Available online at www.sciencedirect.com
ScienceDirect
Procedia Engineering 144 (2016) 1195 – 1204
Procedia Engineering
www.elsevier.com/locate/procedia

12th International Conference on Vibration Problems, ICOVP 2015

Effect of gravity and magnetism on surface wave propagation in heterogeneous earth crust

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Abstract

This paper aims to study the propagation of surface wave in two initially stressed heterogeneous magnetoelastic transversely isotropic media lying over a transversely isotropic half-space under the action of gravity. Heterogeneities of both the layers are caused due to exponential variation in elastic parameters. Dispersion relation is obtained in closed form by using Whittaker's asymptotic expansion. Magnetoelastic coupling parameters, heterogeneity, horizontal compressive initial stress and gravity parameters have remarkable effect on the phase velocity of surface wave. The obtained dispersion relation is found to be in well agreement with the classical Love-wave equation. Comparative study and graphical illustration has been made to exhibit the outcomes.

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Keywords: Heterogeneity; magnetoelastic; transversely isotropic; surface wave; Dispersion equation; initial stress.

1. Introduction

The crust is heterogeneous, which makes the study of wave propagation much practical considering the heterogeneous layers. There are different types of vertical heterogeneity persist in crustal layers in the form of

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Peer-review under responsibility of the organizing committee of ICOVP 2015
doi:10.1016/j.proeng.2016.05.101

exponential function, linear function etc. The study of wave propagation in layered elastic media with different boundaries helps to understand and predict the seismic behaviour at the different margins of earth, which makes it applicable in the field of geophysics, civil, mechanical, and other engineering branches. Many researchers had widely studied the theory of Love wave /shear wave propagation in a medium where the velocity, rigidity and density are functions of depth. Kar [1] worked on the propagation of Love-type waves in a non-homogeneous internal stratum of finite thickness lying between two semi-infinite isotropic media. Love waves in different heterogeneous layered media were studied by Gogna [2]. Chattopadhyay [3] discussed the propagation of SH-waves in a sandwiched heterogeneous layer lying between two semi-infinite homogeneous elastic media where the heterogeneity in the sandwiched layer was taken in the form of linearly varying function of depth in the rigidity and density was kept constant. Due to the presence of many physical factors, a large amount of initial stress evolves in a medium which have a pronounced influence on the propagation of waves as shown by Biot [4]. Chattopadhyay and Singh [5] have discussed the propagation of a crack due to magnetoelastic shear waves in a self-reinforced medium. In this paper we have discussed the propagation of shear waves in two initially stressed non-homogeneous magnetoelastic transversely isotropic media. The both media are lying over a transversely isotropic half-space which is under gravity. The dispersion equation is obtained in closed form.

2. Formulation and solution of the problem

We consider two heterogeneous magnetoelastic transversely isotropic media (M_1 and M_2) of thickness H_1 and H_2. The layers are under horizontal initial stresses P_1 and P_2 overlying a semi-infinite transversely isotropic medium under Gravity. The *x*-axis is taken in the direction of wave propagation, *z*-axis as vertically downwards and origin has taken at the interface of layers and half-space. The rigidity and density of both the layers are taken in the form of exponential variation with heterogeneities parameter a and b in transversely isotropic media.

The rigidity and density for transversely isotropic half-space under gravity are considered as $\{L_3,\ N_3\}$, ρ_3 and $P_3 = -\rho_3 g z$ (Fig.1). If u_j, v_j and w_j are the displacement components in x, y and *z* directions respectively, then for shear wave propagating along the *x*- direction and causing displacement in the *y*- direction only, we consider the displacement components as

$u_j = 0,\ w_j = 0,\ v_j = v_j(x,z,t)$ and $\partial/\partial y \equiv 0,$ where $j = 1,\ 2,\ 3.$

The only non-vanishing equation of motion for the propagation of shear wave in initially stressed heterogeneous magnetoelastic transversely isotropic medium is

$$\frac{\partial}{\partial x}\left[N_i\frac{\partial v_j}{\partial x}\right]+\frac{\partial}{\partial z}\left[L_i\frac{\partial v_j}{\partial z}\right]+\left(\frac{P_i}{2}+\mu_e^{(i)}H_0^2\cos^2\varphi\right)\frac{\partial^2 v_j}{\partial x^2}+\mu_e^{(i)}H_0^2\sin^2\varphi\frac{\partial^2 v_j}{\partial z^2}+\mu_e^{(i)}H_0^2\sin 2\varphi\frac{\partial^2 v_j}{\partial x\partial z}=\rho_i\frac{\partial^2 v_j}{\partial t^2}. \tag{1}$$

