Simulation of an asymmetric lineshape of a resonant dip observed in a switching-current measurement of a superconducting flux qubit

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Abstract

We performed a simulation of the switching current of a DC superconducting quantum interference device (SQUID) magnetometer that reads out the state of a superconducting flux quantum bit (qubit). Three configurations for coupling the SQUID and qubit were considered. The asymmetric lineshape of a resonant dip observed in a switching-current measurement under continuous microwaves was reproduced by the simulation taking into account the effect of the current in the SQUID arms. The flux shift due to the SQUID current was also calculated.

Keywords: superconducting qubit; DC-SQUID; Josephson junction; resonance

1. Introduction

Many experimental results over the last decade have suggested that a superconducting flux qubit [1] has a relatively long coherence time among various solid-state qubit implementations. A very recent experiment has demonstrated a free induction decay time of 2.5 μs and its improvement up to 23 μs using a spin-echo technique [2]. A conventional measurement of coherence time in a flux qubit involves a time-resolved measurement of a resonant peak and dip [3].

For the observation of the peak and dip, typically, a current-bias pulse is applied to a DC-SQUID (hereafter just “SQUID”) magnetometer that is coupled to a qubit immediately after a resonant microwave pulse is applied. Then, the dependence of the switching probability of the SQUID on microwave frequency or magnetic flux reveals a resonant peak and dip attributable to the transition between energy levels.

When the relaxation time is very short, the excited state decays rapidly, causing the observed resonant peak and dip to become small. When the current-bias pulse is further associated with a significant preceding distortion (preshoot), the resonance will be undetectable. In our study of a tunable flux qubit, we were unable to observe resonance in the switching-probability measurement when the microwave pulse was applied more than 10 ns before the current-bias pulse [4,5]. This can be attributed to a short relaxation time and the preshoot associated with the twisted pair cables used in the measurement. Even in such a situation, observation of a clear resonant peak and dip was possible with a switching-current measurement using a slow ramp of the SQUID bias current, as in the first measurement of resonant transition in a flux qubit [6]. Using this technique, we measured the excitation energies between the ground state and the three lowest excited states in a tunable flux qubit, which were in reasonable agreement with calculated results [5].

In the switching-current measurement we found an asymmetric lineshape of a resonant peak and dip for some of the samples we studied, as shown in Fig. 1. The resonant peak was usually much smaller than the resonant dip. This can be attributed to the probabilistic nature of the switching of the Josephson junction [7]. To investigate the origin of the
lineshape's asymmetry, we performed a classical simulation for the switching current of the readout SQUID. We considered three configurations for coupling the qubit and SQUID. The simulation reproduced the asymmetric lineshape of a resonant peak and dip when the coupling between the qubit and SQUID was relatively strong. The flux shift due to the SQUID bias current was calculated for different coupling configurations. We note that this flux shift is utilized in the switching-probability measurement to observe resonance at the symmetry point [3], wherein the level splitting between the ground state and the first excited state is at a minimum and the longest coherence time is thus expected. The estimate of the flux shift presented here is thus useful in designing a sample and in analyzing the data obtained in the switching-probability measurement.

2. Classical equations for the coupled system of qubit and SQUID

Our model consists of a three-Josephson-junction (3-JJ) flux qubit and a SQUID that share a line segment. We assume that the mutual inductance between the qubit and the SQUID is dominated by the kinetic inductance [8] associated with the shared line. This is also the case for the samples we studied [4,5]. We consider three configurations for the coupling between the qubit and the SQUID, as shown in Fig. 2. In many experiments on 3-JJ flux qubits, the qubit is inside the SQUID loop and coupled symmetrically with both arms of the SQUID (case I) [2,3]. We also consider the configurations wherein the qubit is dominantly coupled with one of the arms (cases II and III) [4,5]. In the case of multiple qubits coupled by an inter-qubit coupler, the coupling schemes of cases II and III have an advantage in that the interaction between the inter-qubit coupler and readout SQUIDs can be made weak, which can result in less complicated analysis of the system.

