Computers and Mathematics with Applications 63 (2012) 695–707

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Wire coating analysis with Oldroyd 8-constant fluid by Optimal Homotopy Asymptotic Method

Rehan Ali Shah [a,*], Saeed Islam [a], A.M. Siddiqui [b], T. Haroon [a]

[a] *COMSATS Institute of Information Technology, Park Road, Islamabad, Pakistan*
[b] *Pennsylvania State University, York Campus, Edgecombe 17403, USA*

ARTICLE INFO

Article history:
Received 7 December 2010
Received in revised form 16 November 2011
Accepted 16 November 2011

Keywords:
Oldroyd 8-constant fluid
Wire coating
Pressure type die
Optimal homotopy asymptotic method

ABSTRACT

In this study the wire coating in a pressure type die with the bath of Oldroyd 8-constant fluid with pressure gradient is investigated. The non-linear ordinary differential equation in dimensionless form is obtained, which is solved for the velocity profile using the Optimal Homotopy Asymptotic Method (OHAM). The effect of Dilatant constant α, the Psendoplastic constant β, and the pressure gradient on velocity distribution and shear stress is studied. Shear stress is examined under the effect of the viscosity parameter η_0. Moreover, the volume flow rate and average velocity is carefully studied with changing the domain (thickness) of the polymer and varying the parameter α, β and the pressure gradient.

1. Introduction

Wire coating is often used for the purpose of high and low voltage and protection against corrosion. The wire coating is performed by dragging the wire in a molten polymer inside the coating unit. Due to the shear stress between the wire and the molten polymer the wire is coated. The thickness of the coated wire is the same as the thickness of the die at the exit. A typical wire coating unit consists of a pay off device, preheater, extruder device with a cross head die, cooling device, and a take-up reel as shown in Fig. 1. The pay off device is a reel stand carrying a reel of uncoated wire. The preheater is used to give a temperature to the wire, while the extruder device fitted with a cross-head contains a canonical die. The cooling device is used for cooling the wire. The take-up reel is used for winding the coated wire on a rotating reel.

Wire coating is an important industrial process in which different types of polymer are used. The coating depends on the geometry of the die, the viscosity of the fluid, the temperature of the wire and the polymer used for coating the wire.

Akhter and Hashmi [1,2] have studied wire coating using power law fluid and have investigated the effect of the change in viscosity. Siddiqui et al. [3] studied wire coating extrusion in a pressure-type die in the flow of a third grade fluid. Fenner and Williams [4] carried out an analysis of the flow in the tapering section of a pressure type die. Sajjid et al. [5] studied the wire coating with Oldroyd 8-constant fluid without pressure gradient using the Homotopy Analyses Method (HAM), and give the solution for the velocity field in the form of a series.

We investigate the Oldroyd 8-constant fluid flow under pressure and examine carefully the velocity distribution, shear stress, volume flow rate, average velocity and the effect of velocity distribution while, changing the thickness of fluid under the same geometry with the Optimal Homotopy Asymptotic Method (OHAM) and obtained satisfactory results. The effect of Dilatant constant α, the Psendoplastic constant β, and the pressure gradient on velocity distribution and shear stress is studied. Shear stress is also examined by changing the viscosity parameter η_0. Here, we use a new homotopy approach,

* Corresponding author.
E-mail address: mmrehan79@yahoo.com (R.A. Shah).

0898-1221/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2011.11.033

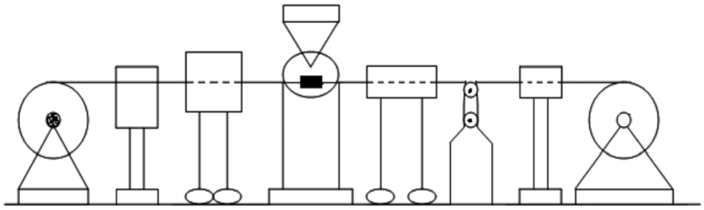

Fig. 1. Typical wire coating process.

namely OHAM to solve the nonlinear differential equation. Marinca and Herişanu [6–8] proposed this homotopy technique called the Optimal Homotopy Asymptotic Method (OHAM) and it proved to be a reliable approach to strongly nonlinear problems. In a series of papers by Marinca and Herişanu [9–11] and Islam et al. [12,13] have shown that this method is a more powerful tool than other perturbation tools for nonlinear problems.

2. Basic equation

Basic equations which govern the flow of an incompressible fluid neglecting the thermal effects are:

$$\nabla \cdot \underline{u} = 0, \tag{1}$$

$$\rho \frac{D\underline{u}}{Dt} = \operatorname{div}\underline{T} + \rho \underline{f}, \tag{2}$$

where $\underline{u}$ is the velocity vector of the fluid, $\underline{T}$ is the Cauchy stress tensor, ρ is the constant density, $\underline{f}$ is the body force per unit mass and $\frac{D}{Dt}$ is the material derivative.

The Rheological equation of state for an Oldroyd 8-constant model is given by

$$\underline{T} = -P\underline{I} + \underline{S}, \tag{3}$$

where P denotes the pressure, $\underline{I}$ is the identity unit tensor and the extra stress tensor $\underline{S}$ is defined as

$$\begin{aligned} &\underline{S} + \lambda_1 \overset{\nabla}{\underline{S}} + \frac{1}{2}(\lambda_1 - \mu_1)\left(\underline{A}_1\, \underline{S} + \underline{S}\, \underline{A}_1\right) + \frac{1}{2}\mu_0 \left(\operatorname{tr} \underline{S}\right) \underline{A}_1 + \frac{1}{2}\upsilon_1 \left(\operatorname{tr} \underline{S}\, \underline{A}_1\right) \underline{I} \\ &\quad = \eta_0 \left(\underline{A}_1 + \lambda_2 \overset{\nabla}{\underline{A}}_1 + (\lambda_2 - \mu_2)\, \underline{A}_1^2 + \frac{1}{2}\upsilon_2 \left(\operatorname{tr} \underline{A}_1^2\right) \underline{I}\right). \end{aligned} \tag{4}$$

Here, the constants $\eta_0, \lambda_1, \lambda_2$ are respectively, zero shear viscosity, relaxation time and retardation time. The other five constants $\mu_0, \mu_1, \mu_2, \upsilon_1, \upsilon_2$ are associated with non-linear terms.

The upper contra-variant convected derivative designed by ∇ over $\underline{S}$ and $\underline{A}_1$ is defined as follows

$$\overset{\nabla}{\underline{S}} = \frac{D\underline{S}}{Dt} - \left[\left(\nabla \underline{u}\right)^T \underline{S} + \underline{S}\left(\nabla \underline{u}\right)\right] \tag{5}$$

$$\overset{\nabla}{\underline{A}}_1 = \frac{D\underline{A}_1}{Dt} - \left[\left(\nabla \underline{u}\right)^T \underline{A}_1 + \underline{A}_1\left(\nabla \underline{u}\right)\right] \tag{6}$$

$$\text{where } \underline{A}_1 = \left(\nabla \underline{u}\right) + \left(\nabla \underline{u}\right)^T \text{ and } \frac{D\underline{S}}{Dt} = \left[\frac{\partial}{\partial t} + \left(\underline{u} \cdot \nabla\right)\right]\underline{S}. \tag{7}$$

It should be noted that the model (4) includes as special cases the following

(i) If $\eta_0 = \lambda_1 = \lambda_2 = \mu_0 = \mu_1 = \mu_2 = \upsilon_1 = \upsilon_2 = 0$, we recover the Newtonian model.
(ii) If $\eta_0 = \lambda_1 = \mu_0 = \mu_1 = \mu_2 = \upsilon_1 = \upsilon_2 = 0$ and $\lambda_2 = \lambda_2$, the second grade fluid model is obtained.
(iii) If $\eta_0 = \lambda_2 = \mu_0 = \mu_2 = \upsilon_1 = \upsilon_2 = 0$ and $\lambda_1 = \lambda_1$, $\mu_1 = \lambda_1$ then the upper convected Maxwell model is recovered.
(iv) If $\lambda_1 = \lambda_1$ and $\eta_0 = \lambda_2 = \mu_0 = \mu_1 = \mu_2 = \upsilon_1 = \upsilon_2 = 0$, we reach the co-rotational Maxwell model.
(v) If $\lambda_1 = \lambda_1, \lambda_2 = \lambda_2, \mu_1 = \lambda_1, \mu_2 = \lambda_2, \eta_0 = \eta_0$ and $\upsilon_1 = \upsilon_2 = 0$ then the Oldroyd 4-constant model is recovered.
(vi) If $\lambda_1 = \lambda_1, \lambda_2 = \lambda_2, \mu_1 = \lambda_1, \mu_2 = \lambda_2$, and $\eta_0 = \upsilon_1 = \upsilon_2 = 0$, we arrive at the upper convected Jeffery (Oldroyd *B*-model).
(vii) If $\lambda_1 = \lambda_1, \lambda_2 = \lambda_2$ and $\eta_0 = \mu_0 = \mu_1 = \mu_2 = \upsilon_1 = \upsilon_2 = 0$, we gain the co-rotational Jeffery model.

