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Mean-semivariance models for fuzzy portfolio selection

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Abstract

This paper discusses portfolio selection problem in fuzzy environment. In the paper, semivariance is originally presented for fuzzy variable, and three properties of the semivariance are proven. Based on the concept of semivariance of fuzzy variable, two fuzzy mean-semivariance models are proposed. To solve the new models in general cases, a fuzzy simulation based genetic algorithm is presented in the paper. In addition, two numerical examples are also presented to illustrate the modelling idea and the effectiveness of the designed algorithm.

Keywords: Fuzzy portfolio selection; Semivariance; Mean-semivariance model; Fuzzy programming; Optimization

1. Introduction

One of the hottest points in applied finance is portfolio selection which is to select a combination of securities that can best meet the investors' objective. Since the introduction of mean-variance models in [37,38], variance has been widely accepted as a risk measure. Numerous models have been developed based on variance such as models proposed in [10,19,31,35]. Though variance has been a rather popular measure of risk in portfolio selection, it has limitations [16,38]. One distinguished limitation is that analysis based on variance considers high returns as equally undesirable as low returns because high returns will also contribute to the extreme of variance. Then, when probability distributions of security returns are asymmetric, variance becomes a deficient measure of investment risk because the selected portfolio based on variance may have a potential danger to sacrifice too much expected return in eliminating both low and high return extremes. In fact, in reality, there do exist empirical evidences [1,9,11,44] indicating that many security returns are not symmetrically distributed.

To overcome the limitation of the mean-variance models, people began to take asymmetry of return distributions into account. Some scholars employed skewness, i.e., the third central moment, to measure the asymmetry degree of return distributions [9,27,29,31,35], others directly used downside risk measure, i.e., the measure which only gauges the negative deviations from a reference return level, to replace variance. Downside risk measure separates undesirable downside fluctuations of security returns from the desirable upside fluctuations and only pays attention to returns falling below some predetermined level. Therefore, it matches investors' notion about risk and gains popularity among investors. Experimental study [47] showed that people were strongly in favor of downside risk measure as the measure of investment risk. Rom and Ferguson [42] also reported that the downside risk concept enjoys an increasing popularity.

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There are many forms of downside risk measure, such as the famous one introduced in [43] in the safety first criterion and those proposed in [3,8,12,17,21,30,41,42]. Semivariance is one of the best-known downside risk measures. It was introduced in [38]. Because it is direct, clear and comparatively simple in reflecting investors' intuition about risk, it receives much attention. Many scholars such as Mao [36], Choobineh and Branting [7], Markowitz [39], Kaplan and Alldredge [26], Grootveld and Hallerbach [16], and Huang [24] researched the properties and computation problem of semivariance. Their researches show that semivariance has merits in measuring risk.

These researches discussed portfolio selection in stochastic environment. As discussed in [22,23,25], in some situations in which it is difficult to use probability theory, investors can make use of fuzzy set to reflect the vagueness and ambiguity of security returns. In fact, many scholars have made achievements in extending Markowitz's stochastic mean-variance idea to fuzzy environment in different ways. For example, Tanaka and Guo [45,46] quantified mean and variance of a portfolio through fuzzy probability and possibility distributions. Zhang and Nie [49] adopted Tanaka's [46] definition of possibility grade and assumed admissible errors on the expected return and risk of the asset. Arenas-Parra et al. [2] proposed a fuzzy goal programming model based on expected intervals defined in [18]. Carlsson et al. [6] used their own definitions of mean and variance of fuzzy numbers [5], and found the optimum portfolio by maximizing utility. Bilbao-Terol et al. formulated a fuzzy compromise programming problem [4]. In particular, Huang [25] quantified portfolio return and risk by the expected value and variance based on credibility measure, and proposed two new fuzzy mean-variance models for portfolio selection with fuzzy returns. In addition, Huang [22] presented two types of fuzzy chance-constrained models to find optimal portfolio. However, so far, there is no research on fuzzy portfolio selection taking semivariance as risk measure. For the similar reasons discussed in stochastic portfolio selection, when the membership functions of fuzzy returns are asymmetric, the fuzzy variance may also become a deficient risk measure because it also eliminates both low and high return extremes. Since semivariance is a direct, clear, comparatively simple and very popular measure to gauge downside risk, in this paper, we will extend stochastic mean-semivariance idea to fuzzy environment. We will first define semivariance of fuzzy variable and discuss three properties of it, and then discuss fuzzy portfolio selection problem using the fuzzy semivariance as the risk measure.

