Applied Mathematics Letters 24 (2011) 2030–2033

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

Fuzzy topological entropy of fuzzy continuous functions on fuzzy topological spaces

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ARTICLE INFO

Article history:
Received 30 July 2010
Received in revised form 23 May 2011
Accepted 31 May 2011

Keywords:
Fuzzy continuous function
Fuzzy dynamical system
Fuzzy compact sets
Fuzzy topological entropy

ABSTRACT

In this paper, we introduce a new type of fuzzy topological entropy of a fuzzy continuous function $\psi : X \to X$, where X is an arbitrary fuzzy topological space, and investigate several of its properties. We also establish that, for fuzzy compact spaces, this new concept of fuzzy topological entropy coincides with the fuzzy topological entropy of İsmail Tok (2005) [11].

1. Introduction

An important application of covering properties of topological spaces is topological entropy. In 1965, Adler et al. [1] introduced the concept of topological entropy of continuous functions for a compact dynamical system on general topological spaces; subsequently, this was generalized by Liu et al. [2] for an arbitrary dynamical system. This concept is of great interest among mathematicians, especially topologists. Already, many researchers have studied the concept of topological entropy. Some of the research works are found in the papers of Bowen [3], Cánovas and Rodríguez [4], Goodwyn [5,6], Kwietniak and Oprocha [7], and Thomas [8]. Bowen [9] generalized the concept of topological entropy due to Adler et al. [1] that is metric dependent. Peña and López [10] introduced the concept of topological entropy for induced hyperspace maps. In 2005, İsmail Tok [11] fuzzified the concept of topological entropy for fuzzy compact topological spaces and introduced the concept of fuzzy topological entropy.

In this paper, the concept of fuzzy topological entropy has been generalized to arbitrary fuzzy topological spaces, and several of its properties are investigated. It has been shown that, for fuzzy compact spaces, this new fuzzy topological entropy coincides with the fuzzy entropy due to İsmail Tok [11].

2. Preliminaries

Throughout this paper, spaces (X, δ) and (Y, σ) (or simply X and Y) represent non-empty fuzzy topological spaces due to Chang [12], and the symbols I and I^X have been used for the unit closed interval $[0, 1]$ and the set of all functions with domain X and codomain I, respectively. The support of a fuzzy set A is the set $\{x \in X : A(x) > 0\}$, and is denoted by $supp(A)$. A fuzzy set with only one non-zero value $p \in (0, 1]$ at only one element $x \in X$ is called a fuzzy point, and is denoted by x_p, and the set of all fuzzy points of a fuzzy topological space is denoted by $Pt(X)$. For any two fuzzy sets A, B of X, $A \leq B$ if and

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doi:10.1016/j.aml.2011.05.037

only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy point x_p is said to be in a fuzzy set A (denoted by $x_p \in A$) if $x_p \leq A$, i.e., if $p \leq A(x)$. The constant fuzzy sets of X with values 0 and 1 are denoted by $\underline{0}$ and $\underline{1}$, respectively. A fuzzy set A is said to be quasi-coincident with B (written as $A\hat{q}B$) [13] if $A(x)+B(x) > 1$ for some $x \in X$. A fuzzy set A is said to be not quasi-coincident with B (written as $A\bar{q}B$) [13] if $A(x) + B(x) \leq 1$, for all $x \in X$. A fuzzy open set A of X is called a fuzzy quasi-neighborhood of a fuzzy point x_p if $x_p\hat{q}A$, and the collection of all fuzzy quasi-neighborhoods of a fuzzy point x_p is denoted by $FQN(X, x_p)$. A collection Σ of fuzzy sets in X is called a fuzzy cover of $\underline{1}$ if $\bigvee\{A : A \in \Sigma\} = \underline{1}$, i.e., if $sup\{U(x) : U \in \Sigma\} = 1$, for all $x \in X$. A fuzzy topological space X is said to be fuzzy compact if every fuzzy cover of $\underline{1}$ by its fuzzy open sets has a finite subcover of $\underline{1}$. It is well known that a function $\psi : X \to Y$ is fuzzy continuous if $\psi^{-1}(U)$ is fuzzy open in X for each fuzzy open set U in Y. In this paper, the cardinalities of an ordinary set K and of the set N of all natural numbers are denoted by $| K |$ and $\aleph_0$, respectively. Let X be a fuzzy compact space and let Σ be an open cover of X. Denote the smallest cardinality of all subcovers (for X) of Σ by $N_X(\Sigma)$, i.e., $N_X(\Sigma) = min\{| \Omega |: \Omega \subset \Sigma$ and Ω is a cover of $X\}$. Since X is fuzzy compact, $N(\Sigma)$ is a positive integer. Let $H_t(\Sigma, X)[H_t(\Sigma)$, for short$] = \log N_X(\Sigma)$. Then, İsmail Tok [11] has shown that, for a fuzzy compact topological space and fuzzy continuous function ψ, $h_t(\psi, \Sigma, X) = \lim_{n\to\infty} \frac{1}{n} H_t(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$ (called the fuzzy topological entropy of the fuzzy continuous function ψ with respect to the open cover Σ) exists. And $h_t(\psi, X) = sup\{h_t(\psi, \Sigma, X) : \Sigma$ is an open cover of $X\}$ is called the fuzzy topological entropy of the fuzzy continuous function ψ.