On simplifying the above Eq. (1), reduces to

$$\left[\frac{N_i}{L_i}+\frac{P_i}{2L_i}+t_H^{(i)}\cos^2\varphi\right]\frac{\partial^2 v_j}{\partial x^2}+\left[1+t_H^{(i)}\sin^2\varphi\right]\frac{\partial^2 v_j}{\partial z^2}+\left[t_H^{(i)}\sin 2\varphi\right]\frac{\partial^2 v_j}{\partial x\partial z}=\frac{1}{\beta_i^2}\frac{\partial^2 v_j}{\partial t^2}, \tag{2}$$

Where

$t_H^{(i)} = \dfrac{\mu_e^{(i)}H_0^2}{L_i},\quad \beta_i = \sqrt{L_i/\rho_i},$

ρ_i are the densities of the media, N_i and L_i can be identified as the shear elastic modulus in the transverse and longitudinal direction of the transversely isotropic media (for all i =1, 2, 3). v_j is the displacement components along y directions for all j=1, 2, 3. $\mu_e^{(i)}$ {for all i=1, 2, 3} are the magnetic permeability for media (M_1, M_2 and M_3) respectively. H_0 be the induced magnetic (primary) field.

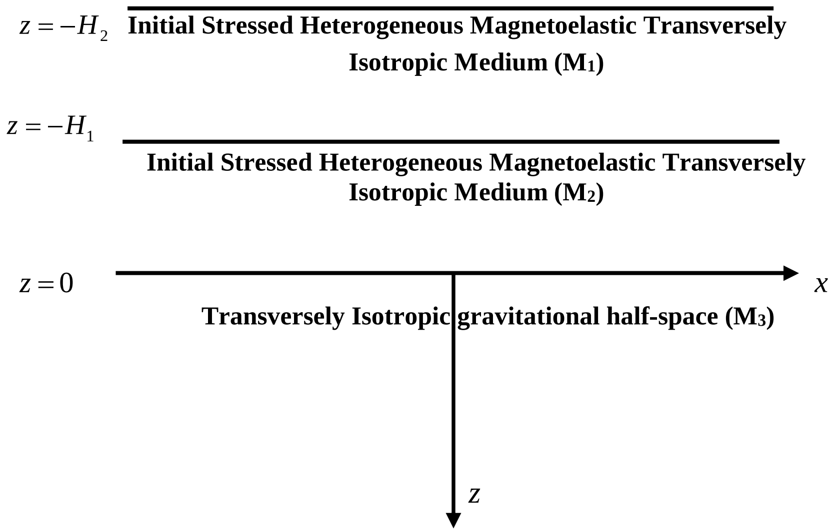

Fig. 1 Geometry of the problem

2.1. The equation of motion for medium (M_1)

Putting $i = j = 1$ in eq. (1) the equation of motion for the medium (M_1) can be written as

$$\frac{\partial}{\partial x}\left[N_1\frac{\partial v_1}{\partial x}\right]+\frac{\partial}{\partial z}\left[L_1\frac{\partial v_1}{\partial z}\right]+\left(\frac{P_1}{2}+\mu_e^{(1)}H_0^2\cos^2\varphi\right)\frac{\partial^2 v_1}{\partial x^2}+\mu_e^{(1)}H_0^2\sin^2\varphi\frac{\partial^2 v_1}{\partial z^2} + \mu_e^{(1)}H_0^2\sin 2\varphi\frac{\partial^2 v_1}{\partial x\partial z}=\rho_1\frac{\partial^2 v_1}{\partial t^2}. \tag{3}$$

