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Soft topology

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ABSTRACT

The concept of soft sets is introduced as a general mathematical tool for dealing with uncertainty. In this work, we define the soft topology on a soft set, and present its related properties. We then present the foundations of the theory of soft topological spaces.

1. Introduction

The concept of soft sets was first introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing with uncertain objects. In [1,2], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on.

After presentation of the operations of soft sets [3], the properties and applications of soft set theory have been studied increasingly [4–14]. The algebraic structure of soft set theory has also been studied in more detail [15–27]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [28,16,29–44,14]. To develop soft set theory, the operations of the soft sets are redefined and a *uni–int* decision making method was constructed by using these new operations [45]. To make easy compaction with the operations of soft sets, soft matrix theory was presented, and the soft *max–min* decision making method was set up [46]. These decision making methods are more practical and can be successfully applied to many problems that contain uncertainties.

Modern topology depends strongly on the ideas of set theory. Therefore, in this work, we introduce a topology on a soft set, so-called "soft topology", and its related properties. We then present the foundations of the theory of soft topological spaces. This may be the starting point for soft mathematical concepts and structures that are based on soft set-theoretic operations.

2. Preliminary

In this section, we present the basic definitions and results of soft set theory which may be found in earlier studies [45,3,1].

Throughout this work, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subseteq E$.

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Definition 1. A soft set F_A on the universe U is defined by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\},$$

where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$.

Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.

Note that the set of all soft sets over U will be denoted by $S(U)$.

Example 1. Suppose that there are six houses in the universe $U = \{h_l, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and that $E = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of decision parameters. The x_i ($i = 1, 2, 3, 4, 5$) stand for the parameters "expensive", "beautiful", "wooden", "cheap", and "in green surroundings", respectively.

Consider the mapping f_A given by "houses (.)", where (.) is to be filled in by one of the parameters $x_i \in E$. For instance, $f_A(e_1)$ means "houses (expensive)", and its functional value is the set $\{h \in U : h$ is an expensive house$\}$.

Suppose that $A = \{x_1, x_3, x_4\} \subseteq E$ and $f_A(x_1) = \{h_2, h_4\}, f_A(x_3) = U$, and $f_A(x_4) = \{h_1, h_3, h_5\}$. Then, we can view the *soft set* F_A as consisting of the following *collection* of approximations:

$$F_A = \{(x_1, \{h_2, h_4\}), (x_3, U), (x_4, \{h_1, h_3, h_5\})\}.$$

Definition 2. Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called an empty set, denoted by F_Φ.

$f_A(x) = \emptyset$ means that there is no element in U related to the parameter $x \in E$. Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.

Definition 3. Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in A$, then F_A is called an A-universal soft set, denoted by $F_{\tilde{A}}$.

If $A = E$, then the A-universal soft set is called a universal soft set, denoted by $F_{\tilde{E}}$.

Definition 4. Let $F_A, F_B \in S(U)$. Then, F_A is a soft subset of F_B, denoted by $F_A \tilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 5. Let $F_A, F_B \in S(U)$. Then, F_A and F_B are soft equal, denoted by $F_A = F_B$, if and only if $f_A(x) = f_B(x)$ for all $x \in E$.

Definition 6. Let $F_A, F_B \in S(U)$. Then, the soft union $F_A \tilde{\cup} F_B$, the soft intersection $F_A \tilde{\cap} F_B$, and the soft difference $F_A \tilde{\setminus} F_B$ of F_A and F_B are defined by the approximate functions

$$f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x), \qquad f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x), \qquad f_{A\tilde{\setminus}B}(x) = f_A(x) \setminus f_B(x),$$

respectively, and the soft complement $F_A^{\tilde{c}}$ of F_A is defined by the approximate function

$$f_{A^{\tilde{c}}}(x) = f_A^c(x),$$

where $f_A^c(x)$ is the complement of the set $f_A(x)$; that is, $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

It is easy to see that $(F_A^{\tilde{c}})^{\tilde{c}} = F_A$ and $F_\Phi^{\tilde{c}} = F_{\tilde{E}}$.

Proposition 1. *Let $F_A \in S(U)$. Then,*

i. $F_A \tilde{\cup} F_A = F_A$, $F_A \tilde{\cap} F_A = F_A$
ii. $F_A \tilde{\cup} F_\Phi = F_A$, $F_A \tilde{\cap} F_\Phi = F_\Phi$
iii. $F_A \tilde{\cup} F_{\tilde{E}} = F_{\tilde{E}}$, $F_A \tilde{\cap} F_{\tilde{E}} = F_A$
iv. $F_A \tilde{\cup} F_A^{\tilde{c}} = F_{\tilde{E}}$, $F_A \tilde{\cap} F_A^{\tilde{c}} = F_\Phi$.

Proposition 2. *Let $F_A, F_B, F_C \in S(U)$. Then,*

i. $F_A \tilde{\cup} F_B = F_B \tilde{\cup} F_A$, $F_A \tilde{\cap} F_B = F_B \tilde{\cap} F_A$
ii. $(F_A \tilde{\cup} F_B)^{\tilde{c}} = F_B^{\tilde{c}} \tilde{\cap} F_A^{\tilde{c}}$, $(F_A \tilde{\cap} F_B)^{\tilde{c}} = F_B^{\tilde{c}} \tilde{\cup} F_A^{\tilde{c}}$
iii. $(F_A \tilde{\cup} F_B) \tilde{\cup} F_C = F_A \tilde{\cup} (F_B \tilde{\cup} F_C)$, $(F_A \tilde{\cap} F_B) \tilde{\cap} F_C = F_A \tilde{\cap} (F_B \tilde{\cap} F_C)$
iv. $F_A \tilde{\cup} (F_B \tilde{\cap} F_C) = (F_A \tilde{\cup} F_B) \tilde{\cap} (F_A \tilde{\cup} F_C)$
$F_A \tilde{\cap} (F_B \tilde{\cup} F_C) = (F_A \tilde{\cap} F_B) \tilde{\cup} (F_A \tilde{\cap} F_C)$.

