathscr{F}}{k} = \{H_1, ..., H_s\}$ be the collection of all the complete k-spaces in $\mathscr{F}$. Suppose now that $Y \subset X$, $E \in \binom{A^Y}{k}$, $U \in \binom{A^X}{n}$, $p_Y(U) = E$, and $\mathscr{F} \subset \binom{U}{l}$ is a family of Y-transverse spaces. Then necessarily $\binom{\mathscr{F}}{k} = \{H_1, ..., H_s\}$ is a family of Y-transverse k-spaces satisfying $p_Y(H_i) = E$, $1 \leqslant i \leqslant s$. For every positive integer m we shall now define the mth Y-amalgamated power $\mathscr{F}^m$ of $\mathscr{F}$. To do this, let $X^{(1)}, ..., X^{(m)}$ be copies of X satisfying $X^{(a)} \cap X^{(b)} = Y$ for $1 \leqslant a < b \leqslant m$. For a subspace U of A^X, let $U^{(a)}$ be the corresponding subspace of $A^{X^{(a)}}$. Then we define

$$\mathscr{F}^m := \{F_1^{(1)} F_2^{(2)} \cdots F_m^{(m)}\colon F_i \in \mathscr{F} \text{ and } p_Y(F_1) = \cdots = p_Y(F_m)\}$$

The next statement can be easily verified.

PROPOSITION 2. $\binom{\mathscr{F}^m}{k} = \{G_1^{(1)} G_2^{(2)} \cdots G_m^{(m)}\colon G_i \in \binom{\mathscr{F}}{k},\ 1 \leqslant i \leqslant m\}$.

Let us now suppose that $m = m(r, s)$ is so large that the conclusion of the Hales–Jewett theorem holds for partitioning the points of $[s]^m$ into r classes.

PROPOSITION 3. *For every partition of $\binom{\mathscr{F}^m}{k}$ into r classes there exists an n-space U' with $\mathscr{F}^m \cap U'$ being Y-isomorphic to $\mathscr{F}$ and such that all its complete k-spaces are in the same class.*

Proof. In view of Proposition 2, a partition of $\binom{\mathscr{F}^m}{k}$ can be regarded as a partition of $[s]^m$ (recall that $\binom{\mathscr{F}}{k} = \{H_1, ..., H_s\}$). By the choice of m we can find a homogeneous line L in $[s]^m$. Let $[m] = C \cup M$ be the corresponding partition into constant and moving coordinates. For $i \in C$, let $G(i)$ be the "value" of the corresponding constant, and let $G^{(i)}$ be the copy of $G(i)$ in $A^{X^{(i)}}$. Thus the line L consists of the following complete k-spaces:

$$L = \{H^{(1)} \cdots H^{(m)}\colon H^{(i)} = G^{(i)},\ i \in C \text{ and } H^{(i)} = H^{(i')} \text{ for } i, i' \in M\}$$

Let us now define a Y-linear map $f\colon A^X \to \bigoplus_i A^{X^{(i)}}$.

For a point $v \in A^X$, let $v' = p_Y(v)$ denote the Y-part of v and $v'' = p_{X-Y}(v)$ the $(X-Y)$-part of v. Then f is defined as follows. The Y-part of $f(v)$ is v'. For $i \in M$, the $(X^{(i)} - Y)$-part of $f(v)$ is v''. Finally for $i \in C$ the $(X^{(i)} - Y)$-part of $f(v)$ is the $(X^{(i)} - Y)$-part of the unique element in $G^{(i)}$ having Y-part v'.

It is easy to check that f is $1-1$ and Y-linear. Let $U' = f(U)$ be the image of U. We must show that $\mathscr{F}^{(m)} \cap \binom{U'}{l}$ is Y-isomorphic to $\mathscr{F}$. Indeed, it is clear that $f(\mathscr{F}) \subseteq \mathscr{F}^m \cap \binom{U'}{l}$. We must show that equality actually holds. Suppose that $\tilde{F} = F_1^{(1)} \cdots F_m^{(m)} \in (\mathscr{F}^m \cap \binom{U'}{l})$ and $p_Y(F_1) = \cdots = p_Y(F_m) := E_0$. We have to show that $F_i = F_j$ for $i, j \in M$, and that for $i \in C$, $F_i^{(i)}$ is the unique l-space in $G^{(i)}$ with $p_Y(F_i^{(i)}) = E_0$.

