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Application of the Laplace decomposition method for solving linear and nonlinear fractional diffusion–wave equations

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ABSTRACT

In this paper, the Laplace decomposition method is employed to obtain approximate analytical solutions of the linear and nonlinear fractional diffusion–wave equations. This method is a combined form of the Laplace transform method and the Adomian decomposition method. The proposed scheme finds the solutions without any discretization or restrictive assumptions and is free from round-off errors and therefore, reduces the numerical computations to a great extent. The fractional derivative described here is in the Caputo sense. Some illustrative examples are presented and the results show that the solutions obtained by using this technique have close agreement with series solutions obtained with the help of the Adomian decomposition method.

1. Introduction

Fractional differential equations have drawn the interest of many researchers [1–5] due to their important applications in science and engineering, such as modelling of anomalous diffusive and sub-diffusive systems, description of fractional random walk and unification of diffusion and wave propagation phenomena. Analysis of the diffusion–wave equation in mathematical physics has been of considerable interest in the literature. The time fractional diffusion–wave equation [6] is obtained from the classical diffusion or wave equation by replacing the first or second order time derivative by a fractional derivative of order α with $0 < \alpha < 1$ or $1 < \alpha < 2$, respectively. It is observed that as α increases from 0 to 2, the process changes from slow diffusion to classical diffusion to diffusion–wave to a classical wave process. Fractional diffusion–wave equations have important applications in mathematical physics [7].

The Laplace decomposition method (LDM) is one of the efficient analytical techniques to solve linear and nonlinear equations [8–15]. LDM is free of any small or large parameters and has advantages over other approximation techniques like perturbation. Unlike other analytical techniques, LDM requires no discretization and linearization. Therefore, results obtained by LDM are more efficient and realistic. This method has been used to obtain approximate solutions of a class of nonlinear ordinary and partial differential equations [8,9,12–14]. See for example, the Duffing equation [10] and the Klein–Gordon equation [11]. In this paper, the LDM is applied to solve fractional partial differential equations. We discuss how to solve fractional diffusion–wave equations using LDM. The results of the present technique have close agreement with approximate solutions obtained with the help of the Adomian decomposition method [16].

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2. Basic definition

Definition 1. A real function $f(t),\ t > 0$ is said to be in the space $C_\alpha,\ \alpha \in \Re$, if there exists a real number p ($> \alpha$), such that $f(t) = t^p f_1(t)$ where $f_1 \in C[0, \infty]$. Clearly $C_\alpha \subset C_\beta$ if $\beta \le \alpha$.

Definition 2. A function $f(t),\ t > 0$ is said to be in the space $C_\alpha^m,\ m \in N \bigcup \{0\}$, if $f^{(m)} \in C_\alpha$.

Definition 3. The left sided Riemann–Liouville fractional integral of order $\mu \ge 0$, of a function $f \in C_\alpha,\ \alpha \ge -1$ is defined as [17,18]

$$I^\mu f(t) = \begin{cases} \dfrac{1}{\Gamma(\mu)} \displaystyle\int_0^t \dfrac{f(\tau)}{(t-\tau)^{1-\mu}} d\tau, & \mu > 0, t > 0, \\ f(t), & \mu = 0. \end{cases} \tag{1}$$

Definition 4. The left sided Caputo fractional derivative of $f, f \in C_{-1}^m,\ m \in N \cup \{0\}$, is defined as [2,19]

$$D_*^\mu f(t) = \frac{\partial^\mu f(t)}{\partial t^\mu} = \begin{cases} I^{m-\mu}\left[\dfrac{\partial^m f(t)}{\partial t^m}\right], & m-1 < \mu < m,\ m \in N, \\ \dfrac{\partial^m f(t)}{\partial t^m}, & \mu = m. \end{cases} \tag{2}$$

Note that [2,19]

