Abstract

The conductivity and temperature of hypervelocity projectiles were studied using a novel magnetic diffusion analysis technique and five color radiometry. A copper shaped charge jet was fired through an electromagnetic coil to saturate the tip with a magnetic field. After leaving the coil, the tip passed through a series of sensing coils that are used to detect the magnetic field’s decay rate, which depends on the geometry of the tip and its electrical conductivity. A mathematical model was then used to calculate the decay rate of the magnetic field from the tip and the signals produced by the sensing coils to determine the electrical conductivity. The resulting conductivity indicated that the tip of the shaped charge jet has a bulk temperature on the order of 1200 K. The experimental data were directly compared to temperature distributions generated by hydrocodes using both the Johnson-Cook and Steinberg-Guinan-Lund strength models. Because the magnetic diffusion technique is extremely versatile, various projectiles over a broad dynamic range can be investigated. Thus, this measurement is compared to radiometry and diffusion measurements of explosively formed penetrators.

Keywords: Shaped charge jet temperature; Johnson-Cook strength model; Steinberg-Guinan-Lund strength model; ALEGRA; Hypervelocity projectiles;

1. Introduction

Direct measurement of the state of matter under extreme conditions provides vital data for shock physics and hydrocode validation. As reliance on modeling with hydrocodes continues to increase, it is critical to provide experimental validation and data for continued enhancement of such codes. We present data on multiple threats and material characterization over a broad dynamic range, which is used to directly test material models as currently implemented in shock physics codes. We have conducted the first direct determination of the electrical conductivity of a shaped charge jet in flight and thus its temperature.

To a large extent, the measurement of the fundamental physical and material properties of a jet after formation has remained elusive. Radiometry measurements performed in the late 1970s showed surface temperatures of a copper jet varying from around 750 K to well above the melt temperature [1]. Thus, the two-color radiometry measurements left significant uncertainty. A “soft catch” method of recovering jet particles showed that the outer portion of a jet was indeed solid but found evidence of melt, such as voids, spheres, and columnar microstructures in the central 10% of the jet diameter [2, 3]. However, precise behavior of the jet material remains uncertain. Therefore, the direct measurement of such properties would provide a significant test of simulations from shock physics codes, such as ALEGRA [4] and CTH [5, 6].
2. Experimental technique

2.1. Magnetic diffusion analysis technique

This method exploits magnetic diffusion into a moving conductor in combination with analysis using an equivalent circuit model [7] and ALEGRA simulations to extract conductivity and temperature characteristics. When a conductor is placed in a magnetic field, eddy currents are established in the conductor to keep the magnetic flux inside the conductor constant. Thus, the rate at which the field penetrates (i.e., diffuses or “soaks”) into the conductor depends on how long the currents last, which is a function of the electrical conductivity (the higher the conductivity, the slower the field penetrates). The total time needed to penetrate the conductor also depends on the material thickness. When the object is removed from the external field, the same diffusion time is required for the “soaked in” field to leave the conductor. This magnetic diffusion process is governed by the following equation:

$$\tau = \alpha \sigma_c d^2,$$

where $\tau$ is the magnetic diffusion time, $\alpha$ is a proportionality constant, $\sigma_c$ is the electrical conductivity, and $d$ is the dimension of the conductor (i.e., diameter of a cylinder or thickness of a sheet). By firing a projectile through an electromagnetic “soak” coil to saturate the projectile with a magnetic field, one can observe the decay of the magnetic flux via sensing coils spaced along the path following the soak coil, as shown in figure 1. With known projectile dimensions, the observed decay gives a direct measurement of the bulk electrical conductivity and thus a corresponding average temperature.

![Diagram of the magnetic diffusion analysis technique](image)

Fig. 1. A schematic of the magnetic diffusion analysis technique. A projectile moves at velocity $v$ through a coil of magnetic field strength $B$. As the projectile, with the then embedded field, leaves the soak coil, the secondary sensing coils pick up the change in flux.