Assume first that $F_i \neq F_j$ for some $i, j \in M$. Then for some $v \in E_0$, the unique elements in the two l-spaces F_i, F_j having Y-part v are distinct. However, this implies that the unique element in $\tilde{F}$ with Y-part v is not in $f(U)$, which is impossible. Next, assume that for some $i \in C$, $F_i^{(i)}$ is not the unique l-space in $G_i^{(i)}$ which projects onto E_0. Then $F_i^{(i)}$ is not a subspace of $G_i^{(i)}$. Therefore we can find $v \in E_0$ such that the unique element of $F_i^{(i)}$ with Y-part v is not in $G_i^{(i)}$. However, then the unique element in $\tilde{F}$ with Y-part v is not in $f(U)$, a contradiction. This also proves that $\binom{f(\mathscr{F})}{k} = L$. Since L is homogeneous, the proof is complete. ∎

The Main Result

Theorem (Induced Restricted Ramsey Theorem for Spaces). *Suppose $\mathscr{F}$ is a family of l-spaces in A^X with $|X| = n$, and $k \geq l$ and r are positive integers. Then there exists a set W and a family $\mathscr{U}$ of l-spaces in A^W so that for any partition of the set of k-spaces $\binom{\mathscr{U}}{k}$ into r classes there always exists an n-space $Z \in \binom{A^W}{n}$ such that*

$$\mathscr{F}' = U \cap \binom{Z}{l} \cong F$$

and with $\binom{\mathscr{F}'}{k}$ homogeneous. Furthermore, if A^X contains no p-space P with $\binom{P}{l} \subseteq \mathscr{F}$ then A^W contains no p-space P' with $\binom{P'}{l} \subseteq \mathscr{U}$.

Note. By $\mathscr{F}' \cong \mathscr{F}$ we mean that $\mathscr{F}'$ is an induced isomorphic copy of F, i.e., there exists a $1-1$ linear map $f \colon A^X \to A^W$ such that

$$f(\mathscr{F}) = \{f(F) \colon F \in \mathscr{F}\} = \mathscr{F}'$$

Proof. To begin with, let t be an integer sufficiently large to guarantee that if the k-spaces of a t-space T are arbitrarily partitioned into r classes then some n-space S of T has $\binom{S}{k}$ homogeneous. Let w_0 be a large integer (to be specified later) and consider a w_0-element set W_0 and a t-element set Y with $Y \subseteq W_0$. Let the set $\binom{A^Y}{n}$ of n-spaces in A^Y be denoted by $\{D_1, D_2, \ldots, D_{\left[\begin{smallmatrix} t \\ n \end{smallmatrix}\right]}\}$ and let the set $\binom{A^Y}{k}$ of k-spaces in A^Y be denoted by $\{E_1, E_2, \ldots, E_{\left[\begin{smallmatrix} t \\ k \end{smallmatrix}\right]}\}$. Choose a collection of n-spaces in A^{W_0}, say H_1, $H_2, \ldots,$

$H_{[{}^{l}_{n}]}$, which are as disjoint as possible, i.e., pairwise disjoint or having pairwise intersection (0, 0,..., 0) either in the case of vector spaces or in the case of parameter sets with the choice $B=\{0\}$, and furthermore, so that $p_Y(H_i)=D_i$, $1\leqslant i\leqslant [{}^{l}_{n}]$. Thus, $p_Y\colon A^{W_0}\to A^Y$ is $1-1$ on each H_i. This is certainly possible if w_0 is taken sufficiently large. For each H_i, let $\mathscr{H}_i\subseteq \binom{H_i}{l}$ be isomorphic to the given family $\mathscr{F}$. We define the *zeroth configuration* $\mathscr{C}_0$ to be the family $\bigcup_{1\leqslant i\leqslant [{}^{l}_{n}]}\mathscr{H}_i$.

Suppose now that for some $j<[{}^{l}_{k}]$ we have defined the jth configuration $\mathscr{C}_j$, consisting of a certain family of Y-transverse l-spaces in A^{W_j} for some $W_j\supseteq Y$ (where w_j will denote $|W_j|$). We will now describe the construction of the family $\mathscr{C}_{j+1}$.