(i) $I_t^\mu f(x,t) = \frac{1}{\Gamma(\mu)} \int_0^t \frac{f(x,t)}{(t-s)^{1-\mu}}\ \ \mu > 0, t > 0,$

(ii) $D_{*t}^\mu f(x,t) = I_t^{m-\mu} \frac{\partial^m f(x,t)}{\partial t^m},\ \ m-1 < \mu \le m.$

Definition 5. The Mittag-Leffler function $E_\alpha(z)$ with $\alpha > 0$ is defined by the following series representation, valid in the whole complex plane [20]:

$$E_\alpha(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma(\alpha n + 1)}. \tag{3}$$

Definition 6. The Laplace transform of $f(t)$

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t) \mathrm{d}t. \tag{4}$$

Definition 7. The Laplace transform $L[f(t)]$ of the Riemann–Liouville fractional integral is defined as [21]

$$L\{I^\alpha f(t)\} = s^{-\alpha} F(s). \tag{5}$$

Definition 8. The Laplace transform $L[f(t)]$ of the Caputo fractional derivative is defined as [21]

$$L\{D^\alpha f(t)\} = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{(\alpha-k-1)} f^{(k)}(0), \quad n-1 < \alpha \le n. \tag{6}$$

3. Laplace decomposition method

The aim of this section is to discuss the use of the Laplace transform algorithm for the linear and nonlinear partial fractional differential equations.

3.1. LDM for linear fractional diffusion equation

Consider the following general form of the linear fractional diffusion equation with the specified initial condition :

$$D_t^\alpha u(\bar{x}, t) = \sum_{i=1}^n N_i(\bar{x}, t) \frac{\partial^2 u(\bar{x}, t)}{\partial x_i^2} + \phi(\bar{x}, t) u^m(\bar{x}, t)\ m = 0, 1, \quad 0 < \alpha < 1, \tag{7}$$

$$u(\bar{x}, 0) = f(\bar{x}).$$

The methodology consists of applying Laplace transform first on both sides of Eq. (7)

$$L[D_t^\alpha u(\bar{x},t)] = L\left[\sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u(\bar{x},t)}{\partial x_i^2}\right] + L[\phi(\bar{x},t)u^m(\bar{x},t)]. \tag{8}$$

Using the differentiation property of Laplace transform and initial condition given in Eq. (8), we get

$$s^\alpha L[u(\bar{x},t)] - s^{\alpha-1}f(\bar{x}) = L\left[\sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u(\bar{x},t)}{\partial x_i^2}\right] + L[\phi(\bar{x},t)u^m(\bar{x},t)], \tag{9}$$

$$L[u(\bar{x},t)] = s^{-1}f(\bar{x}) + s^{-\alpha}L\left[\sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u(\bar{x},t)}{\partial x_i^2}\right] + s^{-\alpha}L[\phi(\bar{x},t)u^m(\bar{x},t)]. \tag{10}$$

The second step in the Laplace decomposition method is that we represent solution as an infinite series given by

$$u(\bar{x},t) = \sum_{m=0}^{\infty} u_m(\bar{x},t), \tag{11}$$

where the component $u_m(\bar{x},t),\ m \geq 0$ will be determined in a recursive manner. For the case $m = 0$, we set

$$u_0(\bar{x},t) = L^{-1}[s^{-1}f(\bar{x})] + L^{-1}[s^{-\alpha}L[\phi(\bar{x},t)]], \tag{12}$$

$$u_{j+1}(\bar{x},t) = L^{-1}\left[s^{-\alpha}L\left[\sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u_j(\bar{x},t)}{\partial x_i^2}\right]\right], j \geq 0. \tag{13}$$

For the case $m = 1$, we set

$$u_0(\bar{x},t) = L^{-1}[s^{-1}f(\bar{x})], \tag{14}$$

$$u_{j+1}(\bar{x},t) = L^{-1}\left[s^{-\alpha}L\left[\sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u_j(\bar{x},t)}{\partial x_i^2}\right]\right] + L^{-1}[s^{-\alpha}L[\phi(\bar{x},t)u_j(\bar{x},t)]]\, j \geq 0. \tag{15}$$

