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# Calculative and experimental analysis of natural and critical frequencies and mode shapes of high-speed rotor for micro gas turbine plant

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## Abstract

The Micro gas turbine plant (MGTP) is used for the decentralized supply to the external electric power consumers. Its nominal and heat capacity are 100 kW and 200 kW respectively. Its critical part is a rotor with the operating speed of 65,000 rpm. It consists of two subsystems: a turbocharger rotor (TCR) and a rotor of the starter-generator (SGR) connected by elastic coupling. One of the design requirements for the rotor is the absence of critical speed in the  $\pm 30\%$  operating speed range.

In this article the natural and the critical frequencies of the rotor are analyzed.

Its natural frequencies evaluated for the whole rotor system and for each of the two subsystems individually. For the TCR such an assessment was obtained through the finite element method (FEM) calculation. Due to the complexity of the SGR design, its natural frequencies were estimated and proved experimentally using LMS modal analysis technology. Also a study of the influence of bearings stiffness on the natural frequencies of the rotor was conducted to identify its acceptable range.

The study of the critical speeds of the MGTP rotor was performed in two stages by calculation: an analytical and numerical solution of the test problem obtained firstly in order to confirm the accuracy of FEM calculation in the Ansys Workbench package followed by the critical speeds evaluated through the FEM calculation on solid 3D model.

Based on the obtained results, some recommendations on the design of the rotor elements are given to ensure the natural frequencies are in the restrained region.

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**Keywords:** Micro gas turbine plant, natural frequencies, natural shapes, critical frequencies, bearings stiffness, finite element method, Campbell diagram.

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## 1. Introduction

### Nomenclature

MGTP	Micro gas turbine plant
TCR	Turbocharger rotor
SGR	Rotor of the starter-generator
SGR	Rotor of the starter-generator
FEM	Finite element method

Low-powered micro gas turbine plants are utilized in industrial enterprises, medical centres; on main gas lines, oil pipe lines, gas distribution stations; in power-hungry regions of Extreme North, Siberia, Far East; to replenish electrical shortage caused with natural disasters and other emergency situations.

The most important part of the MGTP is rotor; its operational frequency is 65,000 rpm. It consists of turbocharger rotor (TCR) and starter-generator rotor (SGR) connected by elastic coupling (Fig. 1).

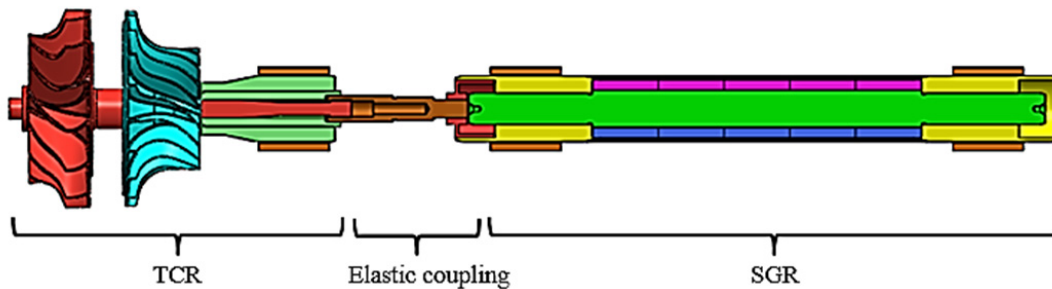


Fig. 1. The MGTP rotor

One of the requirements to high-speed rotor is exclusion of falling its critical speeds in the range of  $\pm 30\%$  of operating speed (45,500–84,500 rpm) [5]. Thus, there rises an objective to develop a rotor with critical speeds, which do not fall in the prohibited range. Works [1, 2, 6, 7, 17-20] are devoted to the same matter.


## 2. Methods

### 2.1. Evaluation of natural frequencies and shapes of MGTP rotor

The evaluation is performed through the finite element method (FEM) in the package Ansys Workbench. Natural frequencies were calculated at free-free boundary conditions. Natural frequencies of bending vibrations of the rotor in the range from 0 to 120,000 rpm (2,000 Hz) are represented in table 1, frequencies and shapes of torsional and longitudinal vibrations are not considered.

As it is seen from the table 1, frequencies corresponding to the third and fourth bending shape lie in the prohibited range 45,500–84,500 rpm.

Table 1. The results of calculating natural frequencies and shapes of the MGTP rotor at its lateral vibrations

Mode number	Mode shape	Natural frequency
1 – 4		0 Hz (0 rpm)
7	First bending	78 Hz (4,692 rpm)
8		87 Hz (5,196 rpm)
11	Second bending	489 Hz (29,322 rpm)
12		499 Hz (29,910 rpm)
14	Third bending	1,165 Hz (69,900 rpm)
17		1,453 Hz (87,180 rpm)
15	Fourth bending	1,302 Hz (78,120 rpm)
16		1,378 Hz (82,680 rpm)

2.2. Evaluation of natural frequencies of MGTP rotor subsystems

To analyze natural frequencies and shapes which fell in the prohibited range it is decided to evaluate natural frequencies of TCR and SGR separately.

