Numerical analysis of drag and lift reduction of square cylinder

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ABSTRACT
Flow around an extended triangular solid (thorn) attached to a square cylinder is investigated numerically. The numerical analysis is carried out at low Reynolds number, $Re = 100 \& 180$ for different non-dimensional thorn lengths ($l' = 0.2, 0.4 \& 0.6$), different inclination angles ($\theta = 5^\circ, 10^\circ, 15^\circ \& 20^\circ$) and two different thorn positions. It is found that drag and lift reduction can be achieved by attaching the thorn on a square cylinder. It is observed that the fluctuation of the drag force as well as the lift force is reduced and there is a comparatively large variation of drag and lift when the thorn is placed at the front stagnation point instead of placing at rear stagnation point. The reduction of drag and lift coefficient are directly proportional to thorn length and thorn inclination angle. It is found that the drag and lift are minimized by $16\% \& 46\%$ for $Re = 100$ respectively, and $22\% \& 60\%$ for $Re = 180$ compared to a square model (without thorn).

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1. Introduction
The analysis of fluid flow around bluff bodies is a subject receiving great attention by the research community due to the vast influence of flow-induced vibrations in a huge number of systems, including solar heating systems, offshore structures, heat exchangers, industrial chimney stacks, etc. Their wakes generate giant unsteady forces that have the potential to violently harm the structure of the bluff body. For this reason, several strategies have been planned over the recent years to regulate the wake vortex dynamics with the objective of weakening the vortex shedding and diminishing the amplitude of the fluctuating lift and also the drag [3,4]. The most pertinent feature of the flow, at moderate values of the Reynolds number ($Re$) is divided into three subcategories. One is the steady flow with two-dimensional characteristic, where the value of $Re$ (based on the free stream velocity and the characteristic dimension of the body) is below 50. Second is the unsteady flow with two-dimensional features which is the instability within the symmetric wake and also the onset of a time-periodic regime, characterized by alternate vortex shedding, called the von Karman vortex street, whose dimensionless amount depends on the $Re$ and the value is near about 180 [18,19,21]. The progression from two dimensional to three dimensional is found to be occurred at a value of near about 180 (in ideal conditions the critical value is about 194) [21]. The three dimensional structure and changeover to turbulent region of flow is developed in the range of $Re$, $190 \leq Re \leq 330$ [12]. They have reported the regime by conducting numerical solution with spectral element method.

The effect of domain height (i.e. the gap between two parallel plates, $H$) containing a circular cylinder on force coefficients and Strouhal number ($St$) has been studied [17] and found that increasing of drag coefficient & $St$ is related to decrease in $H$ and reducing of lift coefficient is associated with decrease in $H$. The numerical analysis of flow past a finite circular cylinder has been simulated numerically the pyramidal wake & three dimensional von Karman vortex street [8]. The value of laminar separation $Re$ ($Re_c$) and separation angle at $Re$, has been reported for the first time for steady flow over a square cylinder at low Reynolds number [15].

In foregoing studies, different strategies such as splitter plate [1,14] and base bleed [2] has been used efficiently to reduce the drag of bluff bodies to make them more consistent with different applications. Due to increment in the wake length, there is a reduction of drag obtained. A new geometrical shape of wavy stagnation face of a square cylinder has been found to be an effective model of noteworthy drag reduction of a square cylinder at different $Re$ [3]. A square cylinder having wavy stagnation face can reduce drag about 3% at $Re = 40$ and about 34% at $Re = 500$
compared to a nonwavv square cylinder. The proposed waviness on
the square cylinder effects on the redistribution of vorticity which
leads to the breakdown of the unsteady and staggered Karman
vortex wake into a steady and symmetric near-wake structure [4].
They found that the waviness of a square cylinder is effective to
reduce drag of 16% at Re = 100. A recent experimental analysis of
fluid flow over a grooved surface circular cylinder has been done
[13] and found a significant reduction in drag at even very
groove depths. An another approach called as control plate to
minimize the drag of a square cylinder over a range of Re 50–200
has been favorably implemented [11] and found a progressive in-

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3. Mathematical formulation

The governing equations for incompressible, 2-D unsteady
laminar flow across a bluff body are the continuity, the x and y
components of the Navier–Stokes equation. Assuming negligible
dissipation, the governing equations of the present numerical study
in a rectangular co-ordinate system are given below:

Continuity equation:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  (1)

x – Momentum equation:

\[ \frac{\partial u}{\partial t} + (u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  (2)

y – Momentum equation:

\[ \frac{\partial v}{\partial t} + (u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \]  (3)