3.2. LDM for nonlinear fractional heat equation

Consider the following general form of the nonlinear fractional heat equation with the indicated initial condition:

$$D_t^\alpha u(\bar{x},t) = \sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u(\bar{x},t)}{\partial x_i^2} + \phi(\bar{x},t)u^m(\bar{x},t), \quad 0 < \alpha < 1, \tag{16}$$
$$u(\bar{x},0) = f(\bar{x}), \quad m = 2, 3, \ldots.$$

The methodology consists of applying Laplace transform first on both sides of Eq. (16)

$$L[D_t^\alpha u(\bar{x},t)] = L\left[\sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u(\bar{x},t)}{\partial x_i^2}\right] + L[\phi(\bar{x},t)u^m(\bar{x},t)]. \tag{17}$$

Using the differentiation property of Laplace transform and initial condition given in Eq. (17), we get

$$s^\alpha L[u(\bar{x},t)] - s^{\alpha-1}f(\bar{x}) = L\left[\sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u(\bar{x},t)}{\partial x_i^2}\right] + L[\phi(\bar{x},t)u^m(\bar{x},t)], \tag{18}$$

$$L[u(\bar{x},t)] = s^{-1}f(\bar{x}) + s^{-\alpha}L\left[\sum_{i=1}^{n} N_i(\bar{x},t)\frac{\partial^2 u(\bar{x},t)}{\partial x_i^2}\right] + s^{-\alpha}L[\phi(\bar{x},t)u^m(\bar{x},t)]. \tag{19}$$

The second step in the Laplace decomposition method is that we represent solution as an infinite series given by

$$u(\bar{x},t) = \sum_{m=0}^{\infty} u_m(\bar{x},t). \tag{20}$$

For obtaining the Adomian decomposition, set

$$L[\phi(\bar{x},t)u^m(\bar{x},t)] = L\left[\sum_{m=1}^{\infty} A_m(u_0,\ldots,u_m)\right], \tag{21}$$

where A_m are Adomian polynomials which depend upon $u_0, u_1, \ldots, u_m$. Substituting (20) and (21) in (19) and applying inverse Laplace transform to (19), our required recursive relation is given below

$$u_0(\bar{x}, t) = L^{-1}[s^{-1}f(\bar{x})], \tag{22}$$

$$u_{j+1}(\bar{x}, t) = L^{-1}\left[s^{-\alpha}L\left[\sum_{i=1}^{n} N_i(\bar{x}, t)\frac{\partial^2 u_j(\bar{x}, t)}{\partial x_i^2}\right]\right] + L^{-1}[s^{-\alpha}L[A_j(u_0, \ldots, u_j)]]\, j \geq 0, \tag{23}$$

where $A_j,\ j \geq 0$ is given by

$$A_j = \frac{1}{j!}\frac{d^j}{d\lambda^j}\left[\phi(\bar{x}, t)\left(\sum_{j=1}^{\infty} u_j(\bar{x}, t)\lambda^j\right)^m\right]_{\lambda=0} = \phi(\bar{x}, t)\frac{1}{j!}\frac{d^j}{d\lambda^j}\left[\left(\sum_{j=1}^{\infty} u_j(\bar{x}, t)\lambda^j\right)^m\right]_{\lambda=0}. \tag{24}$$

3.3. *Applying LDM to linear/nonlinear fractional wave equation*

Consider the following general form of the fractional wave equation with given initial conditions:

$$D_t^\alpha u(\bar{x}, t) = \sum_{i=1}^{n} N_i(\bar{x}, t)\frac{\partial^2 u(\bar{x}, t)}{\partial x_i^2} + \phi(\bar{x}, t)u^m(\bar{x}, t), \quad 1 < \alpha < 2, \tag{25}$$

$$u(\bar{x}, 0) = f(\bar{x}), \frac{\partial u(\bar{x}, 0)}{\partial t} = g(\bar{x}).$$