The evaluation of natural frequencies of the TCR rotor is performed through FEM. The first bending frequency corresponds to the shape of vibrations which is not dangerous for the TCR. The second shape of bending vibrations is the most interesting. However, the frequency, which corresponds to it, is 2,228 Hz (133,680 rpm), and it is far outside the range 45,500–84,500 rpm. Thus, TCR is characterized with rather high bending stiffness.

Due to complexity of the SGR [21] construction its natural frequencies were determined experimentally.

2.3. The experimental evaluation of natural frequencies and shapes of the SGR

In the course of the experiment the rotor was hung on flexible ropes (Fig.2). Vibration excitation was set with a hammer. The experiments took place in the module of Impact Testing of package LMS Test.Lab 13A utilizing the technology of experimental modal analysis [3]. The results of the experiment are given in table 2.

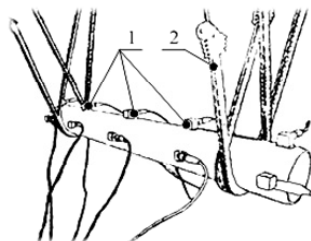


Fig. 2. The experimental unit: 1 – one-component accelerometers x10; 2 – flexible ropes x4

In accordance with the results of the experiment the equivalent model of the rotor of the starter-generator is built with natural frequencies and shapes close to the results of the experiment. The model is built by criteria of equality of masses, lengths and first natural bending frequencies of an equivalent model and its real prototype (Fig. 2) [13, 14, 15]. The results of calculation of natural frequencies on the equivalent model of the SGR are represented in table 3.

Thus, the equivalent model of the SGR is built where both first and second natural frequency and shape of bending vibrations.

Table 2. Experimental bending mode shapes and natural frequencies of SGR

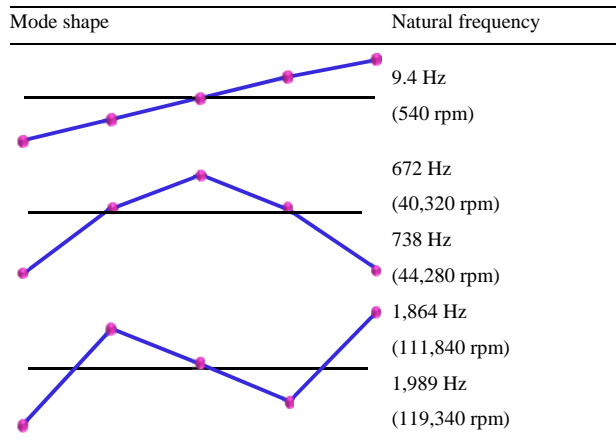


Table 3. Experimental and calculated natural frequencies of SGR

Experiment (Fig. 4, Table 2)	Equivalent rotor (FEM solution)
738 Hz	737 Hz
1,989 Hz	2,032 Hz

### 3. Results and discussions

#### 3.1. The evaluation of influence of bearings stiffness on critical speeds of the rotor of MGTP

A simplified beam FEM model of the rotor (Fig.3) is arranged in the package Ansys Mechanical APDL, and calculation of critical speeds of a rotor [4] is performed in a wide range of stiffness of the assembly [16].

As calculation shows (Fig. 4), at bearings stiffness less than  $10^6$  N/m, flexible assemblies have almost no influence on critical speeds and shapes of the rotor, so it can be considered on free-free conditions. Thus, to meet the requirements by critical speeds it is necessary to use bearing assemblies where stiffness does not exceed  $10^6$  N/m.

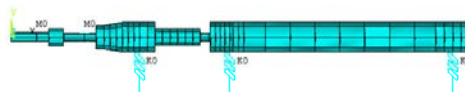


Fig. 3. Beam model of rotor for MGTP

3.2. The calculation of natural frequencies of the MGTP rotor with equivalent SGR

The calculation of natural frequencies of the MGTP rotor is performed in the condition of free hanging (which corresponds to bearings assemblies with stiffness less than 106 N/m) on considering the equivalent model of the SGR. The calculation showed that natural frequencies of the model do not correspond to the design requirements. The conclusion was made that it caused by torsional stiffness.