We denote the magnetic frustrations, i.e., magnetic fluxes in units of the superconducting flux quantum $\Phi_0 = h/2e$, in the qubit loop and SQUID loop as $f_Q$ and $f_{SQ}$, respectively. These are related to externally applied quasistatic magnetic frustrations in the loops, $f_{Q}^{ex}$ and $f_{SQ}^{ex}$, and the currents in each arm of the SQUID, $J_1$ and $J_2$, by

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\begin{align*}
\Delta f_{Q}^{ex} = M(J_1 - J_2) / \Phi_0 & \quad (\text{case I}) \\
\Delta f_{Q}^{ex} = -MJ_1 / \Phi_0 & \quad (\text{case II}) \\
\Delta f_{Q}^{ex} = MJ_2 / \Phi_0 & \quad (\text{case III})
\end{align*}
\]

where $M$ is the kinetic inductance associated with one of the arms used for the coupling, and $I_{\text{circ}}(f_Q)$ is the

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**Fig. 1.** Variation in switching current $I_{SW}$ as a function of magnet current used to apply a uniform magnetic field under the irradiation of continuous microwaves of 20 GHz. The linear background of $I_{SW}$ was subtracted. In contrast to the large asymmetric resonant dip, the resonant peak at $I_{mag} = -3.43$ mA is very small. The arrow indicates the variation in $I_{mag}$ that corresponds to $\Delta f_{Q}^{ex} = 0.02$. Inset: Schematic representation of sample. The crosses represent Josephson junctions.

**Fig. 2.** Schematic representation of the configurations for coupling between qubit and SQUID. The qubit is coupled to both arms of the SQUID (case I) or to one of the arms (cases II and III). $J_1$ and $J_2$ are currents flowing in the left and right SQUID arms, respectively.
circulating current in the qubit loop as a function of \( f_Q \). For simplicity, we neglect the self-inductances of the loops and the contribution of the geometrical inductance to the mutual coupling. The currents in the SQUID arms are obtained as a function of \( f_{SQ} \) and the SQUID bias current \( I_b = J_1 + J_2 \) using the current-phase relations \[9\]. We consider a symmetric DC-SQUID with \( I_0 \) being the critical current of each junction of the SQUID. \( f_{SQ}^{ex} \) and \( f_{SQ}^{in} \) are related to each other by the area ratio \( S_{SQ}/S_Q \) as \( f_{SQ}^{ex} = f_{SQ}^{in} S_{SQ}/S_Q \), where \( S_Q \) and \( S_{SQ} \) are the loop areas of the qubit and SQUID, under the assumption of application of a uniform magnetic field. Here, we consider the case wherein strong monochromatic microwaves are continuously applied to the system, resulting in a saturated resonant peak and dip in \( I_{circ}(f_Q) \), and the SQUID bias current is slowly swept in comparison with qubit dynamics, which are completely neglected in this analysis. We assume that \( I_{circ}(f_Q) \) has the form of a hyperbolic tangent function \[6\] with a resonant peak and dip of Lorentzian lineshapes \[2\].

For given values of \( f_{SQ}^{ex} \) and SQUID bias current \( I_b \), we obtain \( f_{SQ}^{in} \) by solving the above coupled equations. Then, the bare switching current of the SQUID is given as \( I_{sw}^{in}(f_{SQ}) = 2I_b \cos(\pi f_{SQ}) \) \[10\]. In an actual experiment, the SQUID switches to the voltage state before \( I_b \) reaches \( I_{sw}^{(0)} \), because it is affected by thermal fluctuations, quantum fluctuations, and external noises \[10\]. Taking these effects into account, we assume that the switching to the voltage state occurs when \( I_b \) reaches \( I_{sw}^{(0)} \) multiplied by a factor \( \sim 0.6 \). We thus obtain the switching current \( I_{SW} \) as a function of \( f_Q^{ex} \). We note that the conclusion of this study relating to the lineshape does not depend on the precise value of this factor.