The heterogeneity parameters for this medium (M_1) are considered as

$$\left.\begin{array}{l} N_1=N_1^{(0)}e^{az},\ L_1=L_1^{(0)}e^{az},\ \rho_1=\rho_1^{(0)}e^{az}, \\ \mu_e^{(1)}=\mu_{e1}^{(0)}e^{az} \text{ and } P_1=P_1^{(0)}e^{az} \end{array}\right\} \tag{4}$$

where a is the heterogeneity parameter having dimension inverse of length and $\left\{N_1^{(0)},\ L_1^{(0)},\ \rho_1^{(0)}, P_1^{(0)},\ \mu_{e1}^{(0)}\right\}$ are constants of medium M_1. With the help of Eq. (4), the Eq. (3) becomes

$$Q_1 \frac{\partial^2 v_1}{\partial x^2} + R_1 \frac{\partial^2 v_1}{\partial z^2} + S_1 \frac{\partial^2 v_1}{\partial x \partial z} + a \frac{\partial v_1}{\partial z} = \frac{1}{\beta_1^2} \frac{\partial^2 v_1}{\partial t^2}, \tag{5}$$

where

$$Q_1 = \frac{N_1^{(0)}}{L_1^{(0)}} + \frac{P_1^{(0)}}{2L_1^{(0)}} + t_H^{(1)} \cos^2 \varphi, \quad R_1 = 1 + t_H^{(1)} \sin^2 \varphi, \; S_1 = t_H^{(1)} \sin 2\varphi, \; t_H^{(1)} = \frac{\mu_{e1}^{(0)} H_0^2}{L_1^{(0)}}$$

and $\beta_1 = \sqrt{L_1^{(0)} / \rho_1^{(0)}}$ is the shear wave velocity of medium M_1.

Assuming

$$v_1(x,z,t) = V_1(z) e^{ik(x-ct)} \tag{6}$$

With the help of eqns. (5) and (6), we get the displacement components of medium (M_1) as

$$v_1(x,z,t) = e^{-(\alpha_1/2)z} \left[A_1 \cos(T_1 z) + B_1 \sin(T_1 z) \right] e^{ik(x-ct)} \tag{7}$$

where $\alpha_1 = ikS_1 + a$ and $T_1 = k\sqrt{\frac{S_1^2}{4R_1^2} + \frac{\left(c^2/\beta_1^2 - Q_1\right)}{R_1}}$, A_1 and B_1 are arbitrary constants.

2.2. The equation of motion for medium (M_2)

Putting $i = j = 2$ in Eq. (1), the equation of motion for medium (M_2) becomes

$$\frac{\partial}{\partial x}\left[N_2 \frac{\partial v_2}{\partial x} \right] + \frac{\partial}{\partial z}\left[L_2 \frac{\partial v_2}{\partial z} \right] + \left(\frac{P_2}{2} + \mu_e^{(2)} H_0^2 \cos^2 \varphi \right) \frac{\partial^2 v_2}{\partial x^2} + \mu_e^{(2)} H_0^2 \sin^2 \varphi \frac{\partial^2 v_2}{\partial z^2} + \mu_e^{(2)} H_0^2 \sin 2\varphi \frac{\partial^2 v_2}{\partial x \partial z} = \rho_2 \frac{\partial^2 v_2}{\partial t^2}. \tag{8}$$

The heterogeneity parameters in exponential variation for the intermediate layer (M_2) are considered as

$$\left.\begin{aligned} &N_2 = N_2^{(0)} e^{bz}, \; L_2 = L_2^{(0)} e^{bz}, \\ &\rho_2 = \rho_2^{(0)} e^{bz}, \; P_2 = P_2^{(0)} e^{bz} \text{ and } \mu_{e2} = \mu_{e2}^{(0)} e^{bz} \end{aligned}\right\} \tag{9}$$

where b is the heterogeneity parameter of medium M_2 and $\left\{ N_2^{(0)}, L_2^{(0)}, \rho_2^{(0)}, P_2^{(0)}, \mu_{e2}^{(0)} \right\}$ are constants.
In view of Eq. (9), Eq. (8) can be written as

$$Q_2 \frac{\partial^2 v_2}{\partial x^2} + R_2 \frac{\partial^2 v_2}{\partial z^2} + S_2 \frac{\partial^2 v_2}{\partial x \partial z} + b \frac{\partial v_2}{\partial z} = \frac{1}{\beta_2^2} \frac{\partial^2 v_2}{\partial t^2}, \tag{10}$$

where

$$Q_2 = \frac{N_2^{(0)}}{L_2^{(0)}} + \frac{P_2^{(0)}}{2L_2^{(0)}} + t_H^{(2)} \cos^2 \varphi, \quad R_2 = 1 + t_H^{(2)} \sin^2 \varphi, \quad S_2 = t_H^{(2)} \sin 2\varphi, \ t_H^{(2)} = \frac{\mu_{e2}^{(0)} H_0^2}{L_2^{(0)}}$$