3. Fuzzy topological entropy

Definition 3.1. Let (X, δ) be a fuzzy topological space and let $\psi : X \to X$ be a fuzzy continuous function. Then the pair (X, ψ) is called a fuzzy dynamical system. If X is fuzzy compact, (X, ψ) is called a fuzzy compact dynamical system.

Definition 3.2. Let (X, ψ) be a fuzzy dynamical system, let Σ be a fuzzy open cover of X, and let K be a non-empty fuzzy compact subset of X such that $\psi(K) \leq K$. Let $S_K(\Sigma)$ be the smallest cardinality of all subcovers (for K) of Σ, i.e., $S_K(\Sigma) = min\{| \Omega |: \Omega \subset \Sigma, K \leq \vee\Omega\}$. Since K is fuzzy compact, $S_K(\Sigma)$ is a positive integer. Let $H_K(\Sigma) = \log S_K(\Sigma)$.

Let Σ and Ω be two open covers of X. Define their join by $\Sigma \vee \Omega = \{U \wedge V : U \in \Sigma, V \in \Omega\}$. Clearly, the join $\Sigma \vee \Omega$ is an open cover of X. It is well known that Ω is called a refinement of Σ (denoted by $\Sigma \prec \Omega$) if, for each $V \in \Omega$, there exists an $U \in \Sigma$ such that $V \leq U$.

Theorem 3.3. *Let (X, ψ) be a fuzzy dynamical system, let Σ and Ω be open covers of X, and let K be a non-empty compact subset of X such that $\psi(K) \leq K$. Then the following hold.*

(a) $H_K(\Sigma) \geq 0$.
(b) *$\Omega \prec \Sigma$ implies that $H_K(\Sigma) \leq H_K(\Omega)$.*
(c) $H_K(\Sigma \vee \Omega) \leq H_K(\Sigma) + H_K(\Omega)$.
(d) $H_K(\psi^{-1}(\Sigma)) \leq H_K(\Sigma)$. *When $\psi(K) = K$, the equality holds.*

Proof. The proofs of (a), (b), and (c) are straightforward and so we only prove (d).

Let $H_K(\Sigma) = n$. Then there exists a subcover $\{U_1, U_2, \ldots, U_n\}$ (for K) of Σ. Since ψ is fuzzy continuous, $\{\psi^{-1}(U_1), \psi^{-1}(U_2), \ldots, \psi^{-1}(U_n)\}$ is a subcover (for $\psi^{-1}(K)$) of $\psi^{-1}(\Sigma)$. Again, since $\psi(K) \leq K$, i.e., $K \leq \psi^{-1}(K)$, $\{\psi^{-1}(U_1), \psi^{-1}(U_2), \ldots, \psi^{-1}(U_n)\}$ is a finite fuzzy open subcover (for K) of $\psi^{-1}(\Sigma)$. Therefore, $H_K(\psi^{-1}(\Sigma)) \leq n = H_K(\Sigma)$.