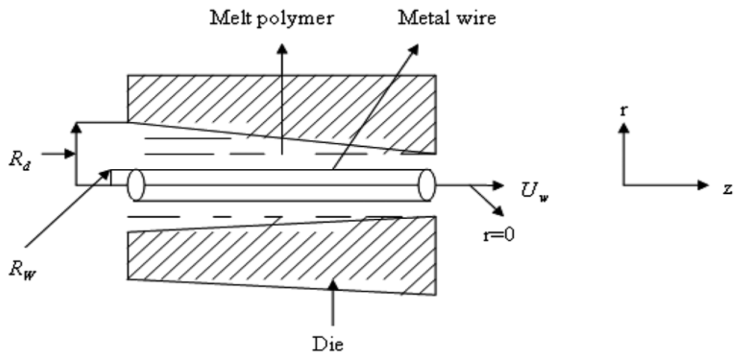

Fig. 2. Schematic profile of wire coating in a pressure type die.

3. Problem formulation

Fig. 2 shows the internal geometry of the die considered here, together with the nomenclature. Here, the wire of radius R_w is dragged with velocity U_w in a pool of an incompressible Oldroyd 8-constant fluid inside an annular die of radius R_d as shown in Fig. 2. The wire and die are concentric. The coordinate system is chosen at the center of the wire, in which r is taken perpendicular to the direction of fluid flow, whereas z is taken in the direction of fluid flow.

Boundary conditions are:

$$\text{At } r = R_w, \quad w = U_w, \\ \text{and at } r = R_d, \quad w = 0. \tag{8}$$

Since the flow is axisymmetric and unidirectional, so the velocity field is defined as

$$\underline{u} = [0, 0, w(r)], \qquad \underline{S} = \underline{S}(r). \tag{9}$$

It is further assumed that the flow is steady, laminar and isothermal. The gravitational force is neglected.

On substituting these expressions (9) in Eqs. (4)–(7), we obtain non-zero components of extra stress $\underline{S}$ as:

$$S_{rr} + (\upsilon_1 - \lambda_1 - \mu_1)\frac{dw}{dr}S_{rz} = \eta_0(\upsilon_2 - \lambda_1 - \mu_1)\left(\frac{dw}{dr}\right)^2 \tag{10}$$

$$S_{rz} - \lambda_1 S_{rr}\frac{dw}{dr} + \frac{1}{2}(\lambda_1 - \mu_1 + \mu_0)(S_{rr} + S_{zz})\frac{dw}{dr} + \frac{\mu_0}{2}S_{zz}\left(\frac{dw}{dr}\right) = \eta_0\left(\frac{dw}{dr}\right) \tag{11}$$

$$S_{zz} + (\lambda_1 - \mu_1 + \upsilon_1)\frac{dw}{dr}S_{rz} = \eta_0(\lambda_2 - \mu_2 + \upsilon_2)\left(\frac{dw}{dr}\right)^2 \tag{12}$$

$$S_{\theta\theta} + \upsilon_1\frac{dw}{dr}S_{rz} = \eta_0\upsilon_2\left(\frac{dw}{dr}\right)^2. \tag{13}$$

On solving (10)–(13), we obtain the explicit expressions for the stress component as:

$$S_{rr} = -(\upsilon_1 - \lambda_1 - \mu_1)\frac{dw}{dr}S_{rz} + \eta_0(\upsilon_2 - \lambda_1 - \mu_1)\left(\frac{dw}{dr}\right)^2 \tag{14}$$

$$S_{\theta\theta} = -\upsilon_1\frac{dw}{dr}S_{rz} + \eta_0\upsilon_2\left(\frac{dw}{dr}\right)^2 \tag{15}$$

$$S_{zz} = -(\lambda_1 - \mu_1 + \upsilon_1)\frac{dw}{dr}S_{rz} + \eta_0(\lambda_2 - \mu_2 + \upsilon_2)\left(\frac{dw}{dr}\right)^2 \tag{16}$$

$$S_{rz} = \eta_0\frac{\left[1 + \alpha\left(\frac{dw}{dr}\right)^2\right]\frac{dw}{dr}}{1 + \beta\left(\frac{dw}{dr}\right)^2} \tag{17}$$

where $\alpha = \lambda_1\lambda_2 + \mu_0\left(\mu_2 - \frac{3}{2}\upsilon_2\right) - \mu_1(\mu_2 - \upsilon_2)$

$$\beta = \lambda_1^2 + \mu_0\left(\mu_1 - \frac{3}{2}\upsilon_1\right) - \mu_1(\mu_1 - \upsilon_1).$$

The constant α is known as the dilatant constant, while the constant β is called the Psendoplastic constant.

Comments: (i) If the ratio $\frac{\alpha}{\beta} = 1$, the shear stress in Eq. (17) reduces to that of a Newtonian fluid.

(ii) If the ratio $\frac{\alpha}{\beta} > 1$, the shear stress in Eq. (17), with moderate values of $\frac{dw}{dr}$ represents dilatant fluids.

(iii) If the ratio $\frac{\alpha}{\beta} < 1$, the shear stress in Eq. (17), with moderate values of $\frac{dw}{dr}$ represents Psendoplastic fluids.

As indicated in Eq. (9), the velocity field $\underline{u}$ and the stress $\underline{S}$ are functions of only r, so the continuity equation (1) is satisfied identically and the dynamic equation (2) reduces to

$$\frac{\partial p}{\partial r} = \frac{1}{r}\frac{d}{dr}\left(rS_{rr}\right) \tag{18}$$

$$\frac{\partial p}{\partial \theta} = 0 \tag{19}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{d}{dr}\left(rS_{rz}\right). \tag{20}$$

From Eq. (19), we have $p = p(r, z)$.

$$r\frac{d^2w}{dr^2} + \frac{dw}{dr} - r\frac{\partial p}{\partial z} + (\alpha+\beta)\left(\frac{dw}{dr}\right)^3 - \beta r\left(\frac{dw}{dr}\right)^2\frac{d^2w}{dr^2} + \alpha\beta r\left(\frac{dw}{dr}\right)^4\frac{d^2w}{dr^2}$$
$$+3\alpha r\left(\frac{dw}{dr}\right)^2\frac{d^2w}{dr^2} + \alpha\beta\left(\frac{dw}{dr}\right)^5 - 2\beta r\frac{\partial p}{\partial z}\left(\frac{dw}{dr}\right)^2 - \beta^2 r\frac{\partial p}{\partial z}\left(\frac{dw}{dr}\right)^4 = 0, \tag{21}$$

The volume flow rate of the coating is

$$Q = \pi U_w\left(R_c^2 - R_w^2\right) \tag{22}$$

where R_c is the radius of the coated wire. On the other hand at the cross-section, within the die, the volume flow rate is

$$Q = \int_{R_W}^{R_D} 2\pi r w(r) dr. \tag{23}$$

The thickness of the coated wire can be obtained from Eqs. (22) and (23).

The force on the total wire surface in the die is

$$\underline{F} = 2\pi R_w L S_{rz}|_{r=R_W}. \tag{24}$$

Let us introduce the following non-dimensional variables and parameters

$$r^* = \frac{r}{R_w}, \qquad w^* = \frac{w}{U_w}, \qquad \alpha^* = \frac{\alpha U_w^2}{R_w^2}, \qquad \beta^* = \frac{\beta U_w^2}{R_w^2}, \qquad p^* = \frac{p}{\mu\left(U_w/R_w\right)}. \tag{25}$$

Hence, Eqs. (8) and (21) after dropping the "$*$" and under the assumption that the pressure gradient in the axial direction is constant i.e. $\frac{\partial p}{\partial z} = \Omega$ takes the following form:

$$r\frac{d^2w}{dr^2} + \frac{dw}{dr} - r\Omega + (\alpha+\beta)\left(\frac{dw}{dr}\right)^3 - \beta r\left(\frac{dw}{dr}\right)^2\frac{d^2w}{dr^2} + \alpha\beta r\left(\frac{dw}{dr}\right)^4\frac{d^2w}{dr^2}$$
$$+3\alpha r\left(\frac{dw}{dr}\right)^2\frac{d^2w}{dr^2} + \alpha\beta\left(\frac{dw}{dr}\right)^5 - r\beta^2\Omega\left(\frac{dw}{dr}\right)^4 - 2r\beta\Omega\left(\frac{dw}{dr}\right)^2 = 0, \tag{26}$$

with the boundary conditions

$$w(1) = 1, \qquad w(\delta) = 0 \quad \text{where } \delta = \frac{R_d}{R_w} > 1. \tag{27}$$

Finally, we solve Eq. (26) with the corresponding boundary conditions (27) by using OHAM.