and $\beta_2 = \sqrt{L_2^{(0)} / \rho_2^{(0)}}$ is the shear wave velocity of intermediate layer M_2.
Using $v_2(x,z,t) = V_2(z) e^{ik(x-ct)}$ in Eq. (10) we get the displacement components of medium M_2 as

$$v_2(x,z,t) = e^{-(\alpha_2/2)z} \left[A_2 \cos(T_2 z) + B_2 \sin(T_2 z) \right] e^{ik(x-ct)} \tag{11}$$

where $\alpha_2 = ikS_2 + b$ and $T_2 = k\sqrt{\frac{S_2^2}{4R_2^2} + \frac{\left(c^2/\beta_2^2 - Q_2\right)}{R_2}}$, A_2 and B_2 are arbitrary constants.

2.3. The equation of motion for half-space (M_3)

The equation of motion for the transversely isotropic half space (M_3) can be written as

$$\left[N_3 - \frac{1}{2}\rho_3 g z \right] \frac{\partial^2 v_3}{\partial x^2} + \left[L_3 - \frac{1}{2}\rho_3 g z \right] \frac{\partial^2 v_3}{\partial z^2} - \frac{1}{2}\rho_3 g \frac{\partial v_3}{\partial z} = \rho_3 \frac{\partial^2 v_1}{\partial t^2}, \tag{12}$$

where N_3, L_3, ρ_3, v_3, g and c are the shear elastic modulus in transverse and longitudinal directions, the density of the medium, displacement component, acceleration due to gravity and common wave velocity of the half-space (M_3).
Using $v_3(x,z,t) = V_3(z) e^{ik(x-ct)}$ in the above eq. (12), we have

$$\frac{d^2 V_3(z)}{dz^2} + \frac{b_1}{(a_1 + b_1 z)} \frac{dV_3(z)}{dz} + k^2 \left[\frac{c_1^2}{(a_1 + b_1 z)} - 1 \right] V_3(z) = 0 \tag{13}$$

Considering $\sigma_1(z) = -2k/b_1(a_1 + b_1 z)$, $s = -c_1^2 k / 2b_1$ and using the transformation $V_3(z) = \psi(z) / \sqrt{a_1 + b_1 z}$ the above Eq. (13) takes the form

$$\frac{d^2 \psi(\sigma_1)}{d\sigma_1^2} + \left[-\frac{1}{4} + \frac{s}{\sigma_1} + \frac{1}{4\sigma_1^2} \right] \psi(\sigma_1) = 0. \tag{14}$$

Eq. (14) is the form of Whittaker's equation and its solution can be written as

$$\psi(\sigma_1) = A_3 W_{-s,0}(-\sigma_1) + B_3 W_{s,0}(\sigma_1) \ , \tag{15}$$

where $W_{s,0}(\sigma_1)$ and $W_{-s,0}(-\sigma_1)$ are Whittaker's functions of first and second kind of order *s* and *0*.
In view of condition $\psi(\sigma_1) \to 0$ as $z \to \infty$ the appropriate solution of eq. (15) becomes

$$\psi(\sigma_1) = A_3 W_{-s,0}(-\sigma_1). \tag{16}$$

Hence the solution for the displacement component of the half space reduces to

$$v_3(x,z,t) = A_3 \left(\frac{2a_1 - gz}{2} \right)^{-1/2} W_{-s,0}\left[-\left(4/G - 2kz \right) \right] e^{ik(x-ct)}, \tag{17}$$

where $a_1 = L_3/\rho_3$, $b_1 = (-g/2)$, $c_1^2 = c^2 + a_1(1 - N_3/L_3)$ and $G = g/ka_1$ is the Universal gravitational constant (Biot's gravity parameter). The asymptotic expansion (Whittaker and Watson [6]) of Whittaker's function for large argument and retaining up to the second term $W_{-s,0}\left[-\left(4/G - 2kz\right)\right]$, may be approximated as

$$W_{-s,0}\left[-\left(4/G - 2kz \right) \right] \sim e^{-(kz - 2/G)} \left(2kz - 4/G \right)^{-s} \left[1 - \frac{(s+0.5)^2}{(2kz - 4/G)} \right]. \tag{18}$$

