Emergency evacuation model and algorithm in the building with several exits

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Abstract

For the problems of evacuation with several exits under the limiting condition of fire smoke and routes capacity, an evacuation model combining of heuristic algorithm and network flow control was established in this study. Taking into account of routes capacity constraints, the aim of emergency evacuation is to minimize the total evacuation time for all people. The optimal evacuation path group, evacuation time and the number of evacuation in the evacuation network can be acquired through updating the evacuation network constantly and finding optimal routes iteratively. An example was presented to show the effectiveness and feasibility of this model and algorithm, and it can be used to explore the method to determine the optimal evacuation plan, while the actual routes inequality was not established.

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Keywords: fire smoke, building with several exits, capacity constraints, evacuation model

Nomenclature

\text{\text{\text{\text{"G\text{"}}(V,E)}}} \text{ evacuation networks}
\begin{align*}
V & \text{ vertex set, including three sub sets } S, D \text{ and } V. \text{ The } S \text{ indicates source node; The } D \text{ indicates set of sink nodes,} \\
& D = \{D_k \mid k = 1,2,\ldots,K\}; \text{ The } V \text{ indicates middle node, } N = \{V_n \mid n = 1,2,\ldots,N\}
\end{align*}
\begin{align*}
E & \text{ set of arcs, and } e_{ij} \text{ indicates the arc between node } i \text{ and node } j \\
l_{ij} & \text{ length of arc } e_{ij} \\
h_{ij}(t) & \text{ walking speed among an arc } e_{ij} \text{ at time } t \\
t_{ij} & \text{ travel time among an arc } e_{ij} \\
c_{ij} & \text{ maximum capacity in a unit time among an arc } e_{ij}, \text{ person/time} \\
P_l & \text{ evacuation path, } l = 1,2,\ldots,L \\
PC_l & \text{ maximum capacity in an evacuation route, depending on minimum of } c_{ij} \text{ among the arcs of the evacuation route} \\
f_t & \text{ number of evacuees passing path in a unit time, person/time} \\
x & \text{ total number of evacuees}
\end{align*}

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1. Introduction

Nowadays, there are much taller and huger buildings in cities, and occupant evacuation problems become more important under building fires. It will render evacuees crowded easily once fire occurs, if occupant can’t evacuate from the building timely, and it is extremely easy to cause serious casualties [1]. Therefore, it is particularly necessary to solve how make an optimal evacuation plan and evacuate occupant effectively in the fire, and it also need further studied as a scientific problem [2].

In recent years, the research on emergency evacuation has been carried out, and many scholars have adopted different methods to study the problem, which can be generally classified into mathematical analysis and computer simulation [3]. Mathematical analysis method is based on the mathematical model, and the actual parameters of evacuation are converted into a mathematical model to be solved, which can be classified into macroscopic models and microscopic models [4]. The microscopic models, such as cellular automaton model, multi lattice model and probability model, consider the evacuees’ individual characteristics and interactions in evacuation process. However, the path selection of evacuees in the evacuation area can’t be solved easily, due the size of the building is large. The macroscopic models ignore the individuals’ behaviors in evacuation process and are based on network flow models, which can solve the problem of emergency evacuation paths [5]. Tjandra [6] proposed a single-source evacuation model, which a type of macroscopic models, to solve the routing problem. However, the capacity limit and priority of the path are also important to the choice of the evacuation paths. Chen [7] considered that evacuation routes could be calculated by Fast flow control algorithm, in order to get multiple optimal evacuation paths. Yang [8] established a mathematical model based on the minimum time of evacuation through giving priority to saturated shortest path. Additionally, some models have been reported on the optimization evacuation routes in case of fire smoke. Xie [9] proposed a shortest path model based on the definition of equivalent length under concentration of fire smoke and crowd density. Yuan [10] proposed that the evacuation speed of each arc in the evacuation network was expressed as a function of evacuation time and smoke diffusion, and built the optimal evacuation route algorithm.

The optimal evacuation paths can be calculated by above evacuation models, considering the uncertainty and dynamic of evacuation in some extent. However, different evacuation models are usually needed to build during different backgrounds. This work mainly considers the characteristic of the personnel density and fire smoke diffusion, and the optimal paths of evacuation in building with several exits are determined by minimizing total evacuation time. Additionally, this work explores the method to make the optimal evacuation plan, while the actual routes inequality is not established, and determines the significance of important arc in the evacuation paths.

2. Evacuation model

2.1. Assumption of the mathematical model

All tables should be numbered with Arabic numerals. Headings should be placed above tables, center justified. Leave one line space between the heading and the table. Only horizontal lines should be used within a table, to distinguish the column headings from the body of the table, and immediately above and below the table. Tables must be embedded into the text and not supplied separately. Below is an example which authors may find useful.