3. Soft topology

In this section, we give a definition of soft topology on a soft set and its related properties.

Definition 7. Let $F_A \in S(U)$. The soft power set of F_A is defined by

$$\tilde{P}(F_A) = \{F_{A_i} : F_{A_i} \tilde{\subseteq} F_A, i \in I \subseteq \mathbb{N}\}$$

and its cardinality is defined by

$$|\tilde{P}(F_A)| = 2^{\sum_{x\in E} |f_A(x)|},$$

where $|f_A(x)|$ is the cardinality of $f_A(x)$.

Example 2. Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$. Then
$F_{A_1} = \{(x_1, \{u_1\})\}$,
$F_{A_2} = \{(x_1, \{u_2\})\}$,
$F_{A_3} = \{(x_1, \{u_1, u_2\})\}$,
$F_{A_4} = \{(x_2, \{u_2\})\}$,
$F_{A_5} = \{(x_2, \{u_3\})\}$,
$F_{A_6} = \{(x_2, \{u_2, u_3\})\}$,
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_3\})\}$,
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$,
$F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$,
$F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$,
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\}$,
$F_{A_{15}} = F_A$,
$F_{A_{16}} = F_\Phi$
are all soft subsets of F_A. So $|\tilde{P}(F_A)| = 2^4 = 16$.

Definition 8. Let $F_A \in S(U)$. A soft topology on F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having the following properties:

i. $F_\Phi, F_A \in \tilde{\tau}$
ii. $\{F_{A_i} \tilde{\subseteq} F_A : i \in I \subseteq \mathbb{N}\} \subseteq \tilde{\tau} \ \Rightarrow\ \tilde{\bigcup}_{i\in I} F_{A_i} \in \tilde{\tau}$
iii. $\{F_{A_i} \tilde{\subseteq} F_A : 1 \le i \le n,\ \ n \in \mathbb{N}\} \subseteq \tilde{\tau} \ \Rightarrow\ \tilde{\bigcap}_{i=1}^{n} F_{A_i} \in \tilde{\tau}$.

The pair $(F_A, \tilde{\tau})$ is called a soft topological space.

Example 3. Let us consider the soft subsets of F_A that are given in Example 2. Then, $\tilde{\tau}_1 = \{F_\Phi, F_A\}$, $\tilde{\tau}_2 = \tilde{P}(F_A)$, and $\tilde{\tau}_3 = \{F_\Phi, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\}$ are soft topologies on F_A.

Definition 9. Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called a soft open set. Clearly, F_Φ and F_A are soft open sets.

Definition 10. Let $(F_A, \tilde{\tau}_1)$ and $(F_A, \tilde{\tau}_2)$ be soft topological spaces. Then, the following hold.
If $\tilde{\tau}_2 \supseteq \tilde{\tau}_1$, then $\tilde{\tau}_2$ is soft finer than $\tilde{\tau}_1$.
If $\tilde{\tau}_2 \supset \tilde{\tau}_1$, then $\tilde{\tau}_2$ is soft strictly finer than $\tilde{\tau}_1$.
If either $\tilde{\tau}_2 \supseteq \tilde{\tau}_1$ or $\tilde{\tau}_2 \subseteq \tilde{\tau}_1$, then $\tilde{\tau}_1$ is comparable with $\tilde{\tau}_2$.

Example 4. Let us consider the soft topologies on F_A that are given in Example 3. Then, $\tilde{\tau}_2$ is soft finer than $\tilde{\tau}_1$ and $\tilde{\tau}_3$, and $\tilde{\tau}_3$ is soft finer than $\tilde{\tau}_1$. So $\tilde{\tau}_1$, $\tilde{\tau}_2$, and $\tilde{\tau}_3$ are comparable soft topologies.

Definition 11. Let $(F_A, \tilde{\tau})$ be a soft topological space and $\tilde{\mathcal{B}} \subseteq \tilde{\tau}$. If every element of $\tilde{\tau}$ can be written as the union of elements of $\tilde{\mathcal{B}}$, then $\tilde{\mathcal{B}}$ is called a soft basis for the soft topology $\tilde{\tau}$. Each element of $\tilde{\mathcal{B}}$ is called a soft basis element.

Example 5. Let us consider Examples 2 and 3. Then, $\tilde{\mathcal{B}} = \{F_\Phi, F_{A_1}, F_{A_2}, F_{A_4}, F_{A_5}\}$ is a soft basis for the soft topology $\tilde{\tau}_2$.

Theorem 1. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $\tilde{\mathcal{B}}$ be a soft basis for $\tilde{\tau}$. Then, $\tilde{\tau}$ equals the collection of all soft unions of elements of $\tilde{\mathcal{B}}$.*

Proof. This is clearly seen from Definition 11. □

Definition 12. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the collection

$$\tilde{\tau}_{F_B} = \{F_{A_i} \tilde{\cap} F_B : F_{A_i} \in \tilde{\tau}, i \in I \subseteq \mathbb{N}\}$$

is called a soft subspace topology on F_B.

Hence, $(F_B, \tilde{\tau}_{F_B})$ is called a soft topological subspace of $(F_A, \tilde{\tau})$.