For objects traveling at sufficient velocities to generate a significant bow shock, it is necessary to perform the experiments in vacuum to avoid the significant electronic noise that would be generated in the sensing coils by the strong shock wave. Additionally, a standoff for shaped charge jets is required such that the jet has begun to particulate, and the tip is separated from the body of the jet. Thus, the jet tip is stable in form and size. Because the technique is extremely sensitive to the dimensions of the projectile, high resolution x-rays and detailed image analysis are required. This data can then be implemented in a mathematical model to fit the decay rate of the magnetic field from the projectile to the signals produced by the sensing coils. We utilized both a two-dimensional (2-D) axisymmetric equivalent circuit model developed by Hummer [7] as well as the magnetohydrodynamic version of the shock physics code ALEGRA.[4]

2.1.1 Equivalent circuit model

The equivalent circuit model is an approach where a conductor is divided up into a number of elements having a uniform current density along the element length. Each element is treated as a part of an equivalent electronic circuit having its own resistance, self-inductance, and mutual inductance with every other element. For cylindrically symmetric problems where the current is azimuthally distributed, it is possible to divide the conductors into a number of concentric rings, each carrying a uniform current density. In this manner, the equivalent circuit model is utilized to find the current distribution in the projectile as it passes through the soak coil and sensing coils. Using this current distribution, the magnetic field of the object and the voltage induced in the sensing coil can be determined. The conductivity is adjusted in the model for the best fit to the observed signals. The temperature is determined from empirical data available relating temperature and resistivity or conductivity for the particular conductor.

2.1.2 Modeling with other transient magnetic codes

A code such as ALEGRA allows the direct insertion of the shape of the particle of interest and the application of a time varying field to replicate the actions of the soak coil. The calculated magnetic flux that the sense coil would observe in the simulation is simply the line integral of the magnetic vector potential at the location of each coil. This can be directly compared to the peak of the integral of the experimental signal from the sensing coils. Again, the conductivity or
temperature can be varied to achieve a best fit to the experimental data. However, a temperature distribution can also be inserted on the object allowing a better comparison to hydrocode simulations such as jet formation.

2.1.3 Magnetic diffusion analysis technique verification and sensitivity

To verify the magnetic diffusion analysis technique, a copper cylinder of 25.4-mm diameter and 15.5-mm length was encapsulated in epoxy (to protect from damage and frictional heating) and shot through a 32-mm tube. In this case, low velocities were used to check the method on an object of known temperature, i.e., room temperature. The cylinder was accelerated using compressed air to a velocity of 7.4 m/s. The tube contained a soak coil with a field of 55 mT at the center and eight pick-up or sensing coils spaced beyond the soak coil. The experimental response of the system is shown in figure 2a. There is a very rapid decay in the embedded field within the cylinder as observed by the sensing coils. Careful planning of the measurement system must be made in order to correlate projectile size and velocity with the desired system response. Magnetic flux response from very slow or small particles is therefore extremely difficult to capture. The response of the same projectile traveling at 91.0 m/s in a slightly different coil configuration captures more data points throughout the decay (as opposed to figure 2a where the majority of the captured data is in the tail of the decay) and is shown in figure 2b.

In this test case, ALEGRA was utilized with a uniform conductivity of $5.8 \times 10^7$ (\si{\Omega m})$^{-1}$, which is the measured room temperature conductivity of the copper rod stock used to make the cylinder. Various copper temperatures were also used in ALEGRA for comparison (note that the copper model in ALEGRA is for pure copper which is slightly more conductive than the measured rod stock value). The solid black lines in figure 3a and 3b result from a simulation using the measured conductivity and matches the experimental data extremely well (the black filled circles represent the peak value from integrating the sensing coil data of figure 2). This is exactly what is expected because the physics that governs the decay of the induced currents in the projectile is the exact same that governs a traditional voltage versus current measurement to determine the conductivity of the rod stock or other conductor for that matter.

The other trends for temperatures from ALEGRA simulations are also shown in figure 3 for comparison. They clearly show that using this method and choosing a “best fit” for an object of unknown temperature (or electrical conductivity) can achieve temperature accuracy well within 50 K.

![Fig. 2](image_url) Decay of the embedded magnetic field in a 295 K copper cylinder with velocities of (a) 7.4 m/s and (b) 91.0 m/s as observed by the sensing coils.

![Fig. 3](image_url) Decay of the embedded magnetic field in a 295 K copper cylinder as observed using the magnetic diffusion analysis technique with velocities of (a) 7.4 m/s and (b) 91.0 m/s.
For an object that does not have uniform temperature, such as a shaped charge jet or explosively formed penetrator (EFP), it is important to consider what portion of the projectile this method samples. Figure 4 shows the currents in an example of a solid jet tip generated by the equivalent circuit model. The majority of the induced current to sustain the field in the jet tip is located in the outer 50 to 60% of the tip. Therefore, the magnetic diffusion technique only samples that outer portion of the jet. For an object of uniform temperature, this method gives a direct measure of conductivity. However, if the temperature distribution is unknown, it can only give an average or bulk temperature of the outer ~50% by volume.