- It is a precautionary evacuation and all people to be evacuated obeying the command by evacuation plan.
- There are multiple exits and only one source, which capacity of every exit is limited.
- The capacity of every exit is transformed into an arc, and the capacities of other nodes are not limited.
- The speed of evacuees is not fixed among every arc in the evacuation network, and travel time among an arc is determined by the length of the arc and the speed of evacuees.
- The evacuation is required to preserve the FIFO (First-In-First-Out) property.
- Returning or cruising is not allowed.

If table footnotes should be used, place footnotes to tables below the table body and indicate them with superscript lowercase letters. Be sparing in the use of tables and ensure that the data presented in tables do not duplicate results described elsewhere in the article.

<table>
<thead>
<tr>
<th>$x_{it}$</th>
<th>number of evacuees for going through $P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{it}$</td>
<td>travel time required for going through $P_i$</td>
</tr>
<tr>
<td>$T$</td>
<td>total evacuation time</td>
</tr>
</tbody>
</table>

$x_{it}$

Number of evacuees for going through $P_i$.

$T_{it}$

Travel time required for going through $P_i$.

$T$

Total evacuation time.
2.2. Mathematical model

The model aims at the safety evacuation of all evacuation personnel in the shortest time, which considering the constraints of fire smoke and path capacity. The definitions of the evacuation model for solving the problem are as follows:

\[
\min T = \min \sum t_{ij}
\]  
\[\text{s.t:}\]
\[
\int_{t_i}^{t_j} h_y(t) dt = l_{ij}
\]
\[
l_{ij} = t_j - t_i
\]
\[
h_y(t) = h_0 \cdot \alpha_y \cdot e^{-\beta y t}
\]
\[
\sum_k \sum_l x_{lj} = x
\]
\[
\sum_{t=0}^{T} \sum_{l \in s} x_{lj}(t) = \sum_{t=0}^{T} \sum_{l \in D} x_{lj}(t) = x
\]
\[
T_{il} = T_{il} + \frac{x_{lj}}{PC_i}
\]
\[
c_y \geq 0, x_{lj} \geq 0, x \geq 0
\]

Eq. (1) is defined as the objective function, which indicates that all evacuees are completed for evacuation within shortest time in the evacuation network. Eq. (2) and (3) are recursive equations for the evacuation time, which evacuees are taken time \( t_{ij} \) at the speed \( h_y(t) \) through the arc \( l_{ij} \). Due the concentrations of smoke is changing during the evacuation paths, Eq. (4) is defined that the speed of evacuees is slowing down, and \( \alpha_y, \beta_y \) are both the coefficient of smoke damage. Eq. (5) and Eq. (6) are defined that evacuation have been done within total evacuation time, and the flow of evacuation network is ensured. Eq. (7) is defined that evacuation time in a path is constituted with the time of passing the arcs and the time of leaving exits. Evacuation parameters can’t be negative in Eq. (8).

3. Algorithm of evacuation model

3.1. Algorithm idea

Based on the network optimization method in Graph Theory, according to the objective function and constraint function in the mathematical model, the single-source-multi-exit evacuation heuristic algorithm is considered by influence of fire smoke and capacity of arcs to solve the NP (Nondeterministic Polynomial) problem. The shortest path to the exit was priority selected in execution process of this algorithm, and the capacity of path is fully utilized.

Because there are multiple exits in evacuation network, if the passing time of all exits is calculated and compared, the complexity of the algorithm will be increased. Thus the concept of the super endpoint is introduced to simplify complexity, for example \( D_a \) is regarded as a super endpoint[11], which is shown in Fig. 1.
The capacity of every exit is transformed into an arc, which capacity of arc is same and length of arc is zero, and each arc once again brings together a new virtual exit node, called a super endpoint. In Fig. 1(a), the $S_1$ is a source, the $D_1$, $D_2$ are exits, and $v$ is defined as a middle node. The numbers in parentheses denote respectively the length and the maximum capacity of each arc, and the maximum capacity of $D_1$ and $D_2$ in a unit time is 8 and 9. In Fig. 1(b), after the introduction of the super endpoint, $D_1$ and $D_2$ are connect with $D_0$, which are converted into a new evacuation network. Additionally, the application of the super endpoint is based that capacity of middle nodes are inexistence and congestion occurs only between the exit node and each arc.

Basic idea of algorithm: Under consideration of the influence of flue gas on the speed of evacuees, optimal path between the source to the super endpoint is calculated through Dijkstra algorithm. The path, capacity and maximum capacity of the path are recorded, while the path capacity of the evacuation network is updated. Optimal paths between the source to the super endpoint is calculated by the cycle in the updating evacuation network, until all of path have been calculated.

3.2. Algorithm step

According to the basic idea of the algorithm, the procedure of the algorithm is constructed as follows:

Setp1: Input transformed evacuation network $G(V, E)$, then make path number $l = 1$, path set $P = \emptyset$, and set of evacuation time $TP = \emptyset$.