The fluid properties (viscosity, \( \mu \), density, \( \rho \)) of the streaming
fluid (air) are dependent on the Reynolds number (Re = \( \frac{\mu U_\infty}{\rho} \)) and
thereby the flow equations are solved. The drag, lift and pressure
coefficients are evaluated by using the viscous and pressure forces
acting on the cylinder. The dimensionless variables are defined as
follows:

\[ u = \frac{\pi}{U_\infty}, \quad v = \frac{\varpi}{V_\infty}, \quad x = \frac{x}{D}, \quad y = \frac{y}{D}, \quad p = \frac{p}{\rho U_\infty^2}, \quad t = \frac{U_\infty t}{D} \]  (4)

Where \( \varpi \) and \( \tau \) are the velocity components in the \( x \) and \( y \) directions respectively, \( p \) is the pressure, \( t \) is the time.

The drag coefficient is given by:

\[ C_D = \frac{F_D}{0.5 \rho U_\infty^2 D} \]  (5)

Where, \( F_D \) is the drag force acting on the cylinder.

The force exerted on the cylinder by the periodic fluctuations of
flow is characterized by the lift coefficient and is given by:

\[ C_L = \frac{F_L}{0.5 \rho U_\infty^2 D} \]  (6)

Where \( F_L \) is the lift force which acts on the cylinder in lateral direction.

The pressure coefficient is a parameter for studying the flow of
incompressible fluids such as water, and also the low-speed flow of
compressible fluids such as air. The relationship between the
dimensionless coefficient and the dimensional numbers is given as:

![Fig. 1. A schematic diagram of the problem description.](image-url)
\[ C_p = \frac{p - p_\infty}{0.5 p_\infty U_\infty^2} \]  

(7)

Where \( p \) is the pressure at the point at which pressure coefficient is being evaluated.

Using the Bernoulli’s equation, the pressure coefficient can be further simplified for incompressible, lossless, and steady flow:

\[ C_p = 1 - \left( \frac{V}{U_\infty} \right)^2 \]  

(8)

Where \( V \) is the velocity of the fluid at the point at which pressure coefficient is being evaluated.

### 3.1. Boundary conditions

The physical boundary condition for the above discussed problem configuration are written as follows:

- The left wall of the computational domain is designed as the inlet. The “velocity inlet” boundary condition is assigned at the inlet boundary with free stream velocity, \( U_\infty \), i.e. \( u = U_\infty; v = 0 \).
- The usual no-slip boundary condition is assigned for flow at the surface of the cylinder and on the upper and lower surface of the domain. i.e. \( u = 0; v = 0 \).
- The extreme right surface of the computational domain is assigned as an outlet. The “pressure outlet” boundary condition is employed at the exit boundary with the zero input value of the static gauge pressure, i.e. \( \frac{\rho u^2}{\rho} = \frac{\rho v^2}{\rho} = 0 \) of Dirichlet type Pressure boundary condition (\( p = 0 \)).

### 4. Grid structure and grid independency study

The whole computational domain is having 9 sub-blocks, in which the central block is having the cylinder model with thorn attached to it (refer Fig. 2). It is observed from the Fig. 2, that the non-uniform triangular & quadrilateral grid structure is incorporated in the central block and in the other remaining blocks respectively. The grid generation package GAMBIT is used to generate the grids for the present computational domain. The expanded view of the central block of the computational domain having the cylinder is shown in Fig. 2, where, in one view, thorn is placed in front stagnation point and in other, thorn is placed in rear stagnation point. The 4 sub-blocks contacted with the 4 sides of the central block are having a finer mesh to capture the wake wall interactions in both \( x \) and \( y \) directions sufficiently. For all the blocks, the grids are becoming coarser non-uniformly towards the boundary wall. The central block owns the triangular element of smallest grid size of 0.043D. The grids for the remaining blocks are assigned as quadrilateral mesh element and increasing linearly in both \( x \) and \( y \) direction from a size of 0.3D to 0.5D.

In order to check the mesh independent, three different mesh sizes (Grid1-25000, Grid2-40000 and Grid3-55000) are adopted for \( l = 0.2 \) and \( Re = 100 \). A detailed grid independent study has been performed and results are obtained from the r.m.s value of lift coefficient (\( Cl_{rms} \)) and mean drag coefficient (\( C_{dmean} \)) but there is no considerable change between Grid2 and Grid3. The results are shown in Table 1. Thus, the grid size 40000 is found to meet the requirements of both grid independence and computation time limit (see Table 1).