The paper is organized as follows. For the better understanding of the paper, we review some preliminary knowledge about fuzzy variable in Section 2. In Section 3, we define semivariance for fuzzy variable and discuss three properties of the semivariance. Then, we propose two fuzzy mean-semivariance models for portfolio selection in Section 4 and summarize a hybrid intelligent algorithm for solving the proposed problems in Section 5. After that, we present two numerical examples to illustrate the potential applications of the new models and the effectiveness of the proposed algorithm in Section 6. Finally, we conclude the paper in Section 7.

2. Preliminaries

Fuzzy set theory was introduced in [48] in 1965 and was well developed. In the fuzzy world, there is a well-known measure, i.e., possibility measure. However, possibility measure is not self-dual, yet the self-dual property is very important both in theory and in practice. As an alternative measure of a fuzzy event, Liu and Liu [34] defined a self-dual credibility measure in 2002. From then on, credibility theory has been well developed based on an axiomatic foundation presented in [33]. In this paper, we define semivariance measure of a fuzzy event based on credibility. For the better understanding of the paper, let us briefly review the necessary knowledge about fuzzy variable.

Let ξ be a fuzzy variable with membership function μ, and r a real number. The credibility of a fuzzy event, characterized by $\xi \leqslant r$, is defined by [34]

$$\mathrm{Cr}\{\xi \leqslant r\} = \frac{1}{2}\left(\sup_{u \leqslant r} \mu(u) + 1 - \sup_{u > r} \mu(u)\right). \tag{1}$$

The value of credibility takes values in [0, 1] [33]. It is easy to verify that the credibility is self-dual, i.e., $\mathrm{Cr}\{\xi \leqslant r\} + \mathrm{Cr}\{\xi > r\} = 1$.

The expected value of a fuzzy variable ξ is defined as [34]

$$E[\xi] = \int_0^{\infty} \mathrm{Cr}\{\xi \geqslant r\}\,\mathrm{d}r - \int_{-\infty}^{0} \mathrm{Cr}\{\xi \leqslant r\}\,\mathrm{d}r, \tag{2}$$

provided that at least one of the two integrals is finite.

Let ξ be a fuzzy variable with finite expected value e. The variance of ξ is defined by [34]

$$V[\xi]=E[(\xi-e)^2]. \tag{3}$$

The properties of variance of fuzzy variable are recorded in [33]. We only mention one property which will be used in the next section. For more expositions on the expected value and variance of fuzzy variable, the interested readers may refer to [33].

Theorem 1 (*Liu and Liu [34]*). *Let ξ be a fuzzy variable with expected value e. Then $V[\xi]=0$ if and only if* $\mathrm{Cr}\{\xi=e\}=1$.

3. Semivariance and its properties

Definition 1. Let ξ be a fuzzy variable with finite expected value e. Then the semivariance of ξ is defined by $S[\xi]=E[[(\xi-e)^-]^2]$, where

$$(\xi-e)^-=\begin{cases}\xi-e & \text{if } \xi\leqslant e,\\ 0 & \text{if } \xi>e.\end{cases} \tag{4}$$

For example, semivariance value of the triangular fuzzy variable $\xi=(-2,1,3)$ is $S[\xi]\approx 1.15$, while variance value of the triangular fuzzy variable $\xi=(-2,1,3)$ is $V[\xi]\approx 1.20$.