Setp2: Make set of time required for going through path $PT_l$, and set of mark point $X$ which $n = 1, T_{(v)} = +\infty, \lambda_{(v)} = N$.

Setp3: While $D_n \in X$, the current optimal path is considered to have been calculated, and the evacuation time of the path is recorded, then go to Setp6. Else, go to Setp4.

Setp4: For $v_j$ of $v_n, v_f \in A$ and $v_j \notin X$, $t_j = T_{(v)}$, $t_j$ can be calculated by $\int_{t_{(v)}}^{t_j} h_j(t) dt = l_j$. If $t_j \geq T_{(v)}$, go to Setp5, else $T_{(v)} = t_j, \lambda_{(v)} = m$, go to Setp5.

Setp5: $\lambda_{(v)} = \min \left\{ T_{(v)} \right\}, P_{T_{(v)}} = T_{(v)}, X_{T_{(v)}} = X \cup \{ v_f \}, n = i, i = i + 1, h_j = h_{(v)} \cdot \alpha_j \cdot e^{-\beta_j \left( T_{(v)} - T_{(v)} \right)}, \text{return to Setp3}.$

Setp6: The path is recorded as $P_i, P = P \cup \{ P_i \}, TP = TP \cup \{ T_{(v)} \}, \text{go to Setp7}.$

Setp7: $PC_i = \min \left\{ c_{ij} \mid e_{ij} \in P_i \right\}, f_i = PC_i, F = F \cup \{ f_i \}, \text{go to Setp8}.$

Setp8: Update the maximum capacity among each arc, and make $c_{ij} = \left\{ \frac{c_{ij} - f_i \cdot e_{ij}}{c_{ij} \cdot e_{ij}} \Phi \right\}^P$. If $c_{ij} = 0$, the arc is saturated and deleted, go to Setp9.

Setp9: If the evacuation network is broken after updating, go to Setp10, else, $l = l + 1$ and return to Setp3.

Setp10: Output the sets of $P, TP$ and $F$.

According to the above algorithm steps, the total number of evacuation routes $M$ can be solved, which satisfy that $T_{p_1} \leq T_{p_2} \leq T_{p_3} \leq \ldots \leq T_{p_M}$. But the total of evacuation routes is not necessarily used in the actual evacuation, and only total of evacuation routes $m (m \leq M)$ can be chosen, thus it’s necessary to group the evacuees while selecting path. In principle, the shorter path to the evacuation should be allocated more evacuees. It’s proposed that the travel time of each path is equal to calculate quantitative evacuees for optimal evacuation plan. The relation function between total number of evacuees and total evacuation time is as follows.
The inequality systems to determine the actual evacuation paths are as follows [12].

\[
x = \sum_{i=1}^{m} x_i \left( T - T_{p_i} \right) = \sum_{i=1}^{m} f_i \left( T - T_{p_i} \right)
\]  
(9)

The inequality systems to determine the actual evacuation paths are as follows [12].

\[
x > \sum_{i=1}^{m} f_i \left( T_{p_i} - T_{p_i} \right), \quad x \leq \sum_{i=1}^{m} f_i \left( T_{p_i} - T_{p_i} \right)
\]  
(10)

The m can be solved by using the known data enumerated in the inequality, and the equation of total evacuation time can be written as:

\[
T = \frac{x + \sum_{i=1}^{m} f_i T_{p_i}}{\sum_{i=1}^{m} f_i}
\]  
(11)

4. Case study

The following example verifies the feasibility and effectiveness of the algorithm. In the evacuation network shown in Fig. 2, suppose we initially have 70 evacuees at source node \(S\), and the maximum capacities of exit nodes \(D_1\) and \(D_2\) in a unit time are 10 and 8. The initial velocity in source node \(S\) is 15, and length of arc \(l_{ij}\), coefficient of smoke damage \(\alpha_{ij}\) and \(\beta_{ij}\), maximum capacity \(c_{ij}\) among the arcs are shown in Table 1.

<table>
<thead>
<tr>
<th>Paths number</th>
<th>(P_1)</th>
<th>(T_{p_i})</th>
<th>(f_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S-2-5-6- (D_1)</td>
<td>15.7</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>S-1-4-7- (D_2)</td>
<td>17.6</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>S-1-5-6- (D_1)</td>
<td>27.4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>S-3-5-6- (D_1)</td>
<td>29.1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>S-3-5-8-7- (D_2)</td>
<td>39.1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>S-3-5-8- (D_1)</td>
<td>40.4</td>
<td>1</td>
</tr>
</tbody>
</table>

The number of chosen paths \(m\) is 4 enumerated after the data is guided into the model, which are as shown in Table 2.
to go through \( P_j \) can be calculated by Eq. (9), which \( x_{p_j} = 41, x_{p_i} = 24, x_{p_j} = 4, x_{p_i} = 1 \), and their total is same as the number of all evacuees, to prove the feasibility of the algorithm.