The value of the upstream length and the downstream length of the present computational domain is adopted by domain independent study. For this domain independent study, several trial and error case for different upstream and downstream length has been numerically simulated and the results are enumerated in the Table 2. It is found that the % change of \( C_{dmean} \) for \( Lu = 5D \) & \( 10D \) is 2.9%, whereas the % variation of \( C_{dmean} \) for \( Lu = 10D \) & \( 15D \) is 0.8%. Hence, by considering the least % change of \( C_{dmean} \), a length of 10D is chosen as the upstream length for the whole numerical solution.

### Table 1

| No. of cells | No. of Nodes | Re=100 | % change | Present value | proceeding value | \(
\times 100\% \) |
<table>
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<td>47000</td>
<td>1.5069</td>
<td>0.1142</td>
<td>0.1142</td>
<td>0.1142</td>
<td>0.12</td>
</tr>
</tbody>
</table>

### Table 2

| \( Lu \) | \( C_{dmean} \) | % change | Present value | proceeding value | \(
\times 100\% \) |
<table>
<thead>
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<tbody>
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<td>5D</td>
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<td></td>
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<td>0.0</td>
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<tr>
<td>10D</td>
<td>1.6950</td>
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<tr>
<td>15D</td>
<td>1.6814</td>
<td>0.8</td>
<td>1.6814</td>
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<td>20D</td>
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<tr>
<td>30D</td>
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<td>1.6947</td>
<td>1.6947</td>
<td>0.12</td>
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<tr>
<td>40D</td>
<td>1.6950</td>
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<td>1.6950</td>
<td>1.6950</td>
<td>0.28</td>
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### Table 3

Comparison of present results with the published literature.

| Studies     | \( C_{dmean} \) | % Error | \( C_{rms} \) | % Error | \(
\times 100\% \) |
<table>
<thead>
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<th></th>
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<td>[15]</td>
<td>1.5287</td>
<td>0.87</td>
<td>0.1928</td>
<td>0.67</td>
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<td>[16]</td>
<td>1.4936</td>
<td>3.24</td>
<td>0.1922</td>
<td>0.36</td>
<td>0.12</td>
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<tr>
<td>Present study</td>
<td>1.5420</td>
<td></td>
<td>0.1915</td>
<td></td>
<td></td>
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</table>

Where, \( V_l \) – Literature values, \( V_p \) – Present values.
In addition, it is monitored that the % change of $C_{d_{mean}}$ at $L_d = 20D$ & $30D$ is 0.28% and for $L_d = 30D$ & $40D$ is 0.02%. Due to the existence of the separation of the flow & the vortex shedding from the cylinder in the downstream region and also for the disturbance of the flow is more in the downstream part [5], the value of $40D$ is assigned as the downstream length for the numerical computation, even though the differences in the results for various downstream distances are relatively negligible.

**Fig. 3.** Time signals of drag coefficients for (a) $\Gamma = 0.2, \theta = 5^\circ$ (b) $\Gamma = 0.2, \theta = 20^\circ$ (c) $\Gamma = 0.4, \theta = 5^\circ$ (d) $\Gamma = 0.4, \theta = 20^\circ$ (e) $\Gamma = 0.6, \theta = 5^\circ$ & (f) $\Gamma = 0.6, \theta = 20^\circ$. 
The present numerical simulation is accomplished by using the finite volume based commercial CFD solver FLUENT 6.3 [6]. In FLUENT all the governing equations, which are the partial differential equations, solved by employing the control volume based collocated grid system approach. In order to solve the governing equations, Semi-Implicit Method for Pressure-Linked Equation (SIMPLE) is selected for the pressure-velocity coupling scheme. The STANDARD scheme is adopted to discretize the pressure term while second order upwind scheme is used for momentum equation discretization.

5. Validation of present results

As the present numerical analysis is performed by the commercial CFD software FLUENT, thus it is required to establish the validity of the numerical results. To authenticate the present numerical study, the mean drag coefficient ($C_{d\text{mean}}$) and r.m.s lift coefficient ($C_{l\text{rms}}$) of a square cylinder have been considered to compare with the earlier studies and tabulated in Table 3. The current predictions are in excellent agreement with the earlier published literature [15,16] for $Re = 100$ and blockage 0.05.