Eq. (25) denotes the linear fractional wave equation for $m = 0$ or 1. Applying the operator Laplace transform on both sides of Eq. (25) and using the differentiation property of Laplace transform and given initial condition, we get

$$L[u(\bar{x}, t)] = s^{-1}f(\bar{x}) + s^{-2}g(\bar{x}) + s^{-\alpha}L\left[\sum_{i=1}^{n} N_i(\bar{x}, t)\frac{\partial^2 u(\bar{x}, t)}{\partial x_i^2}\right] + s^{-\alpha}L[\phi(\bar{x}, t)u^m(\bar{x}, t)]. \tag{26}$$

Case (i) $m = 0$

Following the Laplace decomposition method, define

$$u_0(\bar{x}, t) = L^{-1}[s^{-1}f(\bar{x})] + L^{-1}[s^{-2}g(\bar{x})] + L^{-1}[s^{-\alpha}L[\phi(\bar{x}, t)]], \tag{27}$$

$$u_{j+1}(\bar{x}, t) = L^{-1}\left[s^{-\alpha}L\left[\sum_{i=1}^{n} N_i(\bar{x}, t)\frac{\partial^2 u_j(\bar{x}, t)}{\partial x_i^2}\right]\right], \quad j \geq 0. \tag{28}$$

Case (i) $m = 1, 2, \ldots$

$$u_0(\bar{x}, t) = L^{-1}[s^{-1}f(\bar{x})] + L^{-1}[s^{-2}g(\bar{x})], \tag{29}$$

$$u_{j+1}(\bar{x}, t) = L^{-1}\left[s^{-\alpha}L\left[\sum_{i=1}^{n} N_i(\bar{x}, t)\frac{\partial^2 u_j(\bar{x}, t)}{\partial x_i^2}\right]\right] + L^{-1}[s^{-\alpha}L[A_j(u_0, \ldots, u_j)]]\, j \geq 0, \tag{30}$$

where A_j is the Adomian polynomial defined in Eq. (24).

M-term approximate solution is given by

$$\psi_M = \sum_{m=0}^{M-1} u_m \tag{31}$$

and the exact solution is $u(\bar{x}, t) = \lim_{M\longrightarrow\infty} \psi_M$.

4. Illustrative examples

To give a clear overview of this method, we present some examples below.

Example 1. Consider the fractional diffusion equation

$$D_t^\alpha U = -\frac{1}{3}\sum_{i=1}^{3}\frac{\partial^2 U}{\partial x_i^2} - \infty < x_i < \infty, \quad t > 0,$$

$$U(\bar{x}, 0) = e^{-(x_1+x_2+x_3)}, \quad \alpha \in (0, 1).$$

The Laplace decomposition method leads to the following scheme

$$u_0 = L^{-1}[s^{-1}(e^{-(x_1+x_2+x_3)})], \qquad u_{m+1} = L^{-1}\left[s^{-\alpha}L\left[\left(-\frac{1}{3}\sum_{i=1}^{3}\frac{\partial^2 u_m}{\partial x_i^2}\right)\right]\right], \quad m = 0, 1, \ldots.$$

In the first iteration, we have

$$u_0 = e^{-(x_1+x_2+x_3)}, \qquad u_1 = L^{-1}\left[s^{-\alpha}L\left[\left(-\frac{1}{3}\sum_{i=1}^{3}\frac{\partial^2 u_0}{\partial x_i^2}\right)\right]\right] = -e^{-(x_1+x_2+x_3)}\frac{t^\alpha}{\Gamma(\alpha+1)}.$$