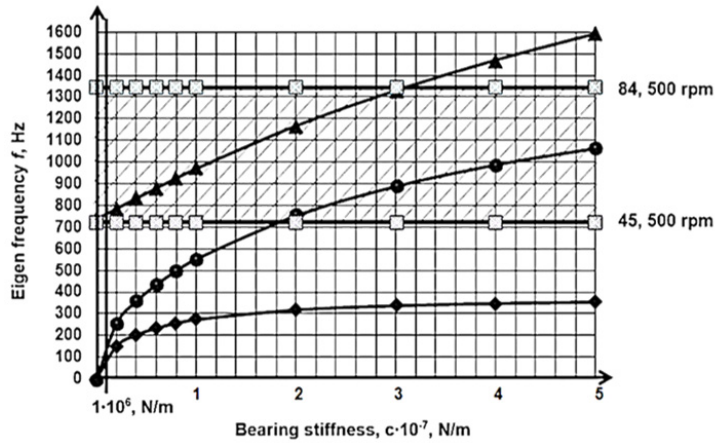


Fig. 4. Influence of bearings stiffness on critical speeds of the rotor of MGTP

The several models were suggested with various elastic connections. The best of them is a MGTP model construction where TCR and SGR are connected with a torsion bar with two membranes (Fig. 5). Natural frequencies and shapes of MGTP rotor with selected version of flexible connection and an equivalent model of SGR are represented in table 4.

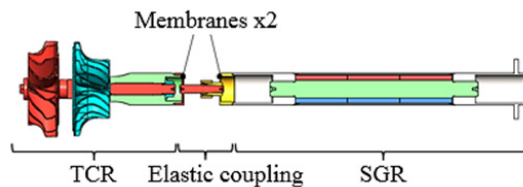


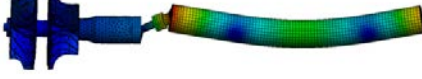

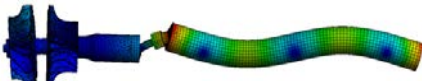


Fig. 5. Model of MGTP rotor with proposed design of elastic coupling

Table 4. Calculated natural frequencies and mode shapes of MGTP rotor

Mode number	Mode shape	Natural frequency
First Bending		51 Hz (3,079 rpm)
		52 Hz (3,129 rpm)
Second Bending		308 Hz (18,453 rpm)
		309 Hz (18,532 rpm)
Third Bending		730 Hz (43,783 rpm)
		731 Hz (43,818 rpm)
Fourth Bending		1,497 Hz (89,808 rpm)
		1,498 Hz (89,862 rpm)
Fifth Bending		1,767 Hz (105,996 rpm)
		1,767 Hz (106,014 rpm)

3.3. Evaluation of critical speeds of 3D model of the MGTP rotor

The problem was resolved through FEM calculation in Ansys Workbench package. Resulting from this solution concerning determination of critical speeds of MGTP rotor Campbell diagram was made [8, 9, 10, 11, 12] (fig. 6). The values of critical speeds of MGTP rotor are given in table 5. Comparison of the results represented in tables 4 and 5 shows that the first, second and third critical speeds of the rotor which are found taking into consideration gyroscopic moment (table 5), are by 7%, 5% and 4% higher than the corresponding natural frequencies (table 4). The calculated critical speeds of rotor (table 5) fall out of the range  $\pm 30\%$  of operational speed (45,500–84,500 rpm) which meets the requirements to the construction.

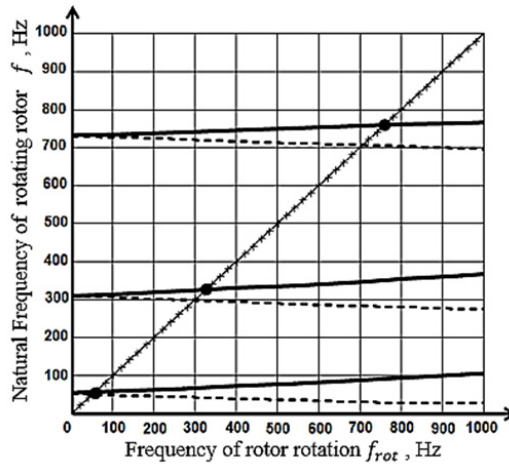


Fig 6. Campbell diagram

Table 5. Critical speeds of MGTP rotor

Number of bending mode	Direct precession	Inverse precession
1	54.4 Hz (3,263 rpm)	49.5 Hz (2,970 rpm)
2	324.2 Hz (19,450rpm)	295.8 Hz (17,746 rpm)
3	757.5 Hz (45,451 pm)	705.4 Hz (42,325 rpm)

#### 4. Conclusion

Thus, based on calculation and experimental approach the recommendations for the construction of MGTP rotor are developed: bearings stiffness must not exceed  $10^6$  N/m; the rotor of turbocharger and the rotor of starter-generator must be connected with flexible element with low bending stiffness; the design of elastic connection should grant critical speeds of the rotor fall out of the range  $\pm 30\%$  of operational speed (45,500–84,500 rpm).

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