Now, suppose that $\psi(K) = K$ and $H_K(\psi^{-1}(\Sigma)) = p$. Let $\{\psi^{-1}(V_1), \psi^{-1}(V_2), \ldots, \psi^{-1}(V_p)\}$ be a finite open subcover (for K) of $\psi^{-1}(\Sigma)$. Therefore, $K \leq \vee\{\psi^{-1}(V_i) : i = 1, 2, \ldots, p\}$, and so $\psi(K) \leq \psi(\vee\{\psi^{-1}(V_i) : i = 1, 2, \ldots, p\}) = \vee\{\psi(\psi^{-1}(V_i)) : i = 1, 2, \ldots, p\} \leq \vee\{V_i : i = 1, 2, \ldots, p\}$. Since $\psi(K) = K$, $\{V_i : i = 1, 2, \ldots, p\}$ is a fuzzy open subcover (for K) of Σ. Hence $H_K(\Sigma) \leq p$, and so, by the first part of (d), $H_K(\psi^{-1}(\Sigma)) = H_K(\Sigma)$. □

Theorem 3.4. *Let (X, ψ) be a fuzzy compact dynamical system, let Σ be a fuzzy open cover of X, and let K be a non-empty compact subset of X such that $\psi(K) \leq K$. Then $\lim_{n\to\infty} \frac{1}{n} H_K(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$ exists.*

Proof. Let $t_n = H_K(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$. From the results given in [14] and [15], it is sufficient to show that $t_{n+p} \leq t_n + t_p$. Theorem 3.3(d) implies that $H_K(\psi^{-1}(\Sigma)) \leq H_K(\Sigma)$, and so $H_K(\psi^{-i}(\Sigma)) \leq H_K(\psi^{-j}(\Sigma))$ for $i = 0, 1, 2, \ldots, n$, and for $j \leq i$. Hence $t_{n+p} = H_K(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n+p-1\}) = H_K((\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) \vee (\vee\{\psi^{-i}(\Sigma) : i = n, n+1, \ldots, n+p-1\}) \leq H_K((\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})) + H_K((\vee\{\psi^{-i}(\Sigma) : i = n, n+1, \ldots, n+p-1\})) = H_K((\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})) + H_K(\psi^{-i}(\vee\{\Sigma : i = n, n+1, \ldots, n+p-1\})) \leq H_K((\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})) + H_K(\psi^{-j}(\vee\{\Sigma : j = 0, 1, 2, \ldots, p-1\})) = H_K((\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})) + H_K((\vee\{\psi^{-j}(\Sigma) : j = 0, 1, 2, \ldots, p-1\}))$. So $t_{n+p} \leq t_n + t_p$. □

Definition 3.5. Let (X, ψ) be a fuzzy topological dynamical system, let $C(X, \psi)$ be the set $\{K \in P(X) - \{\emptyset\} : K$ is fuzzy compact such that $\psi(K) \leq K\}$, and let Σ be a fuzzy open cover of X. Then, for $K \in C(X, \psi)$, $Ent^\star(\psi, \Sigma, K) = \lim_{n\to\infty} \frac{1}{n} H_K(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$ is called the fuzzy topological entropy of ψ on K relative to Σ and $Ent^\star(\psi, K) = sup\{Ent^\star(\psi, \Sigma, K) : \Sigma$ is a fuzzy open cover of $X\}$ is called the fuzzy topological entropy of ψ on K.

Theorem 3.6. *Let (X, ψ) be a fuzzy topological dynamical system, let $\underline{1} \in C(X, \psi)$, and let Σ be a fuzzy open cover of X. Then $Ent^\star(\psi, \Sigma, X) = h_t(\psi, \Sigma, X)$.*

Proof. Since $Ent^\star(\psi, \Sigma, X) = \lim_{n\to\infty} \frac{1}{n} L_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$ and $h_t(\psi, \Sigma, X) = \lim_{n\to\infty} \frac{1}{n} H_t(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$, it is sufficient to show that $S_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) = N_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$. Let $N_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) = s$. Then there exists a finite subcover $\{U_1, U_2, \ldots, U_s\}$ of $\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}$ for X. Then, for each $j \in \{1, 2, \ldots, s\}$, there exist $U_j^0, U_j^1, U_j^2, \ldots, U_j^{n-1} \in \Sigma$ such that $U_j = \wedge\{\psi^{-i}(U_j^i) : i = 0, 1, 2, \ldots, n-1\}$. Let $V_j = \wedge\{\psi^{-i}(U_j^i) : i = 0, 1, 2, \ldots, n-1\}$. Then $U_j \le V_j$ for each $j \in \{1, 2, \ldots, s\}$, and so $\{V_1, V_2, \ldots, V_s\}$ is a finite subcover of $\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}$ for X. Thus $S_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) \le N_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$.