4. Solution by optimal homotopy asymptotic method

4.1. Basic idea

According to OHAM Eq. (26) can be represented by

$$L(u(y)) + g(y) + N(u(y)) = 0, \qquad \mathrm{B}\left(u, \frac{du}{dy}\right) = 0 \tag{28}$$

where L is a linear operator, $u(y)$ is an unknown function, $g(y)$ is a known function, N is a nonlinear operator and B is a boundary operator.

According to OHAM we construct a Homotopy, $\phi(y,p): \mathbb{R}\times[0,1]\to\mathbb{R}$ which satisfies

$$(1-p)[L(\phi(y,p))+g(y)] = H(p)[L(\phi(y,p))+g(y)+N(\phi(y,p))],$$
$$\mathrm{B}\left(\phi(y,p), \frac{\partial\phi(y,p)}{\partial y}\right)=0 \tag{29}$$

where $y\in\mathbb{R}$ and $p\in[0,1]$ is an embedding parameter, $H(p)$ is a nonzero auxiliary function for $p\neq 0$, $H(0)=0$ and $\phi(r,p)$ is an unknown function. The auxiliary function $H(p)$ depends either upon some constants [6–10] or upon some functions depending on a physical parameter [11]. It was shown in the paper [11] that a more complex function $H(p)$ leads to more accurate results.

Obviously, when $p=0$ and $p=1$, $\phi(y,0)=u_\circ(y)$ and $\phi(y,1)=u(y)$ respectively. Thus, as p varies from 0 to 1, the solution $\phi(y,p)$ approaches from $u_\circ(y)$ to $u(y)$, where $u_\circ(y)$ is obtained from Eq. (29) for $p=0$:

$$L(u_\circ(y))+g(y)=0, \qquad B\left(u_\circ, \frac{du_\circ}{dy}=0\right). \tag{30}$$

We choose the auxiliary function $H(p)$ in the form:

$$H(p)=pC_1+p^2C_2+\cdots \tag{31}$$

where $C_1, C_2, \ldots$ are constants to be determined later.

To get an approximate solution, we expand $\phi(y,p,C_i)$ in Taylor's series about p in the following manner:

$$\phi(y,p,C_i)=u_\circ(y)+\sum_{k=1}^{\infty}u_k(y,C_1,C_2,\ldots,C_k)\,p^k. \tag{32}$$

Substituting Eqs. (31) and (32) into Eq. (29) and equating the coefficient of like powers of p, we obtain the following linear equations.

The zeroth order problem is given by Eq. (30) and the first and second order problems are given by Eqs. (33) and (34) respectively:

$$L(u_1(y))+g(y)=C_1N_\circ(u_\circ(y)), \qquad \mathrm{B}\left(u_1, \frac{du_1}{dy}\right)=0 \tag{33}$$

$$L(u_2(y))-L(u_1(y))=C_2N_\circ(u_\circ(y))+C_1\left[L(u_1(y))+N_1(u_1(y),u_1(y))\right], \qquad \mathrm{B}\left(u_2, \frac{du_2}{dy}\right)=0. \tag{34}$$

The general governing equations for $u_k(y)$ are given by:

$$\begin{aligned} L(u_k(y))-L(u_{k-1}(y)) &= C_kN_\circ(u_\circ(y)) \\ &\quad+\sum_{i=1}^{k-1}C_i\left[L(u_{k-i}(y))+N_{k-i}(u_\circ(y),u_1(y),\ldots,u_{k-1}(y))\right], \quad k=2,3,\ldots, \\ \mathrm{B}\left(u_k, \frac{du_k}{dy}\right)&=0 \end{aligned} \tag{35}$$

where $N_m(u_\circ(y),u_1(y),\ldots,u_{k-1}(y))$ is the coefficient of p^m in the expansion of $N(\phi(y,p))$ about the embedding parameter p [6–10].

$$N(\phi(y,p,C_i))=N_\circ(u_\circ(y))+\sum_{m=1}^{\infty}N_m(u_\circ,u_1,u_2,\ldots,u_m)\,p^m. \tag{36}$$

It has been practical that the convergence of the series (32) depends upon the auxiliary constants $C_1, C_2, \ldots$. If it is convergent at $p=1$,

$$\tilde{u}(y,C_1,C_2,\ldots,C_m)=u_\circ(y)+\sum_{i=1}^{m}u_i(y,C_1,C_2,\ldots,C_i). \tag{37}$$

Substitution of Eq. (37) into Eq. (28), results in the following expression for the residual:

$$R(y,C_1,C_2,\ldots,C_m)=L(\tilde{u}(y,C_1,C_2,\ldots,C_m))+g(y)+N(\tilde{u}(y,C_1,C_2,\ldots,C_m)). \tag{38}$$

If $R=0$, then $\tilde{u}$ will be the exact solution. Generally it does not happen, especially in non-linear problems.

There are many methods like the Method of Least Squares, Galerkin's Method, the Ritz Method, and the Collocation Method to find the optimal values of $C_i,\ i = 1, 2, 3, \ldots$. We apply the Method of Least Squares as:

$$J\left(C_1, C_2, \ldots, C_m\right) = \int_a^b R^2\left(y, C_1, C_2, \ldots, C_m\right) dy \tag{39}$$

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = \cdots = \frac{\partial J}{\partial C_m} = 0 \tag{40}$$

where a and b are properly chosen numbers to locate the desired C_i $(i = 1, 2, \ldots, m)$. With these constants known, the approximate solution (of order m) is well-determined.

4.2. Solution of the problem

We construct a homotopy for Eq. (26) with the corresponding boundary conditions given in Eq. (27) according to Eq. (29). Using the given values in the homotopy we obtain zeroth, and first order problem with the boundary conditions given below:

$$p^0: r\frac{d^2 w_0}{dr^2} + \frac{dw_0}{dr} - \Omega r = 0 \tag{41}$$

subject to the boundary conditions

$$w_0(1) = 1, \qquad w_0\left(\delta\right) = 0 \tag{42}$$

$$\begin{aligned} p^1: \quad & r\frac{d^2 w_1}{dr^2} + \frac{dw_1}{dr} - r\frac{d^2 w_0}{dr^2} - \frac{dw_0}{dr} - C_1\left(r\frac{d^2 w_0}{dr^2} + \frac{dw_0}{dr}\right) - \left(\alpha + \beta\right) C_1 \left(\frac{dw_0}{dr}\right)^3 \\ & + \beta C_1 r \left(\frac{dw_0}{dr}\right)^2 \frac{d^2 w_0}{dr^2} - \alpha\beta r C_1 \left(\frac{dw_0}{dr}\right)^4 \frac{d^2 w_0}{dr^2} - 3\alpha r C_1 \left(\frac{dw_0}{dr}\right)^2 \frac{d^2 w_0}{dr^2} \\ & - \alpha\beta C_1 \left(\frac{dw_0}{dr}\right)^5 + \beta^2 r C_1 \Omega \left(\frac{dw_0}{dr}\right)^4 + 2\beta r C_1 \Omega \left(\frac{dw_0}{dr}\right)^2 + \Omega r\left(1 + C_1\right) = 0 \end{aligned} \tag{43}$$