Theorem 2. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then a soft subspace topology on F_B is a soft topology.*

Proof. Indeed, it contains F_Φ and F_B because $F_\Phi \tilde{\cap} F_B = F_\Phi$ and $F_A \tilde{\cap} F_B = F_B$, where $F_\Phi, F_A \in \tilde{\tau}$. Since $\tilde{\tau} = \{F_{A_i} : F_{A_i} \tilde{\subseteq} F_A, i \in I\}$, it is closed under finite soft intersections and arbitrary soft unions:

$$\tilde{\bigcap_{i=1}^{n}}(F_{A_i} \tilde{\cap} F_B) = \left(\tilde{\bigcap_{i=1}^{n}} F_{A_i}\right) \tilde{\cap} F_B$$
$$\tilde{\bigcup_{i\in I}}(F_{A_i} \tilde{\cap} F_B) = \left(\tilde{\bigcup_{i\in I}} F_{A_i}\right) \tilde{\cap} F_B. \quad \square$$

Example 6. Let us consider the soft topology τ_3 on F_A given in Example 3. If $F_B = F_{A_9}$, then $\tilde{\tau}_{F_B} = \{F_\Phi, F_{A_5}, F_{A_8}, F_{A_9}\}$, and so (F_B, τ_{F_B}) is a soft topological subspace of $(F_A, \tilde{\tau}_3)$.

Theorem 3. *Let $(F_A, \tilde{\tau})$ and $(F_A, \tilde{\tau}')$ be soft topological spaces, and $\tilde{\mathcal{B}}$ and $\tilde{\mathcal{B}}'$ be soft bases for $\tilde{\tau}$ and $\tilde{\tau}'$, respectively. If $\tilde{\mathcal{B}}' \subseteq \tilde{\mathcal{B}}$, then $\tilde{\tau}$ is soft finer than $\tilde{\tau}'$.*

Proof. Let $\tilde{\mathcal{B}}' \subseteq \tilde{\mathcal{B}}$. Then, for each $F_B \in \tilde{\tau}'$ and $F_C \in \tilde{B}'$,

$$F_B = \tilde{\bigcup_{F_C \in \tilde{\mathcal{B}}'}} F_C = \tilde{\bigcup_{F_C \in \tilde{\mathcal{B}}}} F_C.$$

Therefore, $F_B \in \tilde{\tau}$; hence $\tilde{\tau}' \subseteq \tilde{\tau}$. $\square$

Theorem 4. *Let $(F_A, \tilde{\tau})$ be a soft topological space. If $\tilde{\mathcal{B}}$ is a soft basis for $\tilde{\tau}$, then the collection $\tilde{\mathcal{B}}_{F_B} = \{F_{A_i} \tilde{\cap} F_B : F_{A_i} \in \tilde{\mathcal{B}}, i \in I \subseteq \mathbb{N}\}$ is a soft basis for the soft subspace topology on F_B.*

Proof. Take as given each $F_{A_i} \in \tilde{\tau}_{F_B}$. From the definition of soft subspace topology, $F_C = F_D \tilde{\cap} F_B$, where $F_D \in \tilde{\tau}$. Because $F_D \in \tilde{\tau}, F_D = \tilde{\cup}_{F_{A_i} \in \tilde{\mathcal{B}}} F_{A_i}$. Therefore,

$$F_C = \left(\tilde{\bigcup_{F_{A_i} \in \tilde{\mathcal{B}}}} F_{A_i}\right) \tilde{\cap} F_B = \tilde{\bigcup_{F_{A_i} \in \tilde{\mathcal{B}}}} (F_{A_i} \tilde{\cap} F_B).$$

Hence $\tilde{\mathcal{B}}_{F_B}$ is a soft basis for soft topology $\tilde{\tau}_{F_B}$ on F_B. $\square$

Theorem 5. *Let $(F_A, \tilde{\tau})$ be a soft topological space, $(F_B, \tilde{\tau}_{F_B})$ be a soft topological subspace, and $F_C \subseteq F_B$. If F_C is soft open in F_B, then F_C is soft open in F_A.*

Proof. This is clearly seen from Definition 12. $\square$

Definition 13. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, F_B is said to be soft closed if the soft set $F_B^{\tilde{c}}$ is soft open.

Theorem 6. *Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, the following conditions hold.*

i. *The universal soft set $F_{\tilde{E}}$ and $F_A^{\tilde{c}}$ are soft closed sets.*
ii. *Arbitrary soft intersections of the soft closed sets are soft closed.*
iii. *Finite soft unions of the soft closed sets are soft closed.*

Proof. i. $F_{\tilde{E}}^{\tilde{c}} = F_\Phi$ and $(F_A^{\tilde{c}})^{\tilde{c}} = F_A$ are soft open sets.

ii. If $\{F_{A_i} : F_{A_i}^{\tilde{c}} \in \tilde{\tau}, i \in I \subseteq \mathbb{N}\}$ is a given collection of soft closed sets, then

$$\left(\tilde{\bigcap_{i\in I}} F_{A_i}\right)^{\tilde{c}} = \tilde{\bigcup_{i\in I}} F_{A_i}^{\tilde{c}}$$

is soft open. Therefore, $\tilde{\cap}_{i\in I} F_{A_i}$ is a soft closed set.

iii. Similarly, if F_{A_i} is soft closed for $i = 1, 2, \ldots, n$, then

$$\left(\tilde{\bigcup_{i=1}^{n}} F_{A_i}\right)^{\tilde{c}} = \tilde{\bigcap_{i=1}^{n}} F_{A_i}^{\tilde{c}}$$

is soft open. Hence, $\tilde{\cup}_{i=1}^{n} F_{A_i}$ is a soft closed set. $\square$

Definition 14. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the soft interior of F_B, denoted F_B°, is defined as the soft union of all soft open subsets of F_B.

Note that F_B° is the biggest soft open set that is contained by F_B.

Example 7. Let us consider the soft topology $\tilde{\tau}_3$ given in Example 3. If $F_B = F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$, then $F_B^\circ = F_\Phi \tilde{\cup} F_{A_2} \tilde{\cup} F_{A_{11}} = F_{A_{11}}$.