### 3. Calculation results and discussion

Calculated curves of \( I_{SW} \) vs. \( f_Q^{ex} \) for \( M = 50 \) pH and 10 pH are plotted in Fig. 3. The other parameters used for the calculation are explained in the figure caption. The height of the qubit step indicating the reversal of the direction of the persistent current is proportional to \( M \). The midpoint of the resonant peak and dip is shifted from \( f_Q^{ex} = 0.5 \) by an amount nearly proportional to \( M \). This flux shift is caused by the currents in the SQUID arms. For the higher value of \( M \), the lineshape of the resonant dip is asymmetric, which is consistent with the observed line shape shown in Fig. 1. The origin of the asymmetry in the lineshape can be understood on the basis of the behavior of the curve of \( I_{sw}^{in} \) vs. \( I_b \) (not shown) from which \( I_{SW} \) was determined.

![Fig. 3. \( I_{SW} \) vs. \( f_Q^{ex} \) curves for \( M = 50 \) pH (solid line) and 10 pH (dotted line), where \( M \) is the kinetic inductance associated with one of the SQUID arms used for coupling. The parameters used for the calculation are \( I_b = 0.7 \) \( \mu A \) and \( S_Q/S_{SQ} = 1.4 \). We assumed that the resonant peak and dip appear at \( f_Q = 0.5 \pm 0.019 \) with intrinsic full width at half the maximum amplitude of 0.0022; we also assumed the maximum circulating current of the qubit to be 0.56 \( \mu A \). The inset shows a comparison of the lineshape of the resonant dips after subtraction of the linear background of \( I_{SW} \).](image1)

![Fig. 4. Comparison among \( I_{SW} \) vs. \( f_Q^{ex} \) curves for different configurations of the coupling between the qubit and the SQUID shown in Fig. 2. The flux shift, i.e., the shift of the midpoint of the resonant peak and dip from \( f_Q^{ex} = 0.5 \) (vertical dashed line), is induced by the SQUID current. This shift depends on the configuration of the coupling as indicated. The curves for cases II and III are offset vertically.](image2)
Curves of $I_{Sw}$ vs. $f_{Q}^{\infty}$ for different configurations, i.e., cases I–III, are compared in Fig. 4. The flux shift due to the currents in the SQUID arms depends on the configuration. For the parameters used for the calculation, the currents in the SQUID arms at the switching of the SQUID are approximately $J_1 = 0.7 \mu A$ and $J_2 = 0.2 \mu A$. Therefore, the flux shift for case III, in which the qubit is dominantly coupled to the right arm in which $J_2$ is flowing, is the smallest.

When a qubit is operated at the symmetry point, where $f_Q = 0.5$, the flux shift due to the SQUID bias current is utilized in order to enable the readout of the qubit state [3]. For this purpose, it is desirable to ensure that the flux shift is larger than the transition width of the qubit step. It should be noted that the flux shift depends on the choice of the arm when only one arm is employed for the coupling.

The observed width of the resonant peak and dip was $(1–10) \times 10^3 \Phi_0$ in terms of the flux in the qubit loop. This value is approximately equal to the variation in the flux shift induced by the SQUID current during a typical 100 μs sweep of the SQUID bias current [4,5]. Because the relaxation time of the qubit measured in our experiment is about three orders of magnitude shorter than this period [4], the continuous-microwave-induced excitation of the qubit prior to the switching of the SQUID should not contribute significantly to the broadening of the resonant peak and dip; the contribution should be less than $\sim 10^{-5} \Phi_0$. This point supports the validity of neglecting the qubit dynamics in our quasistatic simulation. If the rise time of the current-bias pulse becomes less than $\sim 100$ ns, broadening of the resonance from excitation prior to the switching should be observed. This may be the reason for our occasional observation of broad resonance in switching-probability measurements under continuous microwaves.

4. Conclusion

We performed a numerical simulation of the resonant peak and dip in a flux qubit observed in a switching-current measurement using a slowly swept bias current under continuous microwaves. Three different configurations for the coupling between the qubit and the SQUID were considered. Although the intrinsic lineshape of the resonant peak was assumed to be symmetric, the calculated lineshape was asymmetric owing to the inductive effect of the SQUID current on the qubit when the coupling was relatively strong. The calculated asymmetry is consistent with the observed one. The flux shift due to the SQUID bias current, which enables the observation of resonant transition at the symmetry point in the switching-probability measurement, was estimated. The dependence of the flux shift on the choice of the arm used for the coupling should be considered when only one of the SQUID arms is used.

References