![Figure 4](image1.jpg)

**Fig. 4.** Distribution of induced current in the jet tip as predicted by the equivalent circuit model.

2.2. Five color radiometry

Planck's radiation formula gives the intensity of light emitted by an object as a function of wavelength and temperature. However, the light intensity is also governed by the emissivity (simply one for a perfect radiator), which is not known for most explosively driven objects. Here, the radiometer uses the ratios of the light emitted at five differing wavelengths. The temperature is inversely proportional to the natural log of the intensity ratio. This ratio method completely eliminates the need for estimating the emissivity of the source and makes the temperatures obtained independent of changes in emissivity with time and temperature. Multiple ratios give an estimate of error and allow the detection of abnormalities at any one of the given wavelengths such as absorption in the medium between the object and detector. If there is a change in emissivity as a function of wavelength, this uncertainty is contained within the error measured using the multiple ratios.

The light is collected via an interchangeable zoom lens and fed through a beamsplitter assembly to allow the simultaneous measurement of multiple wavelengths as shown in figure 5. Because the amount of light observed from an object is dependent both on its size and time in the field of view, accurate measurements using this technique are limited to explosively formed penetrators (EFPs), which are larger and slower than jets, or other types of scenarios such as explosive fireballs.

![Figure 5](image2.jpg)

**Fig. 5.** Schematic of the five channel radiometer assembly.
3. Temperature of a copper shaped charge jet

3.1. Set up and jet tip radiograph image analysis

A bare shaped charge having a tip velocity of 9.15 km/s is fired in vacuum (on the order of 1 to 4 Pa) through a 300-mm-long, 32-mm-diameter electromagnetic coil having a magnetic field at the center of 40 mT. After leaving the coil, the tip passes through a series of eight pick-up coils, each having 50 turns and a 270-Ω damping resistor. The standoff from the soak coil is chosen to correlate with the beginning of jet particulation and where the jet tip has necked down sufficiently to break away from the body. Thus while traversing the soak coil and the pick-up coils, the jet tip dimensions are quite stable. An x-ray image is taken just after the jet tip has emerged from the soak coil and before entering the pick-up coils, which begin 90 mm from the end of the soak coil. The experimental setup is shown in the radiograph of figure 6.

Detailed characterization of the jet tip profile is obtained from the x-ray images using copper rods with diameters 6.35 and 3.2 mm (slightly larger and slightly smaller than the jet tip) as scales. Line scans across the image of the copper rods are used to determine the number of pixels per millimeter in the image. The rod width is here defined to be the number of pixels along the line scan from the two points where the image intensity changes by >2.5 standard deviations from the mean image background. The average of the scaling obtained from the two rods is then used to determine the pixel size employed for the jet tip dimensions. The image of the jet tip is then truncated such that the pixels with intensities < 2.5 standard deviations from the mean background are given a value of zero intensity (black), while all others are given a large value (white). Thus, a black and white image of the jet tip is obtained, and outliers can then be removed, as depicted in figure 7a. The width of the tip is taken at each pixel along its length to determine a radius to import into the equivalent circuit model. The resulting jet tip profiles used in this study are shown in figure 7b.

![Jet fired in vacuum](image)

**Fig. 6.** Radiograph of the experimental setup as the shaped charge jet just emerges from the soak coil.

![Cu rods as x-ray scale](image)

![Soak Coil](image)

![Pick-up Coils](image)

![Shaped Charge Jet Tip](image)

![Jet is fired in vacuum](image)

![Cu rods as x-ray scale](image)

![Soak Coil](image)

![Pick-up Coils](image)

![Shaped Charge Jet Tip](image)

![Jet is fired in vacuum](image)

![Cu rods as x-ray scale](image)

![Soak Coil](image)

![Pick-up Coils](image)

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![Jet is fired in vacuum](image)

![Cu rods as x-ray scale](image)

![Soak Coil](image)

![Pick-up Coils](image)

![Shaped Charge Jet Tip](image)

**Fig. 7.** (a) Radiograph of the jet tip, a truncated image at the 2.5 standard deviation level, and a final image with the outliers removed, which was used to establish the tip dimensions. (b) Jet tip profiles extracted from x-ray images of jets two different shots, showing nearly identical jet tips.
3.2. Bulk temperature results using equivalent circuit analysis

The data captured by the sense coils are displayed in figure 8a. In this case the jet tip diameter at the widest point is 5.1 ± 0.1 mm. A clear exponential decay is observed in the peak amplitudes. Time zero in this plot references the jet tip reaching the first pick-up coil. Figure 8b shows the best fit to the same experimental data (normalized to a maximum peak value of 1.0 for the first coil) using the equivalent circuit model. The resulting electrical conductivity with the best fit to the B-field decay is $1.28 \times 10^7 (\Omega \text{ m})^{-1}$. By comparing this conductivity with experimental conductivity and temperature dependence data for copper [9], the corresponding bulk temperature of the jet tip is 1190 K. As a comparison the ALEGRA method utilized in section 2.1.3 results in a bulk temperature of 1160 K. Thus, the two methods show significant agreement.