Theorem 7. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. F_B is a soft open set if and only if $F_B = F_B^\circ$.*

Proof. If F_B is an open soft set, then the biggest soft open set that is contained by F_B is equal to F_B. Therefore, $F_B = F_B^\circ$. Conversely, it is known that F_B° is a soft open set, and, if $F_B = F_B^\circ$, then F_B is soft open set. □

Theorem 8. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then,*

i. $(F_B^\circ)^\circ = F_B^\circ$
ii. $F_B \tilde{\subseteq} F_C \Rightarrow F_B^\circ \tilde{\subseteq} F_C^\circ$
iii. $F_B^\circ \tilde{\cap} F_C^\circ = (F_B \tilde{\cap} F_C)^\circ$
iv. $F_B^\circ \tilde{\cup} F_C^\circ \tilde{\subseteq} (F_B \tilde{\cup} F_C)^\circ$.

Proof. i. Let $F_B^\circ = F_D$. Then $F_D \in \tilde{\tau}$ if and only if $F_D = F_D^\circ$. Therefore, $(F_B^\circ)^\circ = F_B^\circ$.
ii. Let $F_B \tilde{\subseteq} F_C$. From the definition of a soft interior, $F_B^\circ \tilde{\subseteq} F_B$ $F_C^\circ \tilde{\subseteq} F_C$. F_C° is the biggest soft open set that is contained by F_C. Hence, $F_B \tilde{\subseteq} F_C \Rightarrow F_B^\circ \tilde{\subseteq} F_C^\circ$.
iii. By the definition of a soft interior, $F_B^\circ \tilde{\subseteq} F_B$ and $F_C^\circ \tilde{\subseteq} F_C$. Then, $F_B^\circ \tilde{\cap} F_C^\circ \tilde{\subseteq} F_B \tilde{\cap} F_C$. $(F_B \tilde{\cap} F_C)^\circ$ is the biggest soft open set that is contained by $F_B \tilde{\cap} F_C$. Hence, $F_B^\circ \tilde{\cap} F_C^\circ \tilde{\subseteq} (F_B \tilde{\cap} F_C)^\circ$. Conversely, $F_B \tilde{\cap} F_C \tilde{\subseteq} F_B$ and $F_B \tilde{\cap} F_C \tilde{\subseteq} F_C$.
Then, $(F_B \tilde{\cap} F_C)^\circ \tilde{\subseteq} F_B^\circ$ and $(F_B \tilde{\cap} F_C)^\circ \tilde{\subseteq} F_C^\circ$. Therefore, $(F_B \tilde{\cap} F_C)^\circ \subseteq F_B^\circ \tilde{\cap} F_C^\circ$.
iv. $F_B^\circ \tilde{\subseteq} F_B$ and $F_C^\circ \tilde{\subseteq} F_C$. Then, $F_B^\circ \tilde{\cup} F_C^\circ \tilde{\subseteq} F_B \tilde{\cup} F_C$. $(F_B \tilde{\cup} F_C)^\circ$ is the biggest soft open set that is contained by $F_B \tilde{\cup} F_C$. Hence, $F_B^\circ \tilde{\cup} F_C^\circ \tilde{\subseteq} (F_B \tilde{\cup} F_C)^\circ$. □

Definition 15. Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, the soft closure of F_B, denoted $\overline{F_B}$, is defined as the soft intersection of all soft closed supersets of F_B.

Note that $\overline{F_B}$ is the smallest soft closed set that containing F_B.

Example 8. Let us consider the soft topology $\tilde{\tau}_3$ that is given in Example 3. If $F_B = F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$, then $F_{A_2}^{\tilde{c}} = \{(x_1\{u_1, u_3\}), (x_2, U), (x_3, U)\}$ and $F_\Phi^{\tilde{c}} = F_{\tilde{E}}$ are soft closed supersets of F_B. Hence, $\overline{F_B} = F_{A_2}^{\tilde{c}} \tilde{\cap} F_{\tilde{E}} = F_{A_2}^{\tilde{c}}$.

Theorem 9. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. F_B is a closed soft set if and only if $F_B = \overline{F_B}$.*

Proof. The proof is trivial. □

Theorem 10. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then, $F_B^\circ \tilde{\subseteq} F_B \tilde{\subseteq} \overline{F_B}$.*

Proof. Indeed, $F_B^\circ = \bigcup\{F_{B_i} : F_{B_i} \in \tilde{\tau}, F_{B_i} \tilde{\subseteq} F_B, i \in I \subseteq \mathbb{N}\}$. Then, $f_{B_i}(x) \subseteq f_B(x)$ and $\cup_{i \in I} f_{B_i}(x) \subseteq f_B(x)$ for all $x \in E$. So $F_B^\circ \tilde{\subseteq} F_B$.
$\overline{F_B} = \bigcap\{F_{A_i} : F_{A_i}^c \in \tilde{\tau}, F_B \tilde{\subseteq} F_{A_i}, i \in J \subseteq \mathbb{N}\}$. Then, $f_B(x) \subseteq f_{A_i}(x)$ and $f_B(x) \subseteq \cap_{i \in J} f_{A_i}(x)$ for all $x \in E$. So $F_B \tilde{\subseteq} \overline{F_B}$. Hence, $F_B^\circ \tilde{\subseteq} F_B \tilde{\subseteq} \overline{F_B}$. □

Theorem 11. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then,*

i. $(\overline{\overline{F_B}}) = \overline{F_B}$
ii. $(\overline{F_B})^{\tilde{c}} = (F_B^{\tilde{c}})^\circ$
iii. $F_C \tilde{\subseteq} F_B \Rightarrow \overline{F_C} \tilde{\subseteq} \overline{F_B}$
iv. $\overline{F_B} \tilde{\cap} \overline{F_C} \tilde{\subseteq} \overline{(F_B \tilde{\cap} F_C)}$
v. $\overline{F_B} \tilde{\cup} \overline{F_C} = \overline{(F_B \tilde{\cup} F_C)}$.