To start, first set $E=E_{j+1}$ and define

$$\mathscr{G}=\{C\in\mathscr{C}_j\colon p_Y(C)\subseteq E\}.$$

Set $s=|\binom{\mathscr{G}}{k}|$ and note that for $G\in\binom{\mathscr{G}}{k}$, we have $P_Y(G)=E$. Fix a labelling of the s members of $\binom{\mathscr{G}}{k}$, say $\binom{\mathscr{G}}{k}=\{H_1,\ldots,H_s\}$. We can consider the set of all sequences $(G_1,\ldots,G_m)$ as a Hales–Jewett cube $[s]^m$. Let $m=m(r,s)$ be the smallest integer so that in every partition of $[s]^m$ into r classes there is a homogeneous line. Let Π be the set of all lines in $[s]^m$ and let Y_π be a copy of Y for each $\pi\in\Pi$. Further, let $h\colon A^Y\to E$ be a *retract*, i.e., h is linear and the restriction of h to E is the identity. Let $W^{(i)}$, $1\leqslant i\leqslant m$, be a copy of W_j and suppose that $W^{(i)}\cap W^{(i')}=Y$ holds for $1\leqslant i<i'\leqslant m$. Finally, assume that the Y_π are pairwise disjoint and disjoint from all the $W^{(i)}$. Set $W_{j+1}=(\bigcup_i W^{(i)})\cup(\bigcup_\pi Y_\pi)$.

Now we are ready to define $\mathscr{C}_{j+1}$. It will be the amalgamation of copies of $\mathscr{C}_j$, one copy $\mathscr{C}^{(\pi)}$ for each line $\pi\in\Pi$, where the distinct copies will overlap only in specific ways.

Let $\pi\in\Pi$ be a line and let I be the set of constant coordinates, M the set of moving coordinates of π, and $G(i)$ the value the constant coordinate $i\in I$. We define a Y-linear map $f_\pi\colon A^{W_j}\to A^{W_{j+1}}$ as follows. Suppose that $v\in A^{W_j}$, and that v has Y-part v_1 and (W_j-Y)-part v_2. Then $w=f_\pi(v)$ is defined to have Y-part v_1 and $(W_j^{(i)}-Y)$-part v_2 for each $i\in M$. Also, the Y_σ-part of w is v_1 for $\sigma=\pi$, and $h(v_1)$ for $\sigma\neq\pi$. Finally, for $i\in I$, the $(W^{(i)}-Y)$-part of w is the (W_j-Y)-part of the unique vector $u\in G(i)$ having Y-part $h(v_1)$. Since $M\neq\emptyset$, f_π is a $1-1$ linear map, and it therefore defines an embedding of $\mathscr{C}_j$ into $A^{W_{j+1}}$. Define $\mathscr{C}^{(\pi)}=f(\mathscr{C}_j)$ and $\mathscr{C}_{j+1}=\bigcup_{\pi\in\Pi}\mathscr{C}^{(\pi)}$. Note that if a point v is contained in some space in both $\mathscr{C}^{(\pi)}$ and $\mathscr{C}^{(\sigma)}$, $\pi\neq\sigma$, then $p_Y(v)\in E$ holds. Let us list (without proofs) two important properties of $\mathscr{C}_{j+1}$, which can be verified in a straightforward way. Define $W=\bigcup_i W^{(i)}$.

PROPOSITION 4. *$p_W(\mathscr{C}_{j+1})=\mathscr{C}_j^m$, the cartesian product of $\mathscr{C}$ m times.*

PROPOSITION 5. *Suppose that K is a complete p-space in $\binom{\mathscr{C}}{p}$ for some p and $p_Y(K) \not\subset E$. Then $K \in \binom{\mathscr{C}^{(\pi)}}{p}$ holds for some $\pi \in \Pi$.*