subject to boundary conditions

$$w_1(1) = 0, \qquad w_1\left(\delta\right) = 0 \tag{44}$$

$$\begin{aligned} p^2: \quad & r\frac{d^2 w_2}{dr^2} + \frac{dw_2}{dr} - r\frac{d^2 w_1}{dr^2} - \frac{dw_1}{dr} - C_1\left(r\frac{d^2 w_1}{dr^2} + \frac{dw_1}{dr}\right) - C_2\left(r\frac{d^2 w_0}{dr^2} + \frac{dw_0}{dr}\right) + \Omega r C_2 \\ & + 2\Omega\beta r C_2 \left(\frac{dw_0}{dr}\right)^2 - \left(\alpha + \beta\right) C_2 \left(\frac{dw_0}{dr}\right)^3 + \Omega\beta^2 C_2 \left(\frac{dw_0}{dr}\right)^4 - \alpha\beta C_2 \left(\frac{dw_0}{dr}\right)^5 \\ & + 4\Omega\beta r C_1 \frac{dw_0}{dr}\frac{dw_1}{dr} - 3\left(\alpha + \beta\right)\left(\frac{dw_0}{dr}\right)^2 \frac{dw_1}{dr} + 4\Omega\beta^2 r C_1 \left(\frac{dw_0}{dr}\right)^3 \frac{dw_1}{dr} \\ & - 5\alpha\beta C_1 \left(\frac{dw_0}{dr}\right)^4 \frac{dw_1}{dr} - 3\alpha r C_2 \left(\frac{dw_0}{dr}\right)^2 \frac{d^2 w_0}{dr^2} + \beta r C_2 \left(\frac{dw_0}{dr}\right)^2 \frac{d^2 w_0}{dr^2} \\ & - \alpha\beta r C_2 \left(\frac{dw_0}{dr}\right)^4 \frac{d^2 w_0}{dr^2} - 6\alpha r C_1 \frac{dw_0}{dr}\frac{dw_1}{dr}\frac{d^2 w_0}{dr^2} + 2\beta r C_1 \left(\frac{dw_0}{dr}\right)^3 \frac{dw_1}{dr}\frac{d^2 w_0}{dr^2} \\ & - 4\alpha\beta C_1 \left(\frac{dw_0}{dr}\right)^3 \frac{dw_1}{dr}\frac{d^2 w_0}{dr^2} - 3\alpha r C_1 \left(\frac{dw_0}{dr}\right)^2 \frac{d^2 w_1}{dr^2} + \beta r C_1 \left(\frac{dw_0}{dr}\right)^2 \frac{d^2 w_1}{dr^2} \\ & - \alpha\beta r C_1 \left(\frac{dw_0}{dr}\right)^4 \frac{d^2 w_1}{dr^2} = 0 \end{aligned} \tag{45}$$

subject to boundary conditions

$$w_2(1) = 0, \qquad w_2\left(\delta\right) = 0. \tag{46}$$

Solving Eqs. (41)–(46) with the corresponding boundary conditions, we obtain the zeroth, first and second order problem solution as follows:

$$w_0(r) = \Lambda_{11} + r^2 \Lambda_{12} + \Lambda_{13} \ln r \tag{47}$$

$$w_1(r) = \frac{1}{r^2}\Lambda_{14} + \Lambda_{15} + r^2 \Lambda_{16} + r^4 \Lambda_{17} + r^6 \Lambda_{18} + \Lambda_{19} \ln r \tag{48}$$

$$w_2(r) = \frac{1}{r^6}\kappa_{10} + \frac{1}{r^4}\kappa_{11} + \frac{1}{r^2}\kappa_{12} + \kappa_{13} + r^2 \kappa_{14} + r^4 \kappa_{15} + r^6 \kappa_{16} + r^8 \kappa_{17} + r^{10} \kappa_{18} + \kappa_{19} \ln r \tag{49}$$

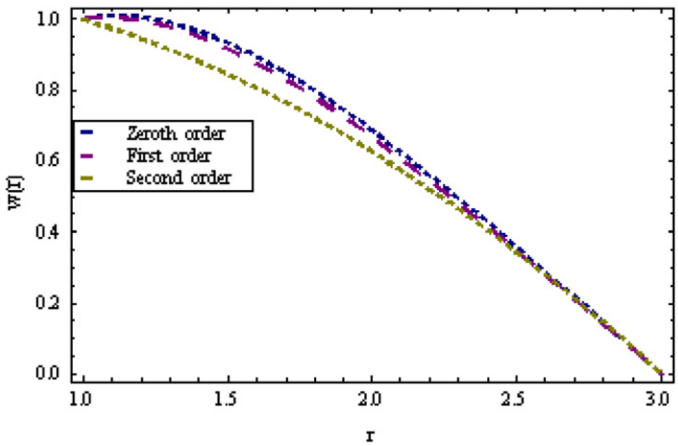

Fig. 3. Comparison of velocity profile for different order problems using OHAM by taking $\alpha = 0.2$, $\beta = 0.4$, and $\Omega = -0.5$, $C_1 = -0.002154869$, $C_2 = -0.0005341298$.

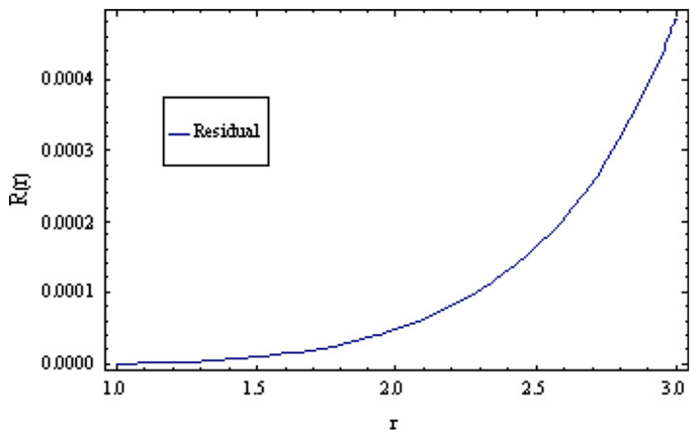

Fig. 4. Residual second order velocity profile by taking $\alpha = 0.2$, $\beta = 0.4$, $\delta = 3$, and $\Omega = -0.5$, $C_1 = -0.002154869$, $C_2 = -0.0005341298$.

where $\Lambda_{11}, \Lambda_{12}, \Lambda_{13}, \Lambda_{14}, \Lambda_{15}, \Lambda_{16}, \Lambda_{17}, \Lambda_{18}, \Lambda_{19}, \kappa_{10}, \kappa_{11}, \kappa_{12}, \kappa_{13}, \kappa_{14}, \kappa_{15}, \kappa_{16}, \kappa_{17}, \kappa_{18}$ and κ_{19} are constant containing the auxiliary constants also are given in Appendix.

The second order approximation is

$$w(r) = w_0(r) + w_1(r) + w_2(r). \tag{50}$$

Substituting Eqs. (47)–(49) in Eq. (50), we obtain that the second order approximate solution for the velocity field is given by

$$\begin{aligned} w(r) = {} & \frac{1}{r^6}\kappa_{10} + \frac{1}{r^4}\kappa_{11} + \frac{1}{r^2}(\Lambda_{14} + \kappa_{12}) + (\Lambda_{11} + \Lambda_{14} + \kappa_{13}) + r^2(\Lambda_{12} + \Lambda_{16} + \kappa_{14}) \\ & + r^4(\Lambda_{17} + \kappa_{15}) + r^6(\Lambda_{18} + \kappa_{16}) + r^8\kappa_{17} + r^{10}\kappa_{18} + \ln r(\Lambda_{13} + \Lambda_{19}) + \kappa_{18} r^6 \ln r. \end{aligned} \tag{51}$$

5. Results and discussion

In the present paper, the solution for velocity field is derived by Optimal Homotopy Asymptotic Method. The solution obtained is discussed under the effect of the Dilatant constant α, the Psendoplastic constant β, the pressure gradient and the viscosity parameter η_0. Fig. 3 shows that as we increase the order of the problem the accuracy increases and the solution converges to the exact solution by choosing the appropriate auxiliary constants and increasing the order.

One can see from Fig. 4 that the accuracy of the solution obtained by the present method is very good. The residual $R(r)$ has a maximum magnitude of 0.0005, which proves the accuracy of the approximate solution. One can observe from Fig. 5 that the velocity decreases as the dilatant parameter α increases, which is a good agreement to the physical behavior of the parameter α (shear thickening).

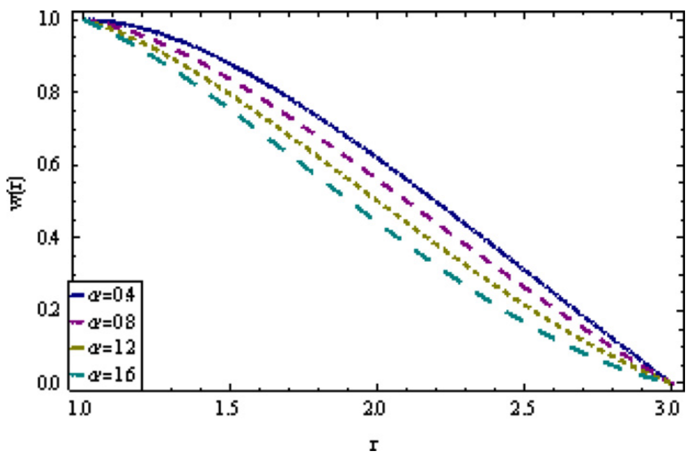

Fig. 5. Velocity profile for different values of dilatant parameter α, taking $\beta = 0.4$, and $\Omega = -0.5$.