3. Boundary conditions

For the shear wave propagation, the following boundary conditions must be satisfied:

$$\left.\begin{array}{ll} i) & \tau_{yz}^{(1)} = 0 \text{ at } z = -H_2, \\ ii) & v_1 = v_2 \ \text{ at } \ z = -H_1, \\ iii) & \tau_{yz}^{(1)} = \tau_{yz}^{(2)} \ \text{ at } \ z = -H_1, \\ iv) & v_2 = v_3 \ \text{ at } \ z = 0, \\ v) & \tau_{yz}^{(2)} = \tau_{yz}^{(3)} \ \text{ at } \ z = 0. \end{array}\right\} \tag{19}$$

Using the displacement components $v_1(x,z,t)$, $v_2(x,z,t)$ and $v_3(x,z,t)$ from eqns. (7), (11) and (17) in the above five boundary conditions of eq. (19), we get

$$\left\{ -\frac{\alpha_1}{2}\cos(T_1 H_2) + T_1 \sin(T_1 H_2) \right\} A_1 + \left\{ \frac{\alpha_1}{2}\sin(T_1 H_2) + T_1 \cos(T_1 H_2) \right\} B_1 = 0, \tag{20}$$

$$e^{\left(\frac{\alpha_1 - \alpha_2}{2}\right) H_1} \left\{ A_1 \cos(T_1 H_1) - B_1 \sin(T_1 H_1) \right\} - \left\{ A_2 \cos(T_2 H_1) - B_2 \sin(T_2 H_1) \right\} = 0, \tag{21}$$

$$\left(L_1^{(0)}/L_2^{(0)}\right)e^{\left\{(b-a)+\left(\frac{\alpha_1-\alpha_2}{2}\right)\right\}H_1}\left[\left\{-\frac{\alpha_1}{2}\cos\left(T_1H_1\right)+T_1\sin\left(T_1H_1\right)\right\}A_1+\left\{\frac{\alpha_1}{2}\sin\left(T_1H_1\right)+T_1\cos\left(T_1H_1\right)\right\}B_1\right]$$
$$=\left[\left\{-\frac{\alpha_2}{2}\cos\left(T_2H_1\right)+T_2\sin\left(T_2H_1\right)\right\}A_2+\left\{\frac{\alpha_2}{2}\sin\left(T_2H_1\right)+T_2\cos\left(T_2H_1\right)\right\}B_2\right], \tag{22}$$

$$A_2=\frac{A_3}{\sqrt{a_1}}\left\{W_{-s,0}\left[-\left(\frac{4}{G}-2kz\right)\right]\right\}_{\text{at } z=0} \tag{23}$$

and

$$\left(L_2^{(0)}/L_3\right)\left\{-\frac{\alpha_2}{2}A_2+T_2B_2\right\}=A_3\left[\frac{d}{dz}\left\{\left(\frac{2a_1-gz}{2}\right)^{-1/2}W_{-s,0}\left[-\left(\frac{4}{G}-2kz\right)\right]\right\}\right]_{\text{at } z=0} \tag{24}$$

where A_1, B_1, A_2, B_2 and A_3 are arbitrary constants. Eliminating arbitrary constants A_1, B_1, A_2, B_2 and A_3 from Eq. (20) to (24), we find the dispersion equation as

$$\frac{L_1^{(0)}e^{(b-a)H_1}}{L_2^{(0)}}\left[\frac{T_2-\left(\frac{\alpha_2}{2}+\frac{\sqrt{a_1}L_3}{L_2^{(0)}}\left[\frac{\frac{d}{dz}\left\{\left(\frac{2a_1-gz}{2}\right)^{-1/2}W_{-s,0}\left[-\left(\frac{4}{G}-2kz\right)\right]\right\}}{W_{-s,0}\left[-\left(\frac{4}{G}-2kz\right)\right]}\right]_{\text{at } z=0}\right)\tan\left(T_2H_1\right)}{1-\left\{\frac{\alpha_1/2-T_1\tan\left(T_1H_2\right)}{\alpha_1/2\tan\left(T_1H_2\right)+T_1}\right\}\tan\left(T_1H_1\right)}\right]=$$

$$\frac{\frac{\left\{T_2\tan\left(T_2H_1\right)-\frac{\alpha_2}{2}\right\}T_2}{\left\{T_2+\frac{\alpha_2}{2}\tan\left(T_2H_1\right)\right\}}+\left(\frac{\alpha_2}{2}+\frac{\sqrt{a_1}L_3}{L_2^{(0)}}\left[\frac{\frac{d}{dz}\left\{\left(\frac{2a_1-gz}{2}\right)^{-1/2}W_{-s,0}\left[-\left(\frac{4}{G}-2kz\right)\right]\right\}}{W_{-s,0}\left[-\left(\frac{4}{G}-2kz\right)\right]}\right]_{\text{at } z=0}\right)}{T_1\left\{T_2+\frac{\alpha_2}{2}\tan\left(T_2H_1\right)\right\}\left\{\tan\left(T_1H_1\right)-\tan\left(T_1H_2\right)\right\}}. \tag{25}$$