Theorem 2. *Let ξ be a fuzzy variable, $S[\xi]$ and $V[\xi]$ the semivariance and variance of ξ, respectively. Then $0\leqslant S[\xi]\leqslant V[\xi]$.*

Proof. Let e be the expected value of fuzzy variable ξ. The non-negativity of variance and semivariance is clear. For any real number r, we have

$$\{\theta|(\xi(\theta)-e)^2\geqslant r\}\supset\{\theta|[(\xi(\theta)-e)^-]^2\geqslant r\},$$

which implies that

$$\mathrm{Cr}\{(\xi-e)^2\geqslant r\}\geqslant \mathrm{Cr}\{[(\xi-e)^-]^2\geqslant r\},\quad \forall r$$

because credibility is an increasing measure [33].

It follows from the definition of variance and semivariance that

$$V[\xi]=\int_0^{+\infty}\mathrm{Cr}\{(\xi-e)^2\geqslant r\}\,\mathrm{d}r\geqslant\int_0^{+\infty}\mathrm{Cr}\{[(\xi-e)^-]^2\geqslant r\}\,\mathrm{d}r=S[\xi]. \qquad \square$$

Theorem 3. *Let ξ be a fuzzy variable with expected value e. Then $S[\xi]=0$ if and only if $\mathrm{Cr}\{\xi=e\}=1$, i.e., $V[\xi]=0$.*

Proof. If $V[\xi]=0$, then it follows from Theorem 2 that $S[\xi]=0$.

If $S[\xi]=0$, then $E[[(\xi-e)^-]^2]=0$. Note that

$$E[[(\xi-e)^-]^2]=\int_0^{+\infty}\mathrm{Cr}\{[(\xi-e)^-]^2\geqslant r\}\,\mathrm{d}r,$$

which implies that $\mathrm{Cr}\{[(\xi-e)^-]^2\geqslant r\}=0$ for any $r>0$ since the value of credibility takes values in $[0,1]$. Because credibility measure is self-dual, we have $\mathrm{Cr}\{[(\xi-e)^-]^2=0\}=1$. Therefore,

$$\mathrm{Cr}\{(\xi-e)^-=0\}=1, \tag{5}$$

which implies that $\xi-e=(\xi-e)^++(\xi-e)^-=(\xi-e)^+$ almost everywhere. Therefore, we have

$$E[(\xi-e)]=E[(\xi-e)^+]=\int_0^{+\infty}\mathrm{Cr}\{(\xi-e)^+\geqslant r\}\,\mathrm{d}r=0,$$

which implies that $\mathrm{Cr}\{(\xi-e)^{+}\geqslant r\}=0$ for any $r>0$. Since credibility is self-dual, we have

$$\mathrm{Cr}\{(\xi-e)^{+}=0\}=1. \tag{6}$$

It follows from Eqs. (5) and (6) that $\mathrm{Cr}\{(\xi-e)=0\}=1$, which means

$$\mathrm{Cr}\{\xi=e\}=1, \quad \text{i.e.,} \quad V[\xi]=0. \qquad \square$$

Theorem 4. *Let ξ be a fuzzy variable with symmetric membership function. Then $S[\xi]=V[\xi]$.*

Proof. Let ξ be a fuzzy variable with symmetric membership function about its expected value e. From the definition of variance, we have

$$V[\xi]=E[(\xi-e)^2]=\int_0^{+\infty}\mathrm{Cr}\{(\xi-e)^2\geqslant r\}\,\mathrm{d}r.$$

Since the membership function of ξ is symmetric about e, we have

$$\mathrm{Cr}\{(\xi-e)^2\geqslant r\}=\mathrm{Cr}\{[(\xi-e)^{-}]^2\geqslant r\}, \quad \forall r.$$