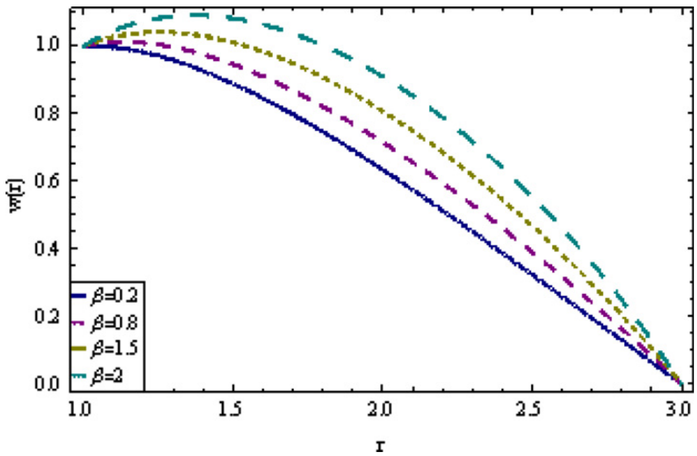

Fig. 6. Velocity profile for different values of viscoelastic parameter β, taking $\alpha = 0.5$, and $\Omega = -0.5$.

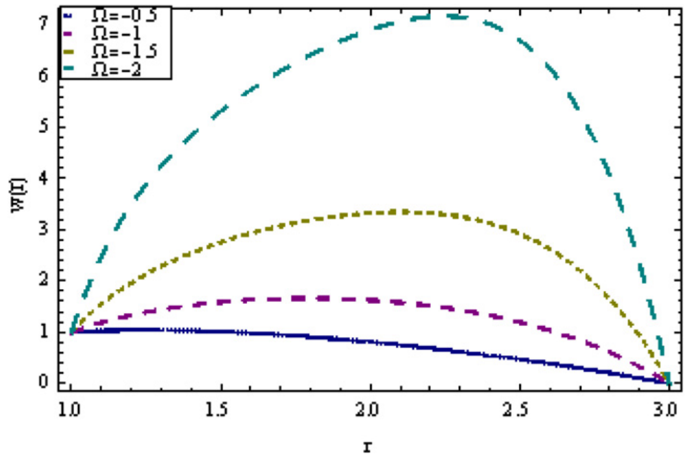

Fig. 7. Velocity profile for different values of pressure gradient, taking $\alpha = 0.4$, and $\beta = 1$.

Fig. 6 depicts that the velocity of the fluid increases as the value of the Psendoplastic constant β increases, which tallies with the physical property of the parameter β (shear thinning). Fig. 7 gives the velocity profile for different values of pressure gradient and one can observe that the velocity increases as the pressure gradient increases in magnitude.

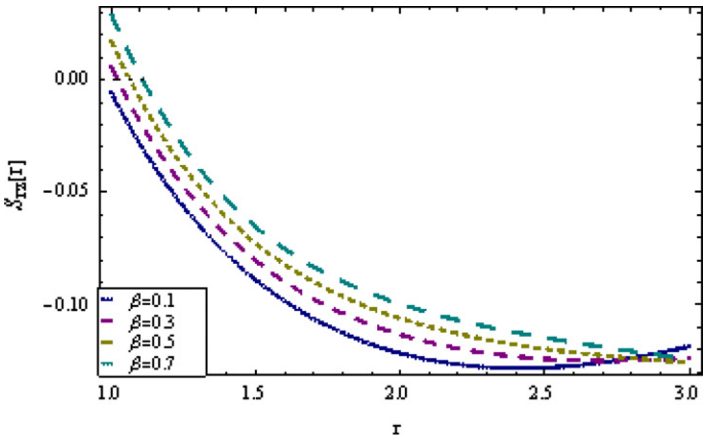

Fig. 8. Profile of shear stress for different values of parameter β, taking $\alpha = 0.2$, $\eta_0 = 0.2$ and $\Omega = -0.5$.

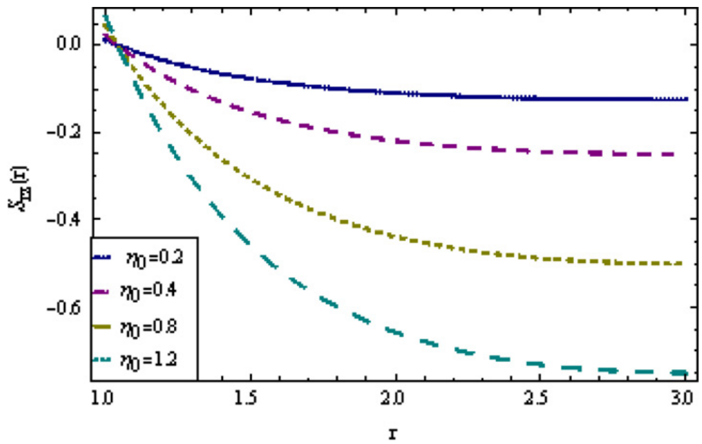

Fig. 9. Profile of shear stress for different values of parameter η_0, taking $\alpha = 0.2$, $\beta = 0.4$ and $\Omega = -0.5$.

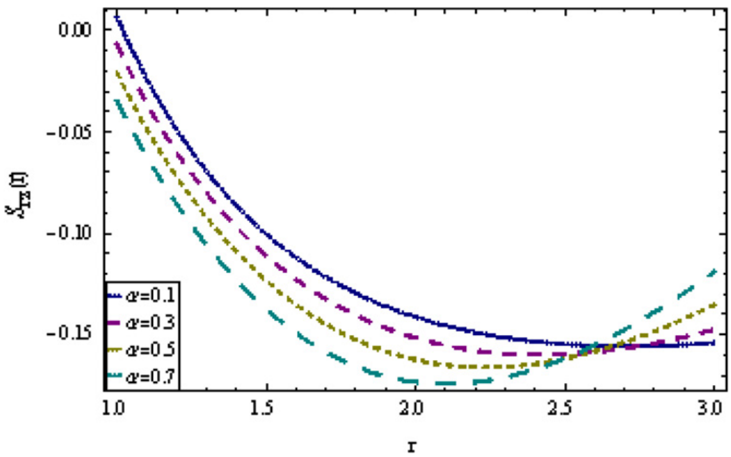

Fig. 10. Profile of shear stress for changing the parameter α, taking $\eta_0 = 0.25$, $\beta = 0.2$ and $\Omega = -0.5$.

Figs. 8–10 shows the profile of shear stress for the Psendoplastic constant β, the dilatant constant α, and the viscosity coefficient η_0, respectively.

Table 1 admits that the average velocity and the volume flow rate both increases as the parameter β increases. One can observe from Table 2 that the volume flow rate and average velocity increases as the thickness of the extrudate polymer increases. Table 3 shows that the average velocity and the volume flow rate both decreases as the parameter α increases.

Table 1
Variation of volume flow rate and average velocity with change of parameter β for $\alpha = 0.2, \delta = 3$, and pressure gradient $= -0.5$.

β	Volume flow rate	Average velocity
0	11.9378	0.47499
0.1	12.1721	0.48431
0.2	12.4141	0.49394
0.3	12.6639	0.50388
0.4	12.9213	0.51412
0.5	13.1866	0.52468
0.6	13.4595	0.53554
0.7	13.7402	0.54671
0.8	14.0286	0.55818
0.9	14.3248	0.56997
1	14.6287	0.58206

Table 2
Variation of volume flow rate and average velocity with change of the radius of wire δ for $\beta = 0.2, \alpha = 0.5$, and $\Omega = -0.5$.

δ	Volume flow rate	Average velocity
2	3.99037	0.413392
2.2	5.07004	0.420272
2.4	6.33704	0.42377
2.6	7.83165	0.432794
2.8	9.59663	0.446594
3	11.6765	0.464592
3.2	14.1165	0.486301
3.4	16.9616	0.511274
3.6	20.2553	0.539086
3.8	24.0379	0.569307
4	28.3451	0.601501

Table 3
Variation of volume flow rate and average velocity with change of parameter α for $\beta = 0.4, \delta = 3$ and $\Omega = -0.5$.