We claim that $\mathscr{C}_{\left[{l \atop k}\right]}$ satisfies the requirements of the Theorem. Set $\mathscr{C} = \mathscr{C}_{\left[{l \atop k}\right]}$ and $E = E_{\left[{l \atop k}\right]}$. Let us suppose that the k-spaces of $\binom{\mathscr{C}}{k}$ are partitioned into r classes. Consider the set of complete k-spaces $K \in \binom{\mathscr{C}}{k}$ satisfying $p_Y(K) = E$. In view of Propositions 5 and 3, for some line $\pi \in \Pi$, the copy $\mathscr{C}^{(\pi)}$ of $\mathscr{C}_{\left[{l \atop k}\right]-1}$ in $\mathscr{C}$ has its complete k-spaces homogeneous. Set $\mathscr{C} = \mathscr{C}^{(\pi)}$ and $E = E_{\left[{l \atop k}\right]-1}$ and apply Propositions 5 and 3 again. Then we obtain an induced copy of $\mathscr{C}_{\left[{l \atop k}\right]-2}$ so that all *its* complete k-spaces which project onto E are homogeneous, as are those which project onto $E_{\left[{l \atop k}\right]}$. Of course, these might be in different classes.

Repeating this altogether $\left[{l \atop k}\right]$ times we obtain an induced copy of $\mathscr{C}_0$ with the property that the color of every complete k-space $K \in \binom{\mathscr{C}_0}{k}$ depends only on $p_Y(K)$. This defines a partition of the k-spaces of A^Y into r classes. By the choice of $t = |Y|$, there must be some n-space $D_i \in \binom{A^Y}{n}$ with all its k-spaces in a single class. Thus, the corresponding family $\mathscr{H}_i = \binom{D_i}{l} \cap C \cong \mathscr{F}$ has all its complete k-spaces $\binom{\mathscr{H}}{k}$ in a single class, as required. It is now straightforward to check that the other requirements of the theorem are satisfied by the choice $\mathscr{U} = \mathscr{C}_{\left[{l \atop k}\right]}$. This completes the proof. ∎

AN APPLICATION TO THE FINITE UNIONS THEOREM

Recall that the special case $A = \{0, 1\}$, $B = \{0\}$, $k = 1$, $l = s$, of the Graham–Rothschild n-parameter set theorem is the following result of Folkman, Rado, and Sanders (cf. [5]).

FINITE UNIONS THEOREM. *Suppose that $n > n_0(s, r)$ and all nonempty subsets of an n-set are partioned into r classes. Then one can find pairwise disjoint sets $S_1, \ldots, S_s$ so that for all $\varnothing \neq I \subseteq [s]$, the sets $\bigcup_{i \in I} S_i$ are in the same class.*

Nešetřil and Rödl [11] gave a direct but somewhat involved proof of the following restricted version. Their result was announced in [10].

RESTRICTED FINITE UNIONS THEOREM. *For all positive integers r and s there exists a family $\mathscr{S}$ of finite sets having the following two properties:*

(i) *If $\mathscr{S} = \mathscr{S}^{(1)} \cup \cdots \cup \mathscr{S}^{(r)}$ is an arbitrary partition then there exists j, $1 \leqslant j \leqslant r$, and pairwise disjoint sets $S_1, \ldots, S_s \in \mathscr{S}^{(j)}$ such that*

$$\bigcup_{i \in I} S_i \text{ is in } \mathscr{S}^{(j)} \text{ for all nonempty sets } I \subseteq [s];$$

(ii) *If $S_1, \ldots, S_{s+1}$ are pairwise disjoint sets then there is a nonempty set $I \subseteq [s+1]$ such that $\bigcup_{i \in I} S_i \notin \mathscr{S}$ (i.e., $\mathscr{S}$ contains no complete $(s+1)$-space).*

To obtain this result from our theorem it is sufficient to set $A = \{0, 1\}$, $B = \{0\}$, $k = l = 1$, F an s-space and $p = s + 1$.

Concluding Remarks

In our main theorem we considered the Ramsey theorem for families of l-subspaces. In other words we considered the pairs (U, Γ) where U is a space and $\Gamma: \binom{U}{l} \to \{0, 1\}$ the mapping describing which l-spaces are the members of our family $\mathscr{F}$. In [12, 13] Prömel proved the induced (but not restricted) version of our main theorem for the case when $\Gamma: \binom{U}{l} \to \{0, 1, \ldots, b\}$ and b is an arbitrary integer. Note that our proof (with no change) actually yields an induced restricted version of Prömel's theorem for an arbitrary b (not just for $b = 1$ as treated in our main theorem). We gave the proof for the case $b = 1$ because it requires somewhat less notation and (we hope) is easier to follow.

Finally, we point out that several special cases of the induced version of the main theorem were previously proved in [1].

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