Proof. i. Let $\overline{F_B} = F_D$. Then, F_D is a soft closed set. Therefore, F_D and $\overline{F_D}$ are equal. Hence, $(\overline{\overline{F_B}}) = \overline{F_B}$.
ii. If we consider the definitions of a soft closure and a soft interior, we obtain

$$(\overline{F_B})^{\tilde{c}} = \left(\tilde{\bigcap_{\substack{F_{A_i} \tilde{\supseteq} F_B \\ F_{A_i}^{\tilde{c}} \in \tilde{\tau}}}} F_{A_i} \right)^{\tilde{c}} = \tilde{\bigcup} F_{A_i}^{\tilde{c}} = (F_B^{\tilde{c}})^\circ.$$

iii. Let $F_C \tilde{\subseteq} F_B$. By the definition of a soft closure, $F_B \tilde{\subseteq} \overline{F}_B$ and $F_C \tilde{\subseteq} \overline{F}_C$. $\overline{F}_C$ is the smallest soft closed set that containing F_C. Then $\overline{F}_C \tilde{\subseteq} \overline{F}_B$.
iv. $\overline{F}_B$ and $\overline{F}_C$ are soft closed sets. So $\overline{F}_B \tilde{\cap} \overline{F}_C$ is a soft closed set. Since $F_B \tilde{\cap} F_C \tilde{\subseteq} \overline{F}_B \tilde{\cap} \overline{F}_C$ and $\overline{(F_B \tilde{\cap} F_C)}$ is the smallest soft closed set that containing $F_B \tilde{\cap} F_C$, $\overline{(F_B \tilde{\cap} F_C)} \tilde{\subseteq} \overline{F}_B \tilde{\cap} \overline{F}_C$.
v. $F_B \tilde{\subseteq} \overline{F}_B$ and $F_C \tilde{\subseteq} \overline{F}_C$. Then, $F_B \tilde{\cup} F_C \tilde{\subseteq} \overline{F}_B \tilde{\cup} \overline{F}_C$. Since $\overline{(F_B \tilde{\cup} F_C)}$ is the smallest soft closed set that containing $F_B \tilde{\cup} F_C$, $\overline{(F_B \tilde{\cup} F_C)} \tilde{\subseteq} \overline{F}_B \tilde{\cup} \overline{F}_C$. Conversely, $F_C \tilde{\subseteq} \overline{F}_C \tilde{\subseteq} \overline{(F_B \tilde{\cup} F_C)}$ and $F_B \tilde{\subseteq} \overline{F}_B \tilde{\subseteq} \overline{(F_B \tilde{\cup} F_C)}$. Therefore, $\overline{F}_B \tilde{\cup} \overline{F}_C \tilde{\subseteq} \overline{(F_B \tilde{\cup} F_C)}$. Hence, $\overline{F}_B \tilde{\cup} \overline{F}_C = \overline{(F_B \tilde{\cup} F_C)}$. □

Theorem 12. *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C \tilde{\subseteq} F_A$. Then the following hold.*

i. *$\alpha \in \overline{F}_B$ if and only if every soft open F_C containing α soft intersects F_B.*
ii. *Supposing the soft topology of F_A is given by a soft basis, then $\alpha \in \overline{F}_B$ if and only if every soft basis element F_D containing α soft intersect F_B.*

Proof. i. The hypothesis is equivalent to $\alpha \notin \overline{F}_B$ if and only if there exists a soft open set F_C containing α that does not soft intersect F_B. If $\alpha \notin \overline{F}_B$, the soft set $F_C = (\overline{F}_B)^{\tilde{c}}$ is a soft open set containing α that does not soft intersect F_B, as required. Conversely, if there exists soft open set F_C containing α which does not soft intersect F_B, then $F_C^{\tilde{c}}$ is a soft closed set containing F_B. By the definition of the soft closure $\overline{F}_B$, the soft set $F_C^{\tilde{c}}$ must contain $\overline{F}_B$; therefore, α cannot be in $\overline{F}_B$.
ii. If $\alpha \in \overline{F}_B$, then every soft open set F_C containing α soft intersects F_B, and so every soft basis element F_D containing α soft intersects F_B. Conversely, if every soft basis element F_D containing α soft intersects F_B, then every soft open set F_C containing α soft intersects F_B. Hence, $\alpha \in \overline{F}_B$. □

Definition 16. Let $(F_A, \tilde{\tau})$ be a soft topological space and $\alpha \in F_A$. If there is a soft open set F_B such that $\alpha \in F_B$, then F_B is called a soft open neighborhood (or soft neighborhood) of α. The set of all soft neighborhoods of α, denoted $\tilde{\mathcal{V}}(\alpha)$, is called the family of soft neighborhoods of α; that is,

$$\tilde{\mathcal{V}}(\alpha) = \{F_B : F_B \in \tilde{\tau}, \alpha \in F_B\}.$$

Example 9. Let us consider the $(F_A, \tilde{\tau}_3)$ topological space in Example 3 and $\alpha = (x_1, \{u_1, u_2\}) \in F_A$. Then, $\tilde{\mathcal{V}}(\alpha) = \{F_A, F_{A_{13}}\}$.

Definition 17. Let $(F_A, \tilde{\tau})$ be a soft topological space, $F_B \tilde{\subseteq} F_A$, and $\alpha \in F_A$. If every neighborhood of α soft intersects F_B in some points other than α itself, then α is called a soft limit point of F_B. The set of all limit points of F_B is denoted by F'_B.

In other words, if $(F_A, \tilde{\tau})$ is a soft topological space, $F_B, F_C \tilde{\subseteq} F_A$, and $\alpha \in F_A$, then $\alpha \in F'_B \Leftrightarrow F_C \tilde{\cap} (F_B \tilde{\setminus} \{\alpha\}) \neq F_\Phi$ for all $F_C \in \tilde{\mathcal{V}}(\alpha)$.

Example 10. Let us consider Example 9. If $F_B = F_{A_{13}}$ and $\alpha = (x_1, \{u_1, u_2\}) \in F_A$, then $\alpha \in F'_B$, since $F_A \tilde{\cap} (F_B \tilde{\setminus} \{\alpha\}) \neq F_\Phi$ and $F_{A_{13}} \tilde{\cap} (F_B \tilde{\setminus} \{\alpha\}) \neq F_\Phi$.