![Fig. 8. Experimental data resulting from a copper shaped charge jet using the magnetic diffusion analysis technique (a) with the best fit from the equivalent circuit model (b).](image)

The magnetic diffusion technique is quite sensitive to the jet tip diameter because the decay time depends on the square of the thickness of the conductor. The dependence of the jet conductivity and temperature with respect to jet tip diameter is shown in table 1. Thus an uncertainty of only 0.1 mm in the tip diameter results in an uncertainty in the average temperature on the order of 50 K for this jet.

<table>
<thead>
<tr>
<th>Jet Tip Diameter (mm)</th>
<th>Average Electrical Conductivity $\times 10^7 (\Omega \text{ m})^{-1}$</th>
<th>Corresponding Temperature (K)</th>
</tr>
</thead>
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<tr>
<td>4.9</td>
<td>1.37</td>
<td>1110</td>
</tr>
<tr>
<td>5.0</td>
<td>1.33</td>
<td>1140</td>
</tr>
<tr>
<td>5.1</td>
<td>1.28</td>
<td>1190</td>
</tr>
</tbody>
</table>

4. Explodively formed penetrator

4.1. Radiometry Measurements

Temperature measurements were also performed using five-color radiometry on two different copper EFPs. EFPs undergo significantly less plastic work than a jet during formation, thus temperatures may be significantly different. A multi-particle EFP and a single particle EFP were tested. Because of light sensitivity and alignment issues only the largest particle or “shuttle cock” from the multi-particle EFP was observed. Figure 9 shows a snap shot from high speed video of the EFP and corresponding temperature traces using multiple wavelength ratios. Because intensities from some wavelengths happened to be very close (i.e., resulting in significant uncertainty due to the natural log of something close to one in the denominator), only six ratios were utilized. The average temperature measured, at the time the EFP shuttle cock was centered within the field of view of the radiometer, is 960 K. However, the temperatures range from 875 to 1030 K.
Fig. 9. Temperature measurements on a shuttle cock from an EFP with corresponding high speed video snap shot.

Data from the single particle EFP are shown in figure 10. The EFP was fired with a stand-off of 2.5 meters with the front of the radiometer lens 400 mm from the shot line. Based on the size and velocity of the EFP particle (76 mm long and 2.2 mm/μs) passing through a 20-mm field of view, the particle should be visible for about 45 μs. Therefore, if it was centered vertically in the field of view, the signal intensity should level out for approximately 22 μs. Only about 10 μs of leveling is observed in figure 10b. Thus, only a portion of the EFP was visible in the field of view, resulting in lower light intensity and higher noise in the temperature measurements. The temperature was averaged during the center 10 μs of the peak intensity, using the same six ratios as the multi-particle EFP. The resulting average temperatures are 910 +/- 100 K, 940 +/- 120K, 880 +/- 80 K, 960 +/- 190 K, 880 +/- 130 K, and 860 +/- 100 K with an average of 900 K. Though there is significant noise, the averaged temperature for each ratio only had a standard deviation of 50 K.

Fig. 10. Single particle EFP temperature measurement (a) with corresponding raw output data from the radiometer photodetectors (b).

While the two EFP experiments agree well with one another and the resulting temperature is significantly lower than a jet tip as expected, it is important to understand the limitations of this method. Radiometry is strictly a surface measurement. Therefore, any heating due to the bow shock of the particle would be observed in this surface measurement. While the shock wave is not strong enough to generate a plasma as in the case of a jet, what contribution it may make to the radiometry surface measurement remains uncertain. Additionally, radiometry is most sensitive to the hottest object in the field of view. If there are other sources of heat present, it could skew the data. Occasionally, burning explosive is observed traveling with the particles in high speed video. Care was taken in this work to utilize long standoffs and precise timing of particle location to minimize such effects and in the case of the single particle EFP, it was shot through a 2 meter tube backfilled with helium to eliminate burning. Such burning would also be at a significantly hotter temperature than an EFP particle or any non-melted projectile.[8]
4.2. Magnetic diffusion analysis of an EFP

A multi-particle EFP was also investigated using the magnetic diffusion technique. The set up for the EFP was very similar to that of the jet temperature measurement. However, the length of the soak coil and spacing of sensing coils were much greater due to larger particle size and thus diffusion time. Additionally, the collapse of the vacuum and the resulting shock wave can traverse the length of the tube more rapidly than the particles, unlike the jet. The details of this experiment will be presented elsewhere. For the purpose of this work, initial results are presented for comparison to the EFP radiometry data and jet temperature.