α	Volume flow rate	Average velocity
0	13.4286	0.534306
0.2	12.6212	0.514124
0.4	12.4141	0.493942
0.6	11.9069	0.473760
0.8	11.3997	0.453578
1	10.8924	0.433396
1.2	10.3852	0.413214
1.4	9.87796	0.393032
1.6	9.37073	0.372850
1.8	8.86350	0.352668
2	8.35627	0.332485

Table 4
Variation of volume flow rate and average velocity with change of the pressure gradient Ω for $\alpha = 0.5, \beta = 0.2$, and $\delta = 3$.

Ω	Volume flow rate	Average velocity
0	8.66613	0.344814
−0.1	9.27270	0.368949
−0.2	9.88806	0.393434
−0.3	10.5012	0.417829
−0.4	11.1010	0.441696
−0.5	11.6765	0.464592
−0.6	12.2162	0.486068
−0.7	12.7088	0.505666
−0.8	13.1424	0.522919
−0.9	13.5050	0.537346
−1	13.7841	0.548451

Table 4 shows that the average velocity and the volume flow rate both increases as the parameter pressure gradient increases.

Appendix

$$\Lambda_{11} = 1 - \frac{\Omega}{4}$$

$$\Lambda_{12} = \frac{\Omega}{4}$$

$$\Lambda_{13} = -\frac{1}{4\ln\delta}\left(4 - \Omega + \Omega\delta^2\right)$$

$$\Lambda_{14} = \left(\Lambda_{12}\Lambda_{13}^4\alpha\beta - \frac{1}{2}\Lambda_{13}^3\alpha + \frac{1}{2}\Lambda_{13}^3\beta - \frac{1}{4}\Lambda_{13}^4\Omega\beta^2\right)C_1$$

$$\begin{aligned}\Lambda_{15} = {} & -\Lambda_{12} - \Lambda_{12}C_1 + \frac{1}{4}\Omega + \frac{1}{4}\Omega C_1 - \frac{3}{2}\Lambda_{13}^3\alpha C_1 - 6\Lambda_{12}^2\Lambda_{13}\alpha C_1 - 2\Lambda_{12}^2\Lambda_{13}\beta C_1 - \frac{1}{2}\Lambda_{13}^3\beta C_1\\ & + \frac{1}{2}\Lambda_{12}^2\Omega\beta C_1 + 2\Lambda_{12}\Lambda_{13}\beta C_1 - \frac{16}{9}\Lambda_{12}^5\alpha\beta C_1 - 8\Lambda_{12}^4\Lambda_{13}\beta C_1 - 24\Lambda_{12}^3\Lambda_{13}^2\alpha\beta C_1\\ & - \Lambda_{12}\Lambda_{13}^4\alpha\beta C_1 + \frac{4}{9}\Lambda_{12}^4\Omega\beta^2C_1 + 2\Lambda_{12}^3\Lambda_{13}\Omega\beta^2C_1 + 6\Lambda_{12}^3\Lambda_{13}^2\beta^2C_1 + \frac{1}{4}\Lambda_{13}^4\Omega\beta^2C_1\end{aligned}$$

$$\begin{aligned}\Lambda_{16} = {} & \Lambda_{12} + \Lambda_{12}C_1 - \frac{1}{4}\Omega - \frac{1}{4}\Omega C_1 + 6\Lambda_{12}^2\Lambda_{13}\alpha C_1 + 2\Lambda_{12}^2\Lambda_{13}\beta C_1 - 2\Lambda_{12}\Lambda_{13}\Omega\beta C_1\\ & + 24\Lambda_{12}^3\Lambda_{13}^2\alpha\beta C_1 - 6\Lambda_{12}^2\Lambda_{13}^3\Omega\beta^2C_1\end{aligned}$$

$$\Lambda_{17} = \left(2\Lambda_{12}^3\alpha - \frac{1}{2}\Lambda_{12}^3\Omega\beta + 6\Lambda_{12}^4\Lambda_{13}\alpha\beta - 2\Lambda_{12}^3\Lambda_{13}\beta^2\Omega\right)C_1$$

$$\Lambda_{18} = \left(\frac{16}{9}\Lambda_{12}^5\alpha\beta - \frac{4}{9}\Lambda_{13}^3\Omega\beta^2\right)C_1$$

$$\begin{aligned}\Lambda_{19} = {} & \frac{1}{\ln\delta}\Bigg(\Lambda_{12} - \frac{1}{4}\Omega - \Lambda_{12}\delta^2 + \frac{1}{4}\Omega\delta^2 + \frac{1}{2\delta^2}\Lambda_{13}^3\alpha C_1 - \frac{1}{2\delta^2}\Lambda_{13}^3\beta C_1 - \frac{1}{\delta^2}\Lambda_{12}^4\Lambda_{13}\alpha\beta C_1\\ & + \frac{1}{4\delta^2}\Lambda_{13}^3\Omega\beta^2C_1 + \Lambda_{12}C_1 - \frac{1}{4}\Omega C_1 + 2\Lambda_{12}^3\alpha C_1 + 6\Lambda_{12}^2\Lambda_{13}\alpha C_1 - \frac{1}{2}\Lambda_{13}^3\alpha C_1 + 2\Lambda_{12}^2\Lambda_{13}\beta\\ & + \frac{1}{2}\Lambda_{13}^3\beta C_1 - \frac{1}{2}\Lambda_{12}^2\Omega\beta C_1 - 2\Lambda_{13}\Lambda_{13}\Omega\beta + 24\Lambda_{12}^3\Lambda_{13}^2\alpha\beta C_1 + \Lambda_{12}\Lambda_{13}^4\alpha\beta C_1 - \frac{4}{9}\Lambda_{12}^4\Omega\beta^2C_1\\ & - 2\Lambda_{12}^3\Lambda_{13}\Omega\beta^2C_1 - 6\Lambda_{12}^2\Lambda_{13}^3\Omega\beta^2C_1 - \frac{1}{\ln\delta}\Lambda_{12}\delta^2C_1 + \frac{1}{2}\Omega\delta^2C_1 - 6\Lambda_{12}^3\Lambda_{13}\alpha\delta^2C_1 - 2\Lambda_{12}^3\Lambda_{13}\beta\delta^2C_1\\ & + 2\Lambda_{12}\Lambda_{13}\Omega\beta\delta^2C_1 - 24\Lambda_{12}^3\Lambda_{13}^2\alpha\beta\delta^2C_1 + 6\Lambda_{12}^2\Lambda_{13}^2\Omega\beta^2C_1 - 2\Lambda_{12}^3\alpha\delta^4C_1 + \frac{1}{2}\Lambda_{12}^2\Omega\beta\delta^4C_1\\ & - 8\Lambda_{12}^4\Lambda_{13}\alpha\beta\delta^4 + 2\Lambda_{13}^2\Omega\beta C_1 - \Lambda_{13}^2\Omega\beta C_1\Bigg)\end{aligned}$$

$$\kappa_{10} = \frac{1}{9}\Lambda_{13}^4\Lambda_{14}\alpha\beta C_1$$

$$\kappa_{11} = \frac{3}{2}\Lambda_{13}^2\Lambda_{14}\alpha C_1 - \Lambda_{13}^2\Lambda_{14}\beta C_1 + \frac{1}{2}\Lambda_{13}^3\Lambda_{14}\Omega\beta^2C_1$$

$$\begin{aligned}\kappa_{12} = {} & \Lambda_{14} + \Lambda_{14}C_1 + 12\Lambda_{12}\Lambda_{13}\Lambda_{14}\alpha C_1 - \frac{3}{2}\Lambda_{13}^2\Lambda_{19}\alpha C_1 - 12\Lambda_{12}\Lambda_{13}\Lambda_{14}\beta C_1 + \frac{3}{2}\Lambda_{13}^2\Lambda_{19}\beta C_1\\ & + 2\Lambda_{13}\Lambda_{14}\Omega\beta C_1 - 24\Lambda_{12}^2\Lambda_{13}^2\Lambda_{14}\alpha\beta C_1 + \Lambda_{13}^2\Lambda_{16}\alpha\beta C + 4\Lambda_{12}\Lambda_{13}^3\Lambda_{19}\alpha\beta C_1\\ & + 12\Lambda_{12}\Lambda_{13}^2\Lambda_{14}\Omega\beta^2C_1 - \Lambda_{13}^3\Lambda_{19}\Omega\beta^2C_1\end{aligned}$$