Theorem 13. *Let $(F_A, \tilde{\tau})$ be a soft topological space, and $F_B \tilde{\subseteq} F_A$. Then,*

$$F_B \tilde{\cup} F'_B = \overline{F}_B.$$

Proof. If $\alpha \in F_B \tilde{\cup} F'_B$, then $\alpha \in F_B$ or $\alpha \in F'_B$. In this case, if $\alpha \in F_B$, then $\alpha \in \overline{F}_B$. If $\alpha \in F'_B$, then $F_C \tilde{\cap} (F_B \tilde{\setminus} \{\alpha\}) \neq F_\Phi$ for all $F_C \in \tilde{\mathcal{V}}(\alpha)$, and so $F_C \tilde{\cap} F_B \neq F_\Phi$ for all $F_C \in \tilde{\mathcal{V}}(\alpha)$; hence, $\alpha \in \overline{F}_B$. Conversely, if $\alpha \in \overline{F}_B$, then $\alpha \in F_B$ or $\alpha \notin F_B$. In this case, if $\alpha \in F_B$, it is trivial that $\alpha \in F_B \tilde{\cup} F'_B$. If $\alpha \notin F_B$, then $F_C \tilde{\cap} (F_B \tilde{\setminus} \{\alpha\}) \neq F_\Phi$ for all $F_C \in \tilde{\mathcal{V}}(\alpha)$. Therefore, $\alpha \in F'_B$, so $\alpha \in F_B \tilde{\cup} F'_B$. Hence, $F_B \tilde{\cup} F'_B = \overline{F}_B$. □

Theorem 14. *Let $(F_A, \tilde{\tau})$ be a soft topological space, and $F_B \tilde{\subseteq} F_A$. Then, F_B is soft closed if and only if $F'_B \tilde{\subseteq} F_B$.*

Proof. $\overline{F}_B = F_B \Leftrightarrow F_B \tilde{\cup} F'_B = F_B \Leftrightarrow F'_B \tilde{\subseteq} F_B$. □

Theorem 15. *Let $(F_A, \tilde{\tau})$ be a soft topological space, and $F_B, F_C \tilde{\subseteq} F_A$. Then,*

i. $F'_B \tilde{\subseteq} \overline{F}_B$
ii. $F_B \tilde{\subseteq} F_C \Rightarrow F'_B \tilde{\subseteq} F'_C$
iii. $(F_B \tilde{\cap} F_C)' \tilde{\subseteq} F'_B \tilde{\cap} F'_C$
iv. $(F_B \tilde{\cup} F_C)' = F'_B \tilde{\cup} F'_C$
v. *F_B is a soft closed set* $\Leftrightarrow F'_B \tilde{\subseteq} F_B$.

Proof. i. From the definition of a soft closure, the proof is trivial.

ii. Let $F_B\tilde{\subseteq}F_C$. Since $F_B\tilde{\setminus}\{\alpha\}\tilde{\subseteq}F_C\tilde{\setminus}\{\alpha\}$, $\overline{F_B\tilde{\setminus}\{\alpha\}}\tilde{\subseteq}\overline{F_C\tilde{\setminus}\{\alpha\}}$, and we obtain $F_B'\tilde{\subseteq}F_C'$.

iii. $F_B\tilde{\cap}F_C\tilde{\subseteq}F_B$ and $F_B\tilde{\cap}F_C\tilde{\subseteq}F_C$. Then $(F_B\tilde{\cap}F_C)'\tilde{\subseteq}F_B'$ and $(F_B\tilde{\cap}F_C)'\tilde{\subseteq}F_C'$. Therefore, $(F_B\tilde{\cap}F_C)'\tilde{\subseteq}F_B'\tilde{\cap}F_C'$.

iv. $\forall\alpha \in (F_B\tilde{\cup}F_C)' \Leftrightarrow \alpha \in \overline{(F_B\tilde{\cup}F_C)\tilde{\setminus}\{\alpha\}}$, therefore

$$\begin{aligned}\overline{(F_B\tilde{\cup}F_C)\tilde{\setminus}\{\alpha\}} &= \overline{(F_B\tilde{\cup}F_C)\tilde{\cap}\{\alpha\}^{\tilde{c}}}\\ &= \overline{(F_B\tilde{\cap}\{\alpha\}^{\tilde{c}})\tilde{\cup}(F_C\tilde{\cap}\{\alpha\}^{\tilde{c}})}\\ &= \overline{(F_B\tilde{\cap}\{\alpha\}^{\tilde{c}})}\tilde{\cup}\overline{(F_C\tilde{\cap}\{\alpha\}^{\tilde{c}})}\\ &= \overline{(F_C\tilde{\setminus}\{\alpha\})}\tilde{\cup}\overline{(F_B\tilde{\setminus}\{\alpha\})}\end{aligned}$$

$\Leftrightarrow \alpha \in F_B'\tilde{\cup}F_C'$. Hence, $(F_B\tilde{\cup}F_C)' = F_B'\tilde{\cup}F_C'$.

v. F_B is a soft closed $\Leftrightarrow F_B = \overline{F}_B \Leftrightarrow F_B = F_B\tilde{\cup}F_B' \Leftrightarrow F_B'\tilde{\subseteq}F_B$. □

Definition 18. Let $(F_A, \tilde{\tau})$ be a soft topological space. If $\forall\alpha_1, \alpha_2 \in F_A$ $(\alpha_1 \neq \alpha_2)$, $\exists F_{B_1} \in \tilde{\mathcal{V}}(\alpha_1)$ and $\exists F_{B_2} \in \tilde{\mathcal{V}}(\alpha_2)$ such that $F_{B_1}\tilde{\cap}F_{B_2} = F_{\Phi}$, then $(F_A, \tilde{\tau})$ is called a soft Hausdorff space.