Therefore,

$$V[\xi]=\int_0^{+\infty}\mathrm{Cr}\{(\xi-e)^2\geqslant r\}\,\mathrm{d}r=\int_0^{+\infty}\mathrm{Cr}\{[(\xi-e)^{-}]^2\geqslant r\}\,\mathrm{d}r=S[\xi]. \qquad \square$$

4. Fuzzy mean-semivariance models

We can see from Theorem 4 that when membership functions of security returns are symmetrical, optimal portfolio can be obtained no matter whether we take the variance or the semivariance as the measurement of risk. However, when membership functions of security returns are asymmetrical, we can see from Theorem 2 that taking semivariance or variance as the measurement of risk will yield different results. Since low part deviation from the expected value implies the possible loss of the investment while high part deviation from the expected value implies the potential return of the investment, we adopt semivariance of fuzzy variable as the measurement of risk to select the optimal portfolio in fuzzy environment.

Let x_i denote the investment proportions in securities i, and ξ_i the fuzzy returns of the ith securities defined as $\xi_i=(p_i'+d_i-p_i)/p_i$, $i=1,2,\ldots,n$, respectively, where p_i' is the estimated closing prices of the securities i in the next year, p_i the closing prices of the securities i at present, and d_i the estimated dividends of the securities i during the coming year. Then, in case when the investors can give a tolerable level of risk, and want to maximize the expected return at the given level of risk, we have the fuzzy mean-semivariance model as follows:

$$\begin{cases} \max & E[x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n], \\ \text{subject to:} & S[x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n]\leqslant\gamma, \\ & x_1+x_2+\cdots+x_n=1, \\ & x_i\geqslant 0, \quad i=1,2,\ldots,n, \end{cases} \tag{7}$$

where γ denotes the maximum risk level the investors can tolerate, E the expected value operator, and S the semivariance of the fuzzy variables.

When the investors preset an expected return level that they feel satisfactory, and want to minimize the risk for this given level of return, the optimization model becomes

$$\begin{cases} \min & S[x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n], \\ \text{subject to:} & E[x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n]\geqslant \alpha, \\ & x_1+x_2+\cdots+x_n=1, \\ & x_i\geqslant 0, \quad i=1,2,\ldots,n, \end{cases} \tag{8}$$

where α denotes the minimum expected investment return that the investors can accept.

Please note that in fuzzy environment, generally speaking, $E[x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n]\neq x_1E[\xi_1]+x_2E[\xi_2]+\cdots+x_nE[\xi_n]$.

5. Fuzzy simulation based genetic algorithm

Here we provide a fuzzy simulation based genetic algorithm (GA) to solve the proposed optimization problems in general cases, i.e., membership functions of fuzzy returns can take any forms of function. The technique of fuzzy simulation was proposed in [34], and has been recorded in [32]. The interested readers can refer to them. GA was proposed in [20] in 1975, and has been well developed since then. GA is a stochastic search method for optimization problems based on the mechanics of "survival of the fittest". It does not require the specific mathematical analysis of optimization problems. By group searching and group information exchanging, GA avoids getting stuck at a local optimal solution and can find the global optimal solution to the complex optimization problems fairly. For the last three decades, by using GA [13,14,15,28,40] researchers have successfully solved many complex optimization problems which are hard to solve by analytic methods. In particular, Liu [32] designed a spectrum of fuzzy simulation based GA for solving general fuzzy programming models. In this paper, we first use the method of fuzzy simulation to compute the expected value and the semivariance value of the fuzzy return. Then we integrate fuzzy simulation into GA to find the optimal solutions. For the detailed introduction of the fuzzy simulation based GA, the interested readers can refer to the paper [22]. The difference mainly lies in initialization operation and the feasibility checking. Here, we introduce in detail the initialization operation and the feasibility checking, and summarize the fuzzy simulation based GA.