$$\begin{aligned}\kappa_{13} = {} & -\Lambda_{14} - \Lambda_{16} - \Lambda_{17} - \Lambda_{18} - \Lambda_{14}C_1 - \Lambda_{16}C_1 - \Lambda_{17}C_1 - \Lambda_{17}C_1 - 12\Lambda_{12}\Lambda_{13}\Lambda_{14}\alpha C_1\\ & - \frac{3}{2}\Lambda_{13}^2\Lambda_{14}\alpha C_1 - 6\Lambda_{12}^2\Lambda_{16}\alpha C_1 - 6\Lambda_{12}^2\Lambda_{18}\alpha C_1 - 12\Lambda_{12}\Lambda_{13}\Lambda_{16}\alpha C_1\\ & - 12\Lambda_{12}\Lambda_{13}\Lambda_{18}\beta C_1 - 6\Lambda_{13}^2\Lambda_{17}\beta C_1 + 6\Lambda_{12}\Lambda_{13}\Lambda_{18}\beta C_1 - 2\Lambda_{12}^4\Lambda_{16}\beta C_1\end{aligned}$$

$$
\begin{aligned}
&+2\Lambda_{13}\Lambda_{14}\Lambda_{16}\Omega\beta C_1-\Lambda_{12}^3\Lambda_{19}\Omega\beta C_1-\frac{3}{2}\Lambda_{12}^2\Lambda_{17}\alpha\beta C_1-\frac{1}{9}\Lambda_{13}^3\Lambda_{19}\alpha\beta C_1\\
&+8\Lambda_{12}\Lambda_{13}^2\Lambda_{16}\alpha\beta C_1-40\Lambda_{12}^2\Lambda_{19}\alpha\beta C_1+2\Lambda_{13}^3\Lambda_{14}\Lambda_{16}\alpha\beta C_1\\
&+6\Lambda_{13}^3\Lambda_{14}\Lambda_{18}\Omega\beta^2C_1-6\Lambda_{12}\Lambda_{13}^3\Lambda_9\Omega\beta^2C_1-\frac{8}{9}\Lambda_{12}^3\Lambda_{18}\Omega\beta^2C_1-2\Lambda_{13}^3\Lambda_{14}\Lambda_{17}\Omega\beta^2C_1
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{14}=\;&\Lambda_{16}+\Lambda_{16}C_1+6\Lambda_{13}^2\Lambda_{17}\alpha C_1+12\Lambda_{12}\Lambda_{13}\Lambda_{16}\alpha C_1+6\Lambda_{12}^2\Lambda_{19}\alpha C_1+4\Lambda_{12}\Lambda_{13}\Lambda_{16}\beta C_1\\
&+2\Lambda_{13}^2\Lambda_{17}\beta C_1+2\Lambda_{12}^2\Lambda_{19}\beta C_1-2\Lambda_{13}\Lambda_{16}\Omega\beta C_1-2\Lambda_{12}\Lambda_{19}\Omega\beta C_1-48\Lambda_{12}^4\Lambda_{14}\alpha\beta C_1\\
&+72\Lambda_{12}^2\Lambda_{13}^2\Lambda_{16}\alpha\beta C_1+48\Lambda_{12}\Lambda_{13}^3\Lambda_{17}\alpha\beta C_1-9\Lambda_{13}^4\Lambda_{18}\alpha\beta C_1+72\Lambda_{12}^3\Lambda_{13}\Lambda_{19}\alpha\beta C_1\\
&+16\Lambda_{12}^3\Lambda_{14}\Lambda_{19}\Omega\beta^2C_1-12\Lambda_{12}\Lambda_{13}^2\Lambda_{16}\Omega\beta^2C_1-4\Lambda_{13}^3\Lambda_{17}\Omega\beta^2C_1-12\Lambda_{12}^2\Lambda_{13}\Lambda_{19}\Omega\beta^2C_1
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{15}=\;&\Lambda_{17}+\Lambda_{17}C_1+6\Lambda_{12}^2\Lambda_{16}\alpha C_1+12\Lambda_{12}\Lambda_{13}\Lambda_{18}\alpha C_1+\frac{9}{2}\Lambda_{13}^2\Lambda_{18}\alpha C_1-\Lambda_{12}\Lambda_{16}\Omega C_1\\
&-\Lambda_{13}\Lambda_{17}\Omega\beta C_1+32\Lambda_{12}^3\Lambda_{13}\Lambda_{16}\alpha\beta C_1+48\Lambda_{12}^3\Lambda_{13}^2\Lambda_{17}\alpha\beta C_1+24\Lambda_{12}\Lambda_{13}^3\Lambda_{18}\alpha\beta C_1\\
&+8\Lambda_{12}^4\Lambda_{19}\alpha\beta C_1-6\Lambda_{12}^2\Lambda_{13}\Lambda_{16}\Omega\beta^2C_1-6\Lambda_{12}\Lambda_{13}^2\Lambda_{17}\Omega\beta^2C_1-\frac{3}{2}\Lambda_{13}^3\Lambda_{18}\Omega\beta^2C_1-2\Lambda_{12}^3\Lambda_{19}\Omega\beta^2C_1
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{16}=\;&\Lambda_{18}+\Lambda_{18}C_1+8\Lambda_{12}^2\Lambda_{17}\alpha C_1+12\Lambda_{12}\Lambda_{13}\Lambda_{18}\alpha C_1-\frac{8}{9}\Lambda_{12}^2\Lambda_{17}\beta C_1-\frac{4}{3}\Lambda_{12}\Lambda_{13}\Lambda_{18}\beta C_1\\
&-\frac{8}{9}\Lambda_{12}\Lambda_{17}\Omega\beta C_1-\frac{2}{3}\Lambda_{13}\Lambda_{18}\Omega\beta C_1+\frac{80}{9}\Lambda_{12}^4\Lambda_{16}\alpha\beta C_1+\frac{320}{9}\Lambda_{12}^3\Lambda_{13}\Lambda_{17}\alpha\beta C_1\\
&+40\Lambda_{12}^3\Lambda_{13}^2\Lambda_{18}\alpha\beta C_1-\frac{16}{9}\Lambda_{12}^3\Lambda_{16}\Omega\beta^2C_1-\frac{48}{9}\Lambda_{12}^2\Lambda_{13}\Lambda_{17}\Omega\beta^2C_1-4\Lambda_{12}\Lambda_{13}^2\Lambda_{18}\Omega\beta^2C_1
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{17}=\;&9\Lambda_{12}^2\Lambda_{18}\alpha C_1-\frac{3}{2}\Lambda_{12}^2\Lambda_{18}\beta C_1-\frac{3}{4}\Lambda_{12}\Lambda_{18}\Omega\beta C_1+12\Lambda_{12}^4\Lambda_{17}\alpha\beta C_1+36\Lambda_{12}^3\Lambda_{13}\Lambda_{18}\alpha\beta C_1\\
&-2\Lambda_{12}^3\Lambda_{17}\beta^2C_1-\frac{9}{2}\Lambda_{12}^2\Lambda_{13}\Lambda_{18}\Omega\beta^2C_1
\end{aligned}
$$

$$
\kappa_{18}=\frac{1}{25}\left(336\Lambda_{12}\alpha-48\Omega\beta\right)\Lambda_{12}^3\Lambda_{18}\beta C_1
$$

$$
\begin{aligned}
\kappa_{19}=\;&\Lambda_{14}+\Lambda_{16}+\Lambda_{17}+\Lambda_{18}+\Lambda_{14}C_1+\Lambda_{16}C_1+\Lambda_{17}C_1+\Lambda_{17}C_1-8\Lambda_{13}\Lambda_{14}\Lambda_{16}\alpha C_1\\
&-\frac{3}{2}\Lambda_{12}^4\Lambda_{13}\alpha C_1-9\Lambda_{13}^2\Lambda_{14}\alpha C_1-2\Lambda_{12}^2\Lambda_{19}\alpha C_1-\Lambda_{12}\Lambda_{13}\Lambda_{18}\alpha C_2-2\Lambda_{12}\Lambda_{13}\Lambda_{16}\alpha C_2\\
&-8\Lambda_{13}^2\Lambda_{18}\alpha C_2-6\Lambda_{13}^4\Lambda_{19}\alpha C_2-2\Lambda_{12}^2\Lambda_{19}\beta C_1-6\Lambda_{12}\Lambda_{14}\Lambda_{19}\beta C_1-\frac{8}{9}\Lambda_{12}\Lambda_{13}\Lambda_{16}\beta C_2\\
&-\Lambda_{13}^2\Lambda_{16}\beta C_2+12\Lambda_{12}\Lambda_{14}\Lambda_{19}\beta C_2-20\Lambda_{13}^4\Lambda_{14}\beta C_2+\frac{1}{2}\Lambda_{12}\Lambda_{14}\Lambda_{19}\Omega\beta C_1-\Lambda_{12}^2\Lambda_{18}\Omega\beta C_2\\
&-2\Lambda_{12}^2\Lambda_{14}\alpha\beta C_1-\frac{1}{9}\Lambda_{12}^3\Lambda_{14}\Lambda_{19}\alpha\beta C_1+4\Lambda_{12}\Lambda_{13}^2\Lambda_{19}\alpha\beta C_2-10\Lambda_{13}^3\Lambda_{16}\alpha\beta C_2\\
&+12\Lambda_{12}^3\Lambda_{14}\Lambda_{17}\alpha\beta C_2+3\Lambda_{13}^3\Lambda_{16}\Lambda_{19}\Omega\beta^2C_1-\Lambda_{12}\Lambda_{13}^3\Lambda_9\Omega\beta^2C_2-\frac{3}{2}\Lambda_{12}^3\Lambda_{16}\Omega\beta^2C_2\\
&-6\Lambda_{12}^3\Lambda_{13}\Lambda_{18}\Omega\beta^2C_2+\Psi_{11}+\Psi_{12}+\Psi_{13}+\Psi_{14}
\end{aligned}
$$