6. Results and discussion

The present numerical result is discussed for $Re = 100$ and 180 for different non-dimensional thorn length $l' = 0.2, 0.4$ & 0.6 and angles, $\theta = 5^\circ, 10^\circ, 15^\circ$ & $20^\circ$. The primary interest of the present numerical investigation is to examine the effect of providing extended solid portion with square cylinder on drag, lift and Strouhal number.

The drag force induced on a bluff body in a cross flow is developed mainly due to the pressure and viscous forces acting on the body. In unsteady flow regime, oscillation on the drag force occurred which is a result of the vortex shedding formation on the flow. Fig. 3(a–f) shows the time signals of drag coefficients for the different thorn length, thorn inclination & thorn position and $Re$. Here only two inclination configurations are disclosed, i.e. $\theta = 5^\circ$ and $20^\circ$. It is evident from the Figs. that the drag coefficients are sinusoidal in nature for all the configurations. It can be also observed from the Figs. that there is a maximum phase difference occurred at higher $Re$ with minimum thorn angle and length. The flow generates a vortex street in the wake region. The periodic shedding of the vortices from the surface of the body induces periodic pressure variation on the body-structure and hence formation of Von-Karman Street occurred. It is clearly evident from the Figs. that the variation of the sinusoidal curve of the drag coefficient is least when the thorn is placed at the front stagnation point rather than at rear for both the $Re$. These variations of drag with different thorn angle and thorn length are plotted in Figs. 4 and 5. It is clearly predictable that at higher values of thorn length and thorn angle, the drag reduction is more effective in both the cases of thorn placements and when the thorn is placed in front, the reduction of drag is found maximum for both Reynolds number conditions. It is found that the drag is minimized by 16% for $Re = 100$ at $l' = 0.6$ & $\theta = 20^\circ$ and 22% for $Re = 180$ compared to square model.
The instantaneous streamlines for different thorn length and angle are presented in Fig. 6 for $Re = 100$ and in Fig. 7 for $Re = 180$. It is clearly understood that at greater thorn length, the separation takes place more quickly and more away from the front surface of the cylinder and for all the cases, there is only one type of vortex shedding flow observed i.e. separation at the thorn tip and no reattachment on the trailing side. It is also evident from the Figs. that in every case, when the thorn is placed in front, the size of the primary vortex is less than the vortex formed when the thorn is placed at the rear. Thus, the cylinder becomes more stable by efficiently, reducing the drag and lift. It is also noticeable from the streamlined contour that the maximum vertical distance covered by the primary and secondary vortex is becoming less by changing the thorn length and angle. They are directly proportional and hence, at a greater thorn angle and length, the most drag and lift reduction is approached. The formation of bubbles is observed on both upper and lower faces of a square cylinder (without thorn) (refer Fig. 8) but after attaching the thorn, that bubbles totally disappear. This phenomenon reduces the lift and makes the square cylinder more stable.

Figs. 9 and 10 exhibits the instantaneous vorticity over the square cylinder attaching the thorn for $Re = 100$ and 180. The Figs. clearly presents that the vortices at near wake region are formed by the pair of shear layers on the both sides of the cylinder for the both Re cases.

The level of unsteadiness can be clearly understood by the magnitude of root mean square of the total lift coefficient ($C_{l_{rms}}$). When a cylinder is in cross flow, the lift force is only generated by vortex shedding due to the movement of the vortex from bottom to top and top to bottom. The dependency of lift coefficient with thorn

![Fig. 6. Instantaneous streamlines for different thorn length and angle at Re = 100.](image1)

![Fig. 7. Instantaneous streamlines for different thorn length and angle at Re = 180.](image2)
length and angle is demonstrated in Figs. 11 and 12 for Re = 100 and 180 respectively. It is clearly informing that $C_{rms}$ suddenly decrease by introducing the thorn by diminishing the upper and lower recirculation zone and keep decreasing by changing the length when placed at front face. But the $C_{rms}$ is increasing up to the value $l' = 0.4$ and then keep decreasing by further increment of length when placed at rear face. It is also clearly evident that length is more effective than the angle to reduce the lift and to keep the cylinder more stable. In both the Re cases; it is found that at a maximum value of thorn length and angle; maximum reduction of lift occurred for the both the condition of thorn placement. Although; when the thorn is placed in front; it is found more
effective than placing at the rear by reducing the lift as 46% and 60% for Re = 100 and 180 respectively.