With the help of Whittaker's asymptotic expansion (18), Eq. (27) reduces to

$$\frac{L_1^{(0)}e^{(b-a)H_1}}{L_2^{(0)}}\left[\frac{1-\left\{\left(\alpha_2/2+Q\right)\tan\left(T_2H_1\right)\right\}/T_2}{1-\left\{\dfrac{\left(\alpha_1/2\right)-T_1\tan\left(T_1H_2\right)}{\left(\alpha_1/2\right)\tan\left(T_1H_2\right)+T_1}\right\}\tan\left(T_1H_1\right)}\right]$$

$$=\left[\frac{\left(T_2\tan\left(T_2H_1\right)+\left\{\left(\alpha_2/2\right)^2\tan\left(T_2H_1\right)/T_2\right\}\right)+Q\left(1+\left\{\left(\alpha_2/2\right)\tan\left(T_2H_1\right)/T_2\right\}\right)}{T_1\left\{\tan\left(T_1H_1\right)-\tan\left(T_1H_2\right)\right\}}\right] \tag{26}$$

where $Q=\left(L_3/L_2^{(0)}\right)\sqrt{a_1}\left\{\dfrac{g}{4a_1}+k\left(\dfrac{Gs}{2}-1+\dfrac{G^2\left(s+0.5\right)^2}{2\left[4+G\left(s+0.5\right)^2\right]}\right)\right\}$.

Equation (26) is the dispersion equation for shear wave in the heterogeneous magnetoelastic transversely isotropic media under initial stresses lying over a transversely isotropic half-space under gravity.

4. Numerical results and discussions

We consider the following data of elastic coefficients of equivalent transversely isotropic models of the Upper Mantle (olivine model and petrofabric model) from Anderson [7]:

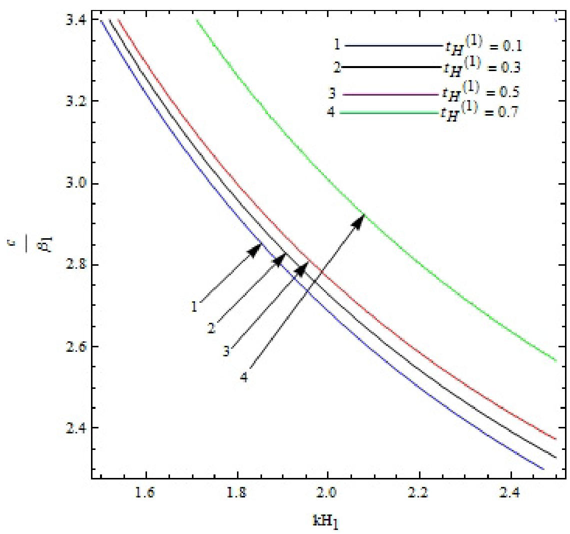

Fig. 2 Variation in dimensionless phase velocity (c/β_1) against dimensionless wave number (kH_1) for different values of transversely isotropic magnetoelastic coupling parameter $\left(t_H^{(1)}\right)$ when both heterogeneous layers (M_1 and M_2) are under initial stress and half space (M_3) is under the action of gravity.