Initialization: In the GA, a solution $\boldsymbol{x}=(x_1, x_2, \ldots, x_n)$ is represented by the chromosome $\mathbf{C}=(c_1, c_2, \ldots, c_n)$, where the genes $c_1, c_2, \ldots, c_n$ are restricted in the interval [0, 1]. The matching between the solution and the chromosome is through

$$x_i = \frac{c_i}{c_1 + c_2 + \cdots + c_n}, \quad i = 1, 2, \ldots, n, \tag{9}$$

which ensures that $x_1 + x_2 + \cdots + x_n = 1$ always holds.

For Model (7), randomly generate a point $\mathbf{C} = (c_1, c_2, \ldots, c_n)$ from the hypercube $[0, 1]^n$. Use fuzzy simulation to calculate the semivariance value $S[x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n]$. Then the feasibility of chromosome $\mathbf{C}=(c_1, c_2, \ldots, c_n)$ is checked as follows: *If* $S[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] > \gamma$ *return* 0;
return 1;
in which 1 means feasible, and 0 non-feasible.

For Model (8), randomly generate a point $\mathbf{C}=(c_1, c_2, \ldots, c_n)$ from the hypercube $[0, 1]^n$. Use fuzzy simulation to calculate the expected value $E[x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n]$. Then the feasibility of chromosome $\mathbf{C}=(c_1, c_2, \ldots, c_n)$ is checked as follows: *If* $E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] < \alpha$ *return* 0;
return 1;
in which 1 means feasible, and 0 non-feasible.

If $\mathbf{C}$ is checked to be feasible, it is taken as an initial chromosome. Otherwise, randomly generate another point $\mathbf{C}$ from the hypercube $[0, 1]^n$ until the point is proven to be feasible and taken as an initial chromosome. Repeat this process *pop_size* times, then initial feasible *pop_size* chromosomes $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_{\text{pop\_size}}$ are produced.

Fuzzy simulation based GA: After initialization, the chromosomes will go through the operations of selection, crossover and mutation. In the crossover and mutation operations, when checking the feasibility of the chromosomes, we check in the similar way described in initialization operation. After finishing the whole round, the new population will be ready for its next round of selection, crossover and mutation. The fuzzy simulation based GA will continue until a given number of cyclic repetitions of the whole round is met. Summarization of the algorithm is as follows:

Step 1. Initialize feasible *pop_size* chromosomes in which fuzzy simulation is used to calculate the semivariance value or the expected value, and to check the feasibility of the chromosomes.

Step 2. Compute the fitness of each chromosome by following steps: Calculate the objective values for all chromosomes by fuzzy simulation first, then give the rank order of the chromosomes according to the objective values. After that, compute the values of the rank-based-evaluation function of the chromosomes, then calculate the fitness of each chromosome according to the rank-based-evaluation function.

Step 3. Select the chromosomes by spinning the roulette wheel. The selection is fitness-proportional.

Step 4. Update the chromosomes by crossover and mutation operations in which fuzzy simulation is used to calculate the semivariance value or the expected value, and to check the feasibility of the chromosomes.

Step 5. Repeat the second to the fourth operations for a given number of cycles.
Step 6. Take the best chromosome as the solution of portfolio selection.

6. Numerical examples

Assume that there are 10 securities. Among them, returns of seven ones are triangular fuzzy variables $\xi_i = (a_i, b_i, c_i)$, $i = 1, 2, \ldots, 7$, respectively. The fuzzy returns of the other three ones take the membership functions μ_i, $i = 8, 9, 10$, where r_i are real numbers. The data set is given in Table 1.

When we solve the following two examples, the parameters in the GA are both set as follows: the population size is 30, the probability of crossover $P_c = 0.3$, the probability of mutation $P_m = 0.2$, the parameter in the rank-based-evaluation function $v = 0.05$.