Example 11. Let $U = \{u_1, u_2, u_3\}$ and $E = \{x_1, x_2, x_3\}$. Clearly, $F_E = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\}), (x_3, \{u_1, u_3\})\} \in S(U)$. If $F_{E_1} = \{(x_1, \{u_1, u_2\}), (x_3, \{u_1\})\}$ and $F_{E_2} = \{(x_2, \{u_2, u_3\}), (x_3, \{u_3\})\}$, then $\tilde{\tau} = \{F_{\Phi}, F_E, F_{E_1}, F_{E_2}\}$ is a soft topology on F_E. Hence, $(F_E, \tilde{\tau})$ is a soft Hausdorff space.

Theorem 16. *Every finite point soft set in a soft Hausdorff space is a soft closed set.*

Proof. Let $(F_A, \tilde{\tau})$ be a soft Hausdorff space. It suffices to show that every point $\{\alpha_1\}$ is soft closed. If α_2 is a point of F_A different from α_1, then α_1 and α_2 have disjoint soft neighborhoods F_{B_1} and F_{B_2}, respectively. Since F_{B_1} does not soft intersect $\{\alpha_2\}$, point α_1 cannot belong to the soft closure of the set $\{\alpha_2\}$. As a result, the soft closure of the set $\{\alpha_1\}$ is $\{\alpha_1\}$ itself, so it is soft closed. □

Definition 19. Let $(F_A, \tilde{\tau})$ be a soft topological space, and $F_B\tilde{\subseteq}F_A$. Then, the soft boundary of F_B, denoted F_B^b, is defined by

$$F_B^b = \overline{F}_B\tilde{\cap}\overline{F_B^{\tilde{c}}}.$$

Example 12. Let us consider Example 8. For F_B, $\overline{F}_B = F_{A_2}^{\tilde{c}}$ and $\overline{F_B^{\tilde{c}}} = F_{\tilde{E}}$. Then, $F_B^b = \overline{F}_B\tilde{\cap}\overline{F_B^{\tilde{c}}} = F_{A_2}^{\tilde{c}}$.

Theorem 17. *Let $(F_A, \tilde{\tau})$ be a soft topological space, and $F_B, F_C\tilde{\subseteq}F_A$. Then,*

i. $F_B^b\tilde{\subseteq}\overline{F}_B$

ii. $F_B^b = (F_B^{\tilde{c}})^b$

iii. $F_B^b = \overline{F}_B\tilde{\setminus}F_B^{\circ}$.

Proof. i. From the definition of a soft boundary, the proof is trivial.

ii. Take as given $\alpha \in F_B^b \Leftrightarrow F_C\tilde{\cap}F_B \neq F_{\Phi}$ and $F_C\tilde{\cap}F_B^{\tilde{c}} \neq F_{\Phi}$ for all $F_C \in \tilde{\mathcal{V}}(\alpha) \Leftrightarrow F_C\tilde{\cap}F_B^{\tilde{c}} \neq F_{\Phi}$ and $F_C\tilde{\cap}(F_B^{\tilde{c}})^{\tilde{c}} \neq F_{\Phi}$ for all $F_C \in \tilde{\mathcal{V}}(\alpha)$. Hence, $F_B^b = (F_B^{\tilde{c}})^b$.

iii. Consider the definitions of a soft closure and a soft interior:

$$\begin{aligned}\overline{F}_B\tilde{\setminus}F_B^{\circ} &= \overline{F}_B\tilde{\cap}(F_B^{\circ})^{\tilde{c}}\\ &= \overline{F}_B\tilde{\cap}\left(\tilde{\bigcup_{\substack{F_{B_i}\tilde{\subseteq}F_B\\ F_{B_i}\in\tilde{\tau}}}} F_{B_i}\right)^{\tilde{c}}\\ &= \overline{F}_B\tilde{\cap}\left(\tilde{\bigcap}F_{B_i}^{\tilde{c}}\right)\\ &= \overline{F}_B\tilde{\cap}(\overline{F_B^{\tilde{c}}})\\ &= F_B^b.\end{aligned}$$

□

4. Conclusion

The concept of soft sets was first introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing with uncertainty. Recently, many scholars have studied the properties and applications of soft set theory, but not the topology. Topology is a major area of mathematics. Therefore, in this work, we have defined the soft topology on a soft set and have presented its related properties. We then presented the foundations of the theory of soft topological spaces. This paper may be the starting point for soft mathematical concepts and structures that are based on soft set-theoretic operations. Hence we expect that some research teams will be actively working on soft topological structures.

