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Stochastic interdependence, possibility and probabilistic causality

1. Preamble: interactions with Lotfi Zadeh

I first met Lotfi Zadeh when I started work at GWU around 1970. He was an advisor to GWU's School of Engineering and Applied Science, and was a revered and well liked figure. Lotfi gave us a seminar wherein he challenged the Kolmogorov Axioms of Probability. Since I was hired at GWU to teach probability and statistics to engineering students, I felt compelled to react to the challenge. Lotfi gracefully listened to me, and in his characteristic calm demeanor defended his viewpoints using arguments that fell on deaf ears. These ears remained deaf until about 1998 when (the late) Tom Bement, Jane Booker and Sallie Keller, who were then at Los Alamos, asked me to look at "fuzzy sets", "membership functions" and the "possibility theory", on the grounds that they were being forcefully touted by the engineering community.

Subsequent to the GWU seminar, I saw Lotfi several times at different venues, and he always inquired how up to date my Farsi was. I kept reminding Lotfi that I was not an Iranian, but that my ancestors, about 1300 years ago, did hail from Iran. It seems that Lotfi never holds a grudge against anyone, even those who challenge him publicly.

Lotfi's initial papers on fuzzy sets deepened my appreciation of Kolmogorov's sample space based architecture of the probability theory, and opened my eyes to the "law of the excluded middle" which underlies the theory. By the same token, his notion of a membership function convinced me of the power and flexibility of the likelihood function. All this enabled me, with Jane Booker, to develop an encompassing approach to probability assessment [1], and in so doing, we were able to make the case that possibility, as an alternative to probability, may not be unnecessary. But Lotfi's work also had (for me) an important by-product. It motivated me to study Art Dempster's work on "belief functions", because Art, like Lotfi, was looking at another limitation of the Kolmogorov measure theoretic architecture. Specifically, Kolmogorov induces probability measures of random variables by requiring that the latter be a many-to-one-map. There are many scenarios which call for a one-to-many-map, and to account for these, belief functions are introduced. Alyson Wilson, with whom I ended up writing a joint paper [2], played a key role in me coming to grips with Art's work. Lotfi's work has had an impact on my professional life, not only vis à vis the two publications cited above, but more so because the paper with Booker has often been referenced in philosophy literature. I will now go on to the topic of this paper, namely causality and interdependence.

2. On causality and interdependence: background

Lotfi's attitude to engineering has been philosophical, and this sets him aside from many of his engineering colleagues whose main concerns appear to be focused on seeking solutions to problems. So the question **I** pose here is: "How can one satisfactorily articulate interdependence and causality using the calculus of possibility?" I do not have an answer to this question. My hope is that there is someone out there that can produce an answer and, if not, at least generate a discussion. With that in mind, I give below an appreciation of causality and independence, using the calculus of personal probability.

From a philosophical perspective, it is the notion of causality that gave birth to the notion of dependence, even though the two turn out to be different. Remember, the famous quote: "*correlation is not causation.*" The empire of causality can be conceptualized as a network of giants, with Aristotle as the source node, and a yet to be discovered sink node. The intervening nodes are Bacon (an empiricist), Descartes (axiomatizer), Newton (the suspicious genius), Berkeley (the non-Bayesian) and Hume (the skeptic), as a junction node that has spawned the likes of Kant (an apriorist), Mill (the logician), Popper (the refuter) and Khun (the revolutionist). Against this background of giants appear two modern day giants: de Finetti and Suppes upon whose ideas I lean. Their disposition to interdependence and causality is probabilistic, and to appreciate this, we need to come to terms with the personalistic view of probability. The personalistic view of probability translates to a personalistic view of causality as well [3].

The personalistic view of probability was espoused by the likes of Bayes, Laplace, Poisson, Borel, Keynes, de Finetti, Ramsey and Savage, and has been most convincingly articulated by Lindley. In doing so, the above part company with Venn,

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von Mises, Einstein, Jeffreys and Kolmogorov, but not with Bohr and Schrodinger.

3. Deterministic causality: difficulties with

Since the notion of dependence sprouts from the notion of causality, it behoves one to say a few words about the difficulties associated with causality, as evidenced by the following quotes of Newton, Hume and Kant, respectively:

- "The cause of gravity I do not pretend to know".
- "The ultimate cause of the phenomena we observe are beyond the reach of human inquiry. We have no means of knowing that future instances will conform to our past experience. Our predictions may have turned out right before, but that is no conclusive argument that they will continue to do so. Our belief in some necessary link or connection between causes and effects derives mainly from our *subjective expectations*. Our belief in real causal connections in nature is founded on subjective habit or expectation, not reason".
- "The concept of causality is something we possess *a priori*, in advance of particular observations, and which has a certain kind of objective validity".