Conversely, let $S_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) = p$. Then there exists a finite subcover $\{G_1, G_2, \ldots, G_p\}$ of $\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}$ for X. Then, for each $j \in \{1, 2, \ldots, p\}$, there exist $G_j^0, G_j^1, G_j^2, \ldots, G_j^{n-1} \in \Sigma$ such that $G_j = \wedge\{\psi^{-i}(G_j^i) : i = 0, 1, 2, \ldots, n-1\}$. Then $\underline{1} = \vee\{\wedge\{(\psi^{-i}(G_j^i)) : i = 0, 1, 2, \ldots, n-1\}, j = 1, 2, \ldots, p\}$. Hence $N_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) \le p = S_X(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$. This completes the proof. □

Theorem 3.7. *Let (X, ψ) be a fuzzy topological dynamical system, let $K_1, K_2 \in C(X, \psi)$ with $K_1 \le K_2$, and let Σ be a fuzzy open cover of X. Then*

(i) *$Ent^\star(\psi, \Sigma, K_1) \le Ent^\star(\psi, \Sigma, K_2)$ and*
(ii) *$Ent^\star(\psi, K_1) \le Ent^\star(\psi, K_2)$.*

Proof. (i) Let $S_{K_2}(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) = p$. Then there exists a fuzzy open subcover $\{U_1, U_2, \ldots, U_p\}$ of $\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}$ for K_2. Since $K_1 \le K_2$, $\{U_1, U_2, \ldots, U_p\}$ is also a subcover of $\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}$ for K_1, and hence $S_{K_1}(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) \le p = S_{K_2}(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$. So $H_{K_1}(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) \le S_{K_2}(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\})$. Thus $Ent^\star(\psi, \Sigma, K_1) \le Ent^\star(\psi, \Sigma, K_2)$.

(ii) Here, by (i), $Ent^\star(\psi, K_1) = sup\{Ent^\star(\psi, \Sigma, K_1) : \Sigma$ is a fuzzy open cover of $X\} \le sup\{Ent^\star(\psi, \Sigma, K_2) : \Sigma$ is an open cover of $X\} = Ent^\star(\psi, K_2)$. □

Definition 3.8. Let (X, ψ) be a fuzzy topological dynamical system. Then $Ent^\star(\psi) = sup\{Ent^\star(\psi, K) : K \in C(X, \psi)\}$ is called the fuzzy topological entropy of ψ. For the special case $C(X, \psi) = \emptyset$, $Ent^\star(\psi) = 0$ is taken.

İsmail Tok [11] has defined fuzzy topological entropy only on fuzzy compact spaces, and we have defined the fuzzy topological entropy on arbitrary topological spaces. Now we establish that, in the case of fuzzy compact spaces, our definition of fuzzy topological entropy coincides with the fuzzy topological entropy of İsmail Tok [11].

Theorem 3.9. *Let (X, ψ) be fuzzy compact topological dynamical system. Then $Ent^\star(\psi) = h_t(\psi, X)$.*

Proof. Since X is fuzzy compact, $C(X, \psi) \ne \emptyset$. Now, by Theorem 3.7, $Ent^\star(\psi) = Ent^\star(\psi, X)$. Again, by Theorem 3.6, $Ent^\star(\psi, \Sigma, X) = h_t(\psi, \Sigma, X)$, and so $Ent^\star(\psi) = Ent^\star(\psi, X) = h_t(\psi, X)$. □

Theorem 3.10. *Let X be any fuzzy topological space and let $\imath : X \to X$ be the identity function. Then $Ent^\star(\imath) = 0$.*