References

[1] D.A. Molodtsov, Soft set theory—first results, Computers and Mathematics with Applications 37 (1999) 19–31.
[2] D. Molodtsov, V.Y. Leonov, D.V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1 (1) (2006) 8–39.
[3] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555–562.
[4] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications 57 (2009) 1547–1553.
[5] Y. Jiang, Y. Tang, Q. Chen, J. Wang, S. Tang, Extending soft sets with description logics, Computers and Mathematics with Applications 59 (2010) 2087–2096.
[6] D.V. Kovkov, V.M. Kolbanov, D.A. Molodtsov, Soft sets theory-based optimization, Journal of Computer and Systems Sciences International 46 (6) (2007) 872–880.
[7] P. Majumdar, S.K. Samanta, Similarity measure of soft sets, New Mathematics and Natural Computation 4 (1) (2008) 1–12.
[8] M.M. Mushrif, S. Sengupta, A.K. Ray, Texture classification using a novel, in: Soft-Set Theory Based Classification, Algorithm, in: Lecture Notes In Computer Science, 3851, 2006, pp. 246–254.
[9] D. Pei, D. Miao, From soft sets to information systems, in: X. Hu, Q. Liu, A. Skowron, T.Y. Lin, R.R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, in: IEEE, vol. 2, 2005, pp. 617–621.
[10] K. Qin, Z. Hong, On soft equality, Journal of Computational and Applied Mathematics 234 (2010) 1347–1355.
[11] Z. Xiao, K. Gong, S. Xia, Y. Zou, Exclusive disjunctive soft sets, Computers and Mathematics with Applications 59 (2010) 2128–2137.
[12] Z. Xiao, Y. Li, B. Zhong, X. Yang, Research on synthetically evaluating method for business competitive capacity based on soft set, Statistical Research (2003) 52–54.
[13] W. Xu, J. Ma, S. Wang, G. Hao, Vague soft sets and their properties, Computers and Mathematics with Applications 59 (2010) 787–794.
[14] Y. Zou, Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge-Based Systems 21 (2008) 941–945.
[15] U. Acar, F. Koyuncu, B. Tanay, Soft sets and soft rings, Computers and Mathematics with Applications 59 (2010) 3458–3463.
[16] H. Aktaş, N. Çağman, Soft sets and soft groups, Information Sciences 1 (77) (2007) 2726–2735.
[17] F. Feng, Y.B. Jun, X. Zhao, Soft semirings, Fuzzy Sets and Systems: Theory and Applications 56 (10) (2008) 2621–2628.
[18] Y.B. Jun, Soft BCK/BCI-algebras, Computers and Mathematics with Applications 56 (2008) 1408–1413.
[19] Y.B. Jun, C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Information Sciences 178 (2008) 2466–2475.
[20] Y.B. Jun, C.H. Park, Applications of soft sets in Hilbert algebras, Iranian Journal Fuzzy Systems 6 (2) (2009) 55–86.
[21] Y.B. Jun, H.S. Kim, J. Neggers, Pseudo d-algebras, Information Sciences 179 (2009) 1751–1759.
[22] Y.B. Jun, K.J. Lee, A. Khan, Soft ordered semigroups, Mathematical Logic Quarterly 56 (1) (2010) 42–50.
[23] Y.B. Jun, K.J. Lee, C.H. Park, Soft set theory applied to commutative ideals, in BCK–algebras, Journal of Applied Mathematics Informatics 26 (3–4) (2008) 707–720.
[24] Y.B. Jun, K.J. Lee, C.H. Park, Soft set theory applied to ideals in d-algebras, Computers and Mathematics with Applications 57 (2009) 367–378.
[25] Y.B. Jun, K.J. Lee, C.H. Park, Fuzzy soft set theory applied to BCK/BCI-algebras, Computers and Mathematics with Applications 59 (2010) 3180–3192.
[26] C.H. Park, Y.B. Jun, M.A. Öztürk, Soft WS-algebras, Communications of the Korean Mathematical Society. 23 (3) (2008) 313–324.
[27] Q.M. Sun, Z.L. Zhang, J. Liu, Soft sets and soft modules, Proceedings of Rough Sets and Knowledge Technology, Third International Conference, RSKT 2008, 17–19 May, Chengdu, China, pp. 403–409, 2008.
[28] B. Ahmad, A. Kharal, On fuzzy soft sets, Advances in Fuzzy Systems (2009) 1–6.
[29] A. Aygünoglu, H. Aygün, Introduction to fuzzy soft groups, Computers and Mathematics with Applications 58 (2009) 1279–1286.
[30] N. Çağman, F. Çıtak, S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems 1 (1) (2010) 21–35.
[31] F. Feng, Y.B. Jun, X. Liu, L. Li, An adjustable approach to fuzzy soft set based decision making, Journal of Computational and Applied Mathematics 234 (2010) 10–20.
[32] S.J. Kalayathankal, G.S. Singh, A fuzzy soft flood alarm model, Mathematics and Computers in Simulation 80 (2010) 887–893.
[33] A. Kharal, B. Ahmad, Mappings on fuzzy soft classes, Advances in Fuzzy Systems (1-1) (2009).
[34] Z. Kong, L. Gao, L. Wang, S. Li, The normal parameter reduction of soft sets and its algorithm, Computers and Mathematics with Applications 56 (2008) 3029–3037.
[35] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics 9 (3) (2001) 589–602.
[36] P.K. Maji, R. Biswas, A.R. Roy, Intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics 9 (3) (2001) 677–691.
[37] P. Majumdar, S.K. Samanta, Generalised fuzzy soft sets, Computers and Mathematics with Applications 59 (2010) 1425–1432.
[38] A. Mukherjee, S.B. Chakraborty, On intuitionistic fuzzy soft relations, Bulletin of Kerala Mathematics Association 5 (1) (2008) 35–42.
[39] A.R. Roy, P.K. Maji, A fuzzy soft set theoretic approach to decision making problems, Journal of Computational and Applied Mathematics 203 (2007) 412–418.
[40] T. Som, On the theory of soft sets soft relation and fuzzy soft relation, Proc. of the National Conference on Uncertainty: A Mathematical Approach, UAMA-06, Burdwan, 1–9, 2006.
[41] M.J. Son, Interval-valued fuzzy soft sets, Journal of Fuzzy Logic and Intelligent Systems 17 (4) (2007) 557–562.
[42] Z. Xiao, K. Gong, Y. Zou, A combined forecasting approach based on fuzzy soft sets, Journal of Computational and Applied Mathematics 228 (2009) 326–333.
[43] X. Yang, T.Y. Lin, J. Yang, Y. Li, D. Yu, Combination of interval-valued fuzzy set and soft set, Computers and Mathematics with Applications 58 (2009) 521–527.
[44] J. Zhan, Y.B. Jun, Soft BL-algebras based on fuzzy sets, Computers and Mathematics with Applications 59 (2010) 2037–2046.
[45] N. Çağman, S. Enginoglu, Soft set theory and uni–int decision making, European Journal of Operational Research 207 (2010) 848–855.
[46] N. Çağman, S. Enginoglu, Soft matrix theory and its decision making, Computers and Mathematics with Applications 59 (2010) 3308–3314.