4. Personal probability: in the style of de Finetti

Let $\mathcal{D}$ denote an assessor of personal probability, and consider two unknown events, say A and B. Then $P^{\tau}_{\mathcal{D}}(A; \mathcal{H})$ is $\mathcal{D}$'s personal probability of event A, assessed at time τ, when the background information about A that is possessed by $\mathcal{D}$ at time τ is $\mathcal{H}$. In what follows, we suppress $\mathcal{D}$, τ and $\mathcal{H}$, so that $P^{\tau}_{\mathcal{D}}(A; \mathcal{H})$ is simply $P(A)$. de Finetti makes $P(A)$, operational by interpreting it as a 2-sided bet against an individual $\mathcal{E}$. That is, $\mathcal{D}$ stakes an amount, $P(A)$, in exchange of one, should A occur, and is prepared to lose $P(A)$ should A not occur. $\mathcal{D}$ is also prepared to stake $(1 - P(A))$ against event A. That is, $\mathcal{D}$ receives one, if A does not occur, and loses $(1 - P(A))$ if A does occur. $\mathcal{E}$ gets to choose the side of the bet [4], similarly, with event B.

The conditional probability of event A, "given" event B, is denoted $P(A|B)$. It is the amount that $\mathcal{D}$ is willing to stake for event A, supposing that B were to occur. If B were not to occur, the bet is called off. Conditional probabilities are in the subjunctive mood.

In Kolmogorov's axiomatization of probability, there are two rules: Convexity; i.e. $0 \leq P(A) \leq 1$, and additivity; i.e. $P(A \text{ or } B) = P(A) + P(B)$ if A and B are mutually exclusive. The multiplication rule, namely, $P(A \text{ and } B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$, is taken as a definition.

5. Stochastic dependence and its interpretation

First, stochastic dependence is not logical dependence, wherein proposition A can be expressed in terms of proposition B, via de Morgan's set operations like unions, intersections, complements, etc., which make A dependent on B. By contrast, stochastic dependence has an easy to appreciate probabilistic connotation. We say that A and B are *stochastically independent* if $P(A|B) = P(A)$ and $P(A|B^c) = P(A)$, where B^c is the complement of B, mutatis-mutandis for $P(B|A)$ as well. Thus A, stochastically independent of B, implies that B is stochastically independent of A, and vice-versa.

Stochastic independence can be interpreted in a personalistic framework by noting that knowledge of B (or A) does not alter ones betting disposition towards A (or B). De facto, stochastic independence connotes an absence of learning and, like probability, is also personal. The above definition of independence generalizes to multiple events, and it also enables the introduction of *conditional independence* used in belief nets (not to be confused with belief functions).

Stochastic interdependence is the absence of stochastic independence. Thus the judgment of stochastic interdependence is not absolute; it is personal. What is independent to Lotfi may be interdependent to Jane, and vice-versa.

6. Probabilistic causality: Suppes

Unlike interdependence, wherein there is no consideration of time of occurrence of events, and wherein the events commute (i.e. A independent of B implies that B is independent of A), probabilistic causality entails a time order and the events in question do not commute. That is, if event A is the cause of event B, then event B cannot be the cause of event A.

Event A, according to [5], is the *prima facie* cause of event B, if:

(i) A occurs before B in time;
(ii) $P(A) > 0$;
(iii) $P(B|A) > P(B)$ and $P(B|A^c) < P(B)$.

Thus a cause is a probability raising event, whereas interdependence is a probability changing event. A is called a prima facie cause, when A could only be an apparent cause, not a true cause. For articulating true causes, we need the notion of a *genuine cause*. A genuine cause is a prima facie cause that is not a *spurious cause*, where A is a spurious cause of B, if and only if there exists a cause S, where:

(i) S occurs before A;
(ii) $P(A \text{ and } S) > 0$;
(iii) $P(B|A, S) = P(B|S)$;
(iv) $P(B|A, S) \geq P(B|A)$.

Thus a spurious cause is a prima facie cause that can be explained away by conditioning on an earlier event (or a common cause of A and B) that accounts as well for the conditional probability of the effect. Probabilistic causality offers a convenient vehicle to articulate the notion of cascading events [4].

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