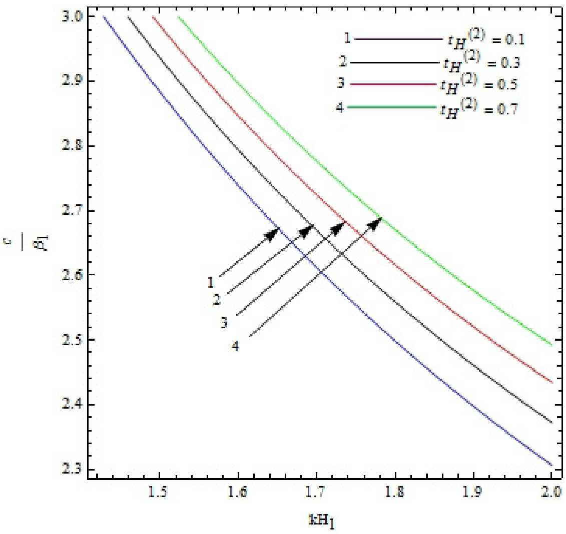

Fig. 3 Variation in dimensionless phase velocity (c/β_1) against dimensionless wave number (kH_1) for different values of transversely isotropic magnetoelastic coupling parameter $(t_H^{(2)})$ when both heterogeneous layers (M_1 and M_2) are under initial stress and half space (M_3) is under the action of gravity.

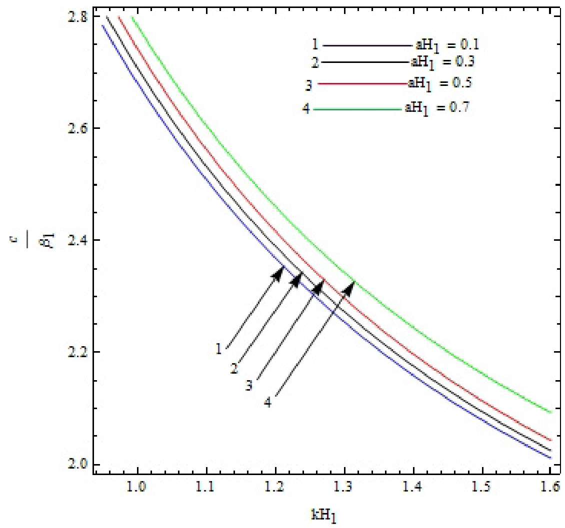

Fig. 4 Variation in dimensionless phase velocity (c/β_1) against dimensionless wave number (kH_1) for different values of heterogeneity parameter (aH_1) when both magnetoelastic transversely isotropic layers (M_1 and M_2) are under initial stress and half space (M_3) is under the action of gravity.

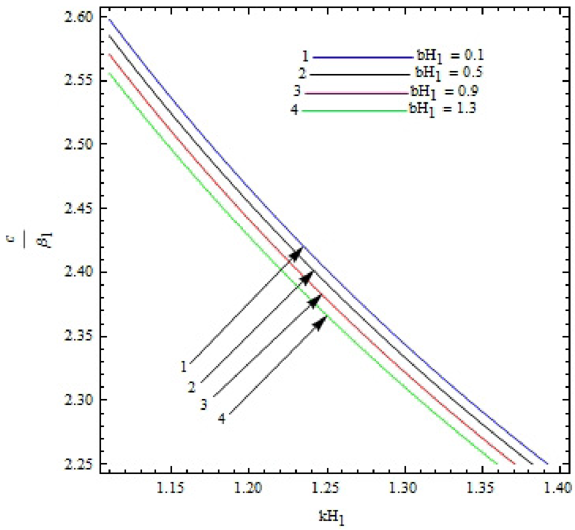

Fig. 5 Variation in dimensionless phase velocity (c/β_1) against dimensionless wave number (kH_1) for different values of heterogeneity parameter (bH_1) when both magnetoelastic transversely isotropic layers (M_1 and M_2) are under initial stress and half space (M_3) is under the action of gravity.

5. Conclusions

The present paper deals with the propagation of shear waves in two initially stressed heterogeneous magnetoelastic transversely isotropic medium overlying a transversely isotropic half-space under the action of gravity has been studied. Closed form expression for dispersion equation in terms of various affecting parameters viz. heterogeneity parameter, transversely isotropic magnetoelastic coupling parameter, horizontal compressive initial stress of the layers and universal gravitational constant of the half-space has been established. The magnetoelastic coupling parameters and heterogeneity parameters of the layers (M_1 and M_2) give the significant effect on the phase velocity of shear waves. The following outcomes can be accomplished through this study:

- The wave number affects phase velocity substantially. More precisely, the phase velocity of dispersion curves decreases with the increase of wave number.
- As heterogeneity grows in the uppermost layer it increases the phase velocity of shear wave. Moreover the heterogeneity grows in the sandwich layer give the adverse effect on the phase velocity of shear wave.

Acknowledgement

The authors convey their sincere thanks to Indian School of Mines, Dhanbad for providing JRF to Ms. Nirmala kumari and also facilitating us with its best facility.

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