Example 1. Suppose that the risk is not allowed to exceed 0.6, then the fuzzy mean-semivariance portfolio selection model is as follows:

$$\begin{cases} \max & E[x_1\xi_1 + x_2\xi_2 + \cdots + x_{10}\xi_{10}], \\ \text{subject to:} & S[x_1\xi_1 + x_2\xi_2 + \cdots + x_{10}\xi_{10}] \leqslant 0.6, \\ & x_1 + x_2 + \cdots + x_{10} = 1, \\ & x_i \geqslant 0, \quad i = 1, 2, \ldots, 10. \end{cases} \tag{10}$$

A run of the hybrid intelligent algorithm with 2000 generations shows that among 10 securities, in order to gain the maximum expected return with the risk not greater than 0.6, the investor should assign his money according to Table 2. The corresponding maximum expected return is 1.60.

To further test the effectiveness of the proposed algorithm, we change the values of parameters in the GA and repeat the solution process. The parameters and the results are shown in Table 3. To compare the results, we give an index called relative error, i.e., (optimal expected return − actual expected return)/optimal expected return×100%, where the optimal expected return is the maximum one of all the six maximum expected returns obtained. It can be seen from Table 3 that the relative errors do not exceed 2%, which shows that the proposed algorithm is robust to the set parameters and effective for solving the fuzzy portfolio selection model.

Table 1
Fuzzy returns of 10 securities

Security i	Fuzzy return	Security i	Fuzzy return
1	(−0.3, 1.8, 2.3)	6	(−0.8, 2.5, 3.0)
2	(−0.4, 2.0, 2.2)	7	(−0.6, 1.8, 3.0)
3	(−0.5, 1.9, 2.7)	8	$\frac{1}{1+(r-1.6)^4}$
4	(−0.6, 2.2, 2.8)	9	$\frac{1}{1+(5r-7.4)^2}$
5	(−0.7, 2.4, 2.7)	10	$e^{-(r-1.6)^2}$

Table 2
Allocation of money to 10 securities

Security i	1	2	3	4	5	6	7	8	9	10
Allocation of money	20.85	11.11	19.11	6.92	9.64	32.38	0.00	0.00	0.00	0.00

Table 3
Comparison solutions of Example 1

pop_size	P_c	P_m	Generation	Expected return value	Percent error
30	0.3	0.5	2000	1.6002	1.00
30	0.3	0.2	2000	1.6039	0.77
50	0.3	0.5	2000	1.6112	0.32
50	0.3	0.2	2000	1.6124	0.25
300	0.5	0.4	2000	1.6164	0.00
500	0.4	0.5	2000	1.6136	0.17

Table 4
Allocation of money to 10 securities (%)

Security i	1	2	3	4	5	6	7	8	9	10
Allocation of money	11.28	9.96	9.97	12.10	20.62	20.87	6.39	3.58	5.23	0.00

Example 2. In case when the minimum expected return the investor can accept is 1.5, the fuzzy mean-semivariance portfolio selection model is set as follows:

$$\begin{cases} \min & S[x_1\xi_1 + x_2\xi_2 + \cdots + x_{20}\xi_{10}], \\ \text{subject to:} & E[x_1\xi_1 + x_2\xi_2 + \cdots + x_{10}\xi_{10}] \geqslant 1.5, \\ & x_1 + x_2 + \cdots + x_{10} = 1, \\ & x_i \geqslant 0, \quad i = 1, 2, \ldots, 10. \end{cases} \tag{11}$$

A run of the fuzzy simulation based GA with 2000 generations shows that among 10 securities, in order to minimize the investment risk with the expected return not less than 1.5, the investor should assign his money according to Table 4. The corresponding minimum semivariance is 0.32.

7. Conclusions

In this paper, concept of semivariance for fuzzy variable is originally presented, and three properties of the semivariance are proven. Taking semivariance of fuzzy returns as risk measure, two fuzzy mean-semivariance models are proposed in the paper. In addition, a fuzzy simulation based GA is presented to provide a general solution to the new model problems. Results of numerical experiments show that the proposed algorithm is effective for solving the fuzzy mean-semivariance models.

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