The response of the sensing coils was captured from the lead particle of the EFP, which was traveling at 3.07 km/s. Choosing the lead particle eliminates noise from overlapping signals of later particles and eliminates ambiguity that might result from a hollow particle like the shuttlecock. X-ray radiographs show that the lead particle was rotating away from the magnetic field axis during flight. Thus, three dimensional (3D) magnetic diffusion calculations were performed in ALEGRA to account for the angular motion (approximately 5.5 degrees during the time the particle transitioned the sensing coils). The best fit to the experimental data for a uniform conductivity approximation results in a bulk temperature between 600 and 650 K, significantly cooler than the surface measurements! Figure 11 shows the experimental peak values from the sensing coils and the results of a 3D ALEGRA simulation with a uniform particle temperature of 600 K.

![Fig. 11. Magnetic diffusion measurements on the lead particle of an EFP with a comparison of a 3D ALEGRA simulation at 600 K.](image)

5. Hydrocode comparison

5.1. Temperature distribution from jet formation using ALEGRA

Of course, the bulk temperature of 1190 K ignores any temperature distribution, which surely exists within the jet. However, it shows clear evidence of a jet tip that is significantly hotter (by approximately 400 K) than radiometry surface measurements attempted in the 1970s [1]. It therefore seems valuable to insert a reasonable temperature distribution for the tip and observe the resulting behavior and comparison to the experimental diffusion data. The detonation of the shaped charge and the resulting jet formation were therefore simulated with ALEGRA using two different copper strength models, as outlined by Niederhaus and Uhlig[10], to obtain tip temperature profiles.

The temperature profiles resulting from 2D-axisymmetric simulations using the Johnson-Cook (JC) and Steinberg-Guinan-Lund (hereafter referred to as just Steinberg) strength models are shown in figure 12. Both simulations were allowed to run to 80 µs, at which point the jet tip has broken away from the body and is dimensionally stable and thus thermally stable. The temperature distributions are significantly different between the two models. The JC model shows a cool core and warmer outer regions (600 to 900 K with very localized regions at the front corners of the tip reaching 1100K). On the other hand, the Steinberg model results in a very hot core (with some temperatures exceeding melt, 1370K) and slightly cooler outer regions (on the order of 1150 K).
Because the purpose of this work is not an all inclusive study of the models, model details will not be presented here but are available in references 5, 11, and 12. It should, however, be noted that the jet velocities, particulation, and necking of the tip agree very well with one another and experimental observations.

The temperature distributions produced by the ALEGRA simulations were then applied to the experimentally observed jet tip and used in simulations as outlined in section 2.1.2 and 2.1.3 of this work. As one would expect because of the large temperature differences between the models, the results diverge appreciably (see figure 13). The lower temperatures and resulting higher conductivity of the JC model allow sustained current levels within the tip. Thus, the decay is much more gradual than the experimental data. However, the temperatures produced in the Steinberg based simulations produce magnetic diffusion nearly identical to the data and well within experimental uncertainties. Additionally, strong experimental evidence of melted cores of jet particles also gives support to the Steinberg results. [2,3]
6. Conclusion

Both magnetic diffusion and radiometry techniques can be effective at acquiring temperature measurements on high speed projectiles. However, both have some limitations, especially for small particles, and neither can sample the center of a particle volume. While radiometry is limited to surface only measurements, magnetic diffusion can sample farther into the projectile (50% or greater) and provide critical tests for temperature distributions predicted by simulations. Additionally, this work shows that magnetic diffusion analysis is a sound method to capture data on projectiles in the hypervelocity range, which is extremely challenging for radiometry due to the low temperature of the projectiles (on the blackbody scale) and detection speed in the infrared regime.

Utilizing these two techniques, temperature measurements of copper shaped charge jets and explosively formed penetrators have been achieved. Validation of the temperature response of the Steinberg strength model as implemented in ALEGRA has also been shown for a shaped charge jet. While the radiometry and magnetic diffusion data on EFPs are significantly different, it is not completely unexpected. The cause is most likely due to shock wave induced surface heating, for which radiometry is sensitive. In conclusion, our measurements show that a jet tip has a bulk temperature on the order of 1200 K, while the particles of an EFP are significantly cooler and most likely lie in the range of 600 K to 900 K.

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