The subsequent terms are

$$u_2 = L^{-1}\left[s^{-\alpha}L\left[\left(-\frac{1}{3}\sum_{i=1}^{3}\frac{\partial^2 u_1}{\partial x_i^2}\right)\right]\right] = e^{-(x_1+x_2+x_3)}\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},$$

$$u_3 = L^{-1}\left[s^{-\alpha}L\left[\left(-\frac{1}{3}\sum_{i=1}^{3}\frac{\partial^2 u_2}{\partial x_i^2}\right)\right]\right] = -e^{-(x_1+x_2+x_3)}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)},$$

$$\vdots$$

$$u_m = L^{-1}\left[s^{-\alpha}L\left[\left(-\frac{1}{3}\sum_{i=1}^{3}\frac{\partial^2 u_{m-1}}{\partial x_i^2}\right)\right]\right] = (-1)^m e^{-(x_1+x_2+x_3)}\frac{t^{m\alpha}}{\Gamma(m\alpha+1)}, \quad m = 2, 3, \ldots.$$

Using the above terms, the solution $U(\bar{x}, t)$ is

$$\begin{aligned} U(\bar{x}, t) &= e^{-(x_1+x_2+x_3)}\left(1 - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - \cdots\right) \\ &= e^{-(x_1+x_2+x_3)}\left(1 + \sum_{m=1}^{\infty}\frac{(-t^\alpha)^m}{\Gamma(m\alpha+1)}\right) = e^{-(x_1+x_2+x_3)}E_\alpha(-t^\alpha). \end{aligned}$$

We note that the result obtained by LDM is the same as ADM solution [16].

Example 2. Consider the two-dimensional fractional wave equation

$$D_t^\alpha U = 2\left(\frac{\partial^2 U}{\partial x_1^2} + \frac{\partial^2 U}{\partial x_2^2}\right) - \infty < x_1, x_2 < \infty, \quad t > 0,$$

$$U(\bar{x}, 0) = \sin x_1 \sin x_2, \frac{\partial U(\bar{x}, 0)}{\partial t} = 0, \quad \alpha \in (1, 2).$$

The Laplace decomposition method leads to the following scheme:

$$u_0 = L^{-1}[s^{-1}(\sin x_1 \sin x_2)], \qquad u_{m+1} = L^{-1}\left[s^{-\alpha}L\left[\left(2\left(\frac{\partial^2 u_m}{\partial x_1^2} + \frac{\partial^2 u_m}{\partial x_2^2}\right)\right)\right]\right], \quad m = 0, 1, \ldots.$$

In the first iteration we have

$$u_1 = L^{-1}\left[s^{-\alpha}L\left[\left(2\left(\frac{\partial^2 u_0}{\partial x_1^2} + \frac{\partial^2 u_0}{\partial x_2^2}\right)\right)\right]\right] = -\frac{4t^\alpha}{\Gamma(\alpha+1)}(\sin x_1 \sin x_2).$$

The subsequent terms are

$$u_2 = L^{-1}\left[s^{-\alpha}L\left[\left(2\left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2}\right)\right)\right]\right] = \frac{4^2 t^{2\alpha}}{\Gamma(2\alpha+1)}(\sin x_1 \sin x_2),$$

$$u_3 = L^{-1}\left[s^{-\alpha}L\left[\left(2\left(\frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_2}{\partial x_2^2}\right)\right)\right]\right] = -\frac{4^3 t^{3\alpha}}{\Gamma(3\alpha+1)}(\sin x_1 \sin x_2),$$

$$\vdots$$

$$u_m = L^{-1}\left[s^{-\alpha}L\left[\left(2\left(\frac{\partial^2 u_{m-1}}{\partial x_1^2} + \frac{\partial^2 u_{m-1}}{\partial x_2^2}\right)\right)\right]\right] = (-1)^m\frac{4^m t^{m\alpha}}{\Gamma(m\alpha+1)}(\sin x_1 \sin x_2), \quad m = 2, 3, \ldots.$$