The range of the number of chosen paths \( m \) is found is \( [2, l-1] \). When the all evacuees Increase or decrease, the \( m \) will change at the same time. When \( x < \sum_{l=1}^{m} f_i \left( T_{p_{wi}} - T_{p_i} \right) \) or \( x < \sum_{l=1}^{m} f_i \left( T_{p_{wi}} - T_{p_i} \right) \), the Eq. (10) will become invalid.

Aiming at this problem, the corresponding solution is put forward. When \( x < \sum_{l=1}^{m} f_i \left( T_{p_{wi}} - T_{p_i} \right) \) and \( m = 2 \), the path whose length is shorter and capacity is huger will be chosen priority. For example, if the number of the evacuees is 6, the path S-2-5-6-Dk is the optimal evacuation path, due to shortest and huger capacity. When \( x > \sum_{l=1}^{m} f_i \left( T_{p_{wi}} - T_{p_i} \right) \) and \( m = l-1 \), the limiting establishment condition of the Eq. (10) should be calculated at first, and maximum number of evacuees can be calculated, which \( \text{Max}(x) = \sum_{l=1}^{m+1} f_i \left( T_{p_{wi}} - T_{p_i} \right) \). The maximum number of evacuees are distributed to the \( P_1, P_2, \cdots, P_m \) by the Eq. (9), and partial evacuation time \( T' \) can be calculated. Additionally, \( T' = T_{pw} \) and Prove as follows.

When \( m = M - 1 \), \( \text{Max}(x) = \sum_{l=1}^{m+1} f_i \left( T_{p_{wi}} - T_{p_i} \right) \) and \( \text{Max}(x) = \sum_{l=1}^{m+1} f_i \left( T' - T_{p_i} \right) \), thus,

\[
\sum_{l=1}^{m} f_i \left( T' - T_{p_i} \right) = \sum_{l=1}^{m+1} f_i \left( T_{p_{wi}} - T_{p_i} \right)
\]

\[
\Rightarrow \sum_{l=1}^{m} f_i T' = \sum_{l=1}^{m+1} f_i T_{p_{wi}} - \sum_{l=1}^{m+1} f_i T_{p_i}
\]

\[
\Rightarrow \sum_{l=1}^{m} f_i T' = \sum_{l=1}^{m+1} f_i T_{p_{wi}} - \sum_{l=1}^{m+1} f_i T_{p_{wi}} = \sum_{l=1}^{m} f_i T_{p_{wi}}
\]

\[
\Rightarrow T' = T_{p_{wi}} = T_{pw}
\]

Thus, the total evacuation time is also depended on surplus evacuees \( \Delta x \) and its corresponding increase time \( \Delta T \). Due \( T' = T_{pw} \), the numerical value of \( \Delta T \) is depended on distribution of surplus evacuees. In order to decrease total evacuation time, each group of evacuees should arrive on super endpoint at the same time, and the allocation of evacuees should be proportional to the flow of each path. For example, if the number of the evacuees is 171 and the maximum number of evacuees is calculated to be 160, which \( \Delta T \) is 40.4 and \( \Delta x \) is 11. The surplus evacuees \( \Delta x \) are allocated by 3, 2, 2, 1, 2, 1 to each path, and increased time is 1. Thus \( T = T' + \Delta T = 40.4 + 1 = 41.4 \), and \( T = T' + \Delta T = 40.4 + 1 = 41.4 \).

In the case, the arc 5-6 and 6-1 are included in three actual evacuation paths, and flows are 6, accounting for 75% of the total evacuation flow. This kind of arc section should be defined as "important arc", which is important in evacuation. Thus, the channels like "important arc" should be checked earnestly, in order to prevent the occurrence of channel jams and failure of lighting facilities, etc. In the evacuation process, the security guards who are responsible for command and guidance should be arranged in the channels like "important arc", to prevent the occurrence of retention of evacuees, thereby affecting the overall evacuation.

5. Conclusions

In this paper, the problems of the evacuation with several exits under limiting condition of fire smoke and routes capacity are studied, and an evacuation model is established by network flow control and heuristic algorithm. Minimizing the total evacuation time for all evacuees is the aim of evacuation model. The optimal evacuation path, evacuation time and the number of evacuees in each evacuation group can be calculated through updating the evacuation network constantly.

In the case study, the result verified the effectiveness and feasibility of this model and algorithm. And it has proposed the method to determine the optimal evacuation plan, when the number of evacuees changes, causing the actual routes inequality invalid.

Through the analysis of the actual evacuees in each evacuation path, finding "important arc" in evacuation network can contribute to provide references on key positions in daily fire inspection and guiding crowd during evacuation process.

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