where

$$
\begin{aligned}
\Psi_{11}=\;&\frac{1}{\ln\delta}\Bigg(\Lambda_{12}\Lambda_{14}\Lambda_{19}\alpha C_1+2\Lambda_{13}^2\Lambda_{17}\alpha C_2+40\Lambda_{12}\Lambda_{14}\Lambda_{19}\beta C_1+6\Lambda_{12}^2\Lambda_{18}\beta C_2\\
&+3\Lambda_{12}^4\Lambda_{16}\beta C_2-4\Lambda_{14}\Lambda_{16}\Omega\beta C_2-\frac{3}{2}\Lambda_{12}\Lambda_{14}\Omega\beta C_2-24\Lambda_{13}^2\Lambda_{14}\alpha\beta C_2\\
&+36\Lambda_{12}^3\Lambda_{13}^2\Lambda_{18}\alpha\beta C_2+18\Lambda_{12}\Lambda_{13}^2\Lambda_{19}\alpha\beta C_2-\frac{1}{9}\Lambda_{12}^3\Lambda_{17}\alpha\beta C_2+12\Lambda_{12}^2\Lambda_{14}\Lambda_{18}\alpha\beta C_2\\
&+32\Lambda_{12}^3\Lambda_{13}\Lambda_{19}\Omega\beta^2C_2-\frac{80}{9}\Lambda_{12}\Lambda_{13}^2\Lambda_{18}\Omega\beta^2C_2-\frac{320}{9}\Lambda_{12}^2\Lambda_{17}\Omega\beta^2C_2\Bigg)
\end{aligned}
$$

$$\Psi_{12} = \frac{\delta^2}{\ln\delta}\left(\Lambda_{12}\Lambda_{13}\Lambda_{16}\alpha C_2 + \frac{32}{9}\Lambda_{12}\Lambda_{14}\Lambda_{16}\beta C_2 + 2\Lambda_{13}^2\Lambda_{19}\beta C_2 + 2\Lambda_{12}^3\Lambda_{17}\beta C_2\right.$$
$$- 2\Lambda_{14}\Lambda_{17}\Omega\beta C_2 - \frac{1}{2}\Lambda_{13}\Lambda_{19}\Omega\beta C_2 + 8\Lambda_{12}^2\Lambda_{13}^2\Lambda_{16}\alpha\beta C_2 + \frac{80}{9}\Lambda_{12}\Lambda_{13}^2\Lambda_{19}\alpha\beta C_2$$
$$\left. + 12\Lambda_{13}^2\Lambda_{16}\Lambda_{19}\alpha\beta C_2 + 2\Lambda_{12}^3\Lambda_{14}\Lambda_{16}\Omega\beta^2 C_2 - 24\Lambda_{13}^4\Lambda_{18}\Omega\beta^2 C_2\right)$$

$$\Psi_{13} = \frac{\delta^4}{\ln\delta}\left(2\Lambda_{12}\Lambda_{14}\Lambda_{18}\alpha C_2 + 16\Lambda_{13}^3\Lambda_{14}\alpha C_2 + \frac{40}{9}\Lambda_{13}\Lambda_{14}\Lambda_{16}\beta C_2 + \Lambda_{13}^2\Lambda_{19}\beta C_2\right.$$
$$- 8\Lambda_{12}\Lambda_{18}\Omega\beta C_2 - \Lambda_{13}\Lambda_{19}\Omega\beta C_2 + 9\Lambda_{12}^3\Lambda_{13}^2\Lambda_{17}\alpha\beta C_2 + 12\Lambda_{12}\Lambda_{13}^2\Lambda_{19}\alpha\beta C_2$$
$$\left. - 3\Lambda_{12}^2\Lambda_{16}\alpha\beta C_2 + 4\Lambda_{13}^3\Lambda_{14}\Lambda_{17}\Omega\beta^2 C_2 - \frac{1}{9}\Lambda_{12}\Lambda_{13}^3\Lambda_{18}\Omega\beta^2 C_2\right)$$

$$\Psi_{14} = \frac{\delta^6}{\ln\delta}\left(\Lambda_{12}\Lambda_{14}\Lambda_{17}\alpha C_2 + \frac{16}{9}\Lambda_{13}^3\Lambda_{16}\Lambda_{19}\alpha C_2 + 8\Lambda_{13}\Lambda_{14}\Lambda_{18}\beta C_2 + 24\Lambda_{12}^2\Lambda_{19}\beta C_2\right.$$
$$\left. - 4\Lambda_{13}\Lambda_{16}\Lambda_{18}\Omega\beta C_2 - 96\Lambda_{13}^4\Lambda_{16}\alpha\beta C_2 + \frac{1}{2}\Lambda_{12}^3\Lambda_{13}^2\Lambda_{17}\alpha\beta C_2 - 6\Lambda_{12}^3\Lambda_{18}\Omega\beta^2 C_2\right).$$

References

[1] S. Akter, M.S.J. Hashmi, Analysis of polymer flow in a canonical coating unit: power law approach, Prog. Org. Coat. 37 (1999) 15–22.
[2] S. Akter, M.S.J. Hashmi, Plasto-hydrodynamic pressure distribution in a tapered geometry wire coating unit, in: Proceedings of the 14th Conference of the Irish Manufacturing Committee, IMC14, Dublin, 1997, pp. 331–340.
[3] A.M. Siddiqui, T. Haroon, H. Khan, Wire coating extrusion in a pressure-type die *n* flow of a third grade fluid, Int. J. Nonlinear Sci. Numer. Simul. 10 (2) (2009) 247–257.
[4] R.T. Fenner, J.G. Williams, Analytical methods of wire coating die design, Trans. Plast. Inst. (London) 35 (1967) 701–706.
[5] M. Sajjid, A.M. Siddiqui, T. Hayat, Wire coating analysis using MHD Oldroyd 8-constant fluid, Internat. J. Engrg. Sci. 45 (2007) 381–392.
[6] V. Marinca, N. Herişanu, Optimal homotopy perturbation method for strongly nonlinear differential equations, Nonlinear Sci. Lett. A 1 (3) (2010) 273–280.
[7] V. Marinca, N. Herişanu, I. Nemes, An optimal homotopy asymptotic method with application to thin film flow, Cent. Eur. J. Phys. 6 (3) (2008) 648–653.
[8] N. Herişanu, V. Marinca, T. Dordea, G. Madescu, A new analytical approach to nonlinear vibration of an electrical machine, Proc. Rom. Acad. Ser. A 9 (3) (2008) 229–236.
[9] V. Marinca, N. Herişanu, C. Bota, B. Marinca, An optimal homotopy asymptotic method applied to steady flow of a fourth-grade fluid past a porous plate, Appl. Math. Lett. 22 (2009) 245–251.
[10] V. Marinca, N. Herişanu, Application of optimal homotopy asymptotic method for solving nonlinear equations arising in heat transfer, Int. Commun. Heat Mass Transfer 35 (2008) 710–715.
[11] N. Herişanu, V. Marinca, Accurate analytical solutions to oscillators with discontinuities and fractional-power restoring force by means of the optimal homotopy asymptotic method, Comput. Math. Appl. 60 (2010) 1607–1615.
[12] S. Islam, Rehan Ali Shah, Ishtiaq Ali, Optimal homotopy asymptotic solutions of Couette and Poiseuille flows of a third grade fluid with heat transfer analysis, Int. J. Nonlinear Sci. Numer. Simul. 11 (6) (2010) 389–400.
[13] Javed Ali, S. Islam, Sirajul Islam, Gul Zaman, The solution of multipoint boundary value problems by the optimal homotopy asymptotic method, Comput. Math. Appl. 59 (2010) 2000–2006.