Fig. 11 shows the drag and lift phase diagram for a complete cycle time period for Re 100 & 180 for different thorn length at thorn angle, $\theta = 20^\circ$. It shows a smooth curve relation for all the variable parameters with two complete cycles of drag which is equal to one complete cycle of lift. In the phase diagram for all the cases, there is the existence of a single orbit which reveals the periodic nature of the flow. It clearly reveals that when thorn is introduced with a square cylinder, there is a huge drop in drag and as well as in lift. It is found, when the thorn is placed in front instead of the rear, it becomes more effective in the reduction of

![Fig. 11. Variation of Clrms with different thorn length and angle at Re = 100.](image1)

![Fig. 12. Variation of Clrms with different thorn length and angle at Re = 180.](image2)

![Fig. 13. The phase cycle diagram dependency on thorn length and thorn angle at (a) Re = 100 and (b) Re = 180.](image3)
drag and lift both for every thorn length and at a constant thorn angle. A great drop in drag is found at $l' = 0.6 \& \theta = 20^\circ$ between front and rear placement of thorn.

The idea about the periodic behavior of the cylinder, when fluid is passing over it can be obtained by analyzing the Fast Fourier transform (FFT) of lift coefficient. The FFT of lift coefficients for $Re = 100 \& 180$ and $l' = 0.6, \theta = 20^\circ$ are demonstrated in Figs. 14 and 15 respectively. It shows there is only one apex for every case of thorn length, thorn angle, position of thorn and Re and similar in nature to each other which describes the flow is definitely periodic.

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**Fig. 14.** Lift Coefficient spectra at $Re = 100$ for $l' = 0.2, 0.4 \& 0.6$ and $\theta = 20^\circ$.

**Fig. 15.** Lift Coefficient spectra at $Re = 180$ for $l' = 0.2, 0.4 \& 0.6$ and $\theta = 20^\circ$. 
in nature of flow and indicates the Strouhal number for main shedding frequency of the primary Karman vortices \([7]\). It is clearly noticeable that there is an enormous drop of amplitude of lift between a square cylinder without thorn and a square cylinder with thorn for every Re at \(l' = 0.6 \& \theta = 20^\circ\).

The variation of the Strouhal number (St) with the variety of thorn length and angle is shown in Figs. 16 and 17 for Re = 100 and 180. It is clearly explained that by introducing the thorn on a square cylinder, the St is increased for all the cases of thorn parameters and for both Re. It is evident from the figures that at \(l' = 0.2 \& \theta = 5^\circ\), the St increase and beyond that it decreases when the thorn is placed at the rear side. The increment in the Strouhal number by introducing thorn can be due to the weak viscous effects in the shear layer around the cylinder which are adjacent to each other and vice versa. But when the thorn is placed at the front side, the value of St goes on increasing for all the value of the thorn length at both Re cases. The increment of the Strouhal number leads to a conclusion of increment of the vortex shedding frequency which is a result of the increase in the distance between the two free shear layers.

7. Conclusion

A numerical study of two dimensional unsteady flow over a triangular extended solid (thorn) attached with square cylinder is reported. Here, the two arrangements of thorn are studied. In one arrangement, the thorn is placed on front stagnation point and in another it is positioned at the rear stagnation point. There are three different arrangements of non dimensional thorn length, \(l' = 0.2, 0.4 \& 0.6\) with four inclination angle of thorn, \(\theta = 5^\circ, 10^\circ, 15^\circ \& 20^\circ\) disclosed. The primary goal of the present analysis is to reduce the drag and lift by attaching thorn with a square cylinder. It is found that, the thorn is more effective when it is placed at the front stagnation point rather than at the rear stagnation point for all the thorn length, thorn angle and Re cases. An enormous fall in drag is observed by attaching the thorn. This reduction in drag is found by weakening the pressure and friction drag on the cylinder. A huge fall is also observed in the lift when the thorn is placed with the square cylinder. This drop of lift is a result of die out of upper and lower vortex formed at square cylinder (without thorn). It is understood from the above numerical simulations that at higher values of thorn length and thorn angle \((l' = 0.6 \& \theta = 20^\circ)\), drag and lift can be reduced up to 16\% and 46\% for Re = 100 respectively, and 22\% and 60\% for Re = 180 compared to square model without thorn. In future, the thorn can be placed on both sides of the cylinder simultaneously and also number of thorns can be used to analyze the effect on reduction of drag and lift.

Acknowledgment:

The authors like to acknowledge the reviewers for making valuable suggestions which have lead us to make remarkable improvements in the article.
References