Proof. Let Σ be any fuzzy open cover of X and let $K \in C(X, \imath)$. Then, by Theorem 3.3, $\vee\{\imath^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\} \prec \Sigma$ implies that $H_K(\vee\{\imath^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) \le H_K(\Sigma)$, and so $Ent^\star(\imath, \Sigma, K) = \lim_{n\to\infty} \frac{1}{n} H_K(\vee\{\imath^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\}) \le \lim_{n\to\infty} \frac{1}{n} H_K(\Sigma) = 0$, i.e., $Ent^\star(\imath, \Sigma, K) = 0$ for each $K \in C(X, \psi)$ and for each fuzzy open cover Σ of X. Therefore, $Ent^\star(\imath, K) = sup\{Ent^\star(\imath, \Sigma, K) : K \in C(X, \imath)\} = 0$, for each $K \in C(X, \psi)$, and hence $Ent^\star(\imath) = sup\{Ent^\star(\imath, \Sigma) : \Sigma$ is any fuzzy open cover of $X\} = 0$. □

Theorem 3.11. *Let X be any fuzzy topological space, let $\psi : X \to X$ be any fuzzy continuous function, and let $p \in \mathbb{Z}^+$. Then $Ent^\star(\psi^p) \ge p.Ent^\star(\psi)$. When $C(X, \psi^p) = C(X, \psi)$, the equality holds.*

Proof. If $C(X, \psi) = \emptyset$, then there is nothing to prove. So we take $C(X, \psi) \ne \emptyset$. It is obvious that $C(X, \psi) \le C(X, \psi^p)$. Let Σ be any fuzzy open cover of X and let $K \in C(X, \psi^p)$. Then $H_K(\vee\{(\psi^p)^{-i}(\vee\{\psi^{-j}(\Sigma)) : j = 0, 1, 2, \ldots, p-1\}, i = 0, 1, 2, \ldots, n-1\}) = H_K(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, np-1\})$. Let $\Omega = \vee\{\psi^{-j}(\Sigma) : j = 0, 1, 2, \ldots, p-1\}$. Then $Ent^\star(\psi^p, \Omega, K) = \lim_{n\to\infty} \frac{1}{n} H_K(\vee\{(\psi^p)^{-i}(\Omega) : i = 0, 1, 2, \ldots, n-1\}) = \lim_{n\to\infty} \frac{1}{n} H_K(\vee\{(\psi^p)^{-i}(\vee\{\psi^{-j}(\Sigma) : j = 0, 1, 2, \ldots, p-1\}) : i = 0, 1, 2, \ldots, n-1\}) = \lim_{n\to\infty} \frac{1}{n} H_K(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, np-1\}) = p.\lim_{n\to\infty} \frac{1}{np} H_K(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, np-1\}) = p.Ent^\star(\psi, \Sigma, K)$. Thus $Ent^\star(\psi^p) \ge Ent^\star(\psi^p, K) \ge Ent^\star(\psi^p, \Omega, K) = p.Ent^\star(\psi, \Sigma, K)$, for every fuzzy open cover Σ of X and for every $K \in C(X, \psi)$, and hence $Ent^\star(\psi^p) \ge p.Ent^\star(\psi)$. □

Now, let $C(X, \psi^p) = C(X, \psi)$. If $C(X, \psi) = \emptyset$, then $Ent^\star(\psi^p) = 0 = Ent^\star(\psi)$. So we consider $C(X, \psi) \neq \emptyset$. Let Σ be any fuzzy open cover of X and let $K \in C(X, \psi) = C(X, \psi^p)$. Since $\vee\{(\psi^p)^{-i}(\Sigma) : i = 0, 1, 2, \ldots, n-1\} \prec \vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, np-1\}$, $Ent^\star(\psi^p, \Omega, K) = \lim_{n\to\infty} \frac{1}{n} H_K(\vee\{(\psi^p)^{-i}(\vee\{\psi^{-j}(\Sigma) : j = 0, 1, 2, \ldots, p-1\}) : i = 0, 1, 2, \ldots, n-1\}) \leq p.\lim_{n\to\infty} \frac{1}{np} H_K(\vee\{\psi^{-i}(\Sigma) : i = 0, 1, 2, \ldots, np-1\}) = p.Ent^\star(\psi, \Omega, K)$. So $Ent^\star(\psi^p, \Omega, K) \leq p.Ent^\star(\psi, \Omega, K)$, for every fuzzy open cover Σ of X and for every $K \in C(X, \psi)$, and hence $Ent^\star(\psi^p) \leq p.Ent^\star(\psi)$. Therefore, by the first part of the theorem, $Ent^\star(\psi^p) = p.Ent^\star(\psi)$.

Acknowledgment

The authors are grateful to the learned referee for his constructive suggestions, which improved the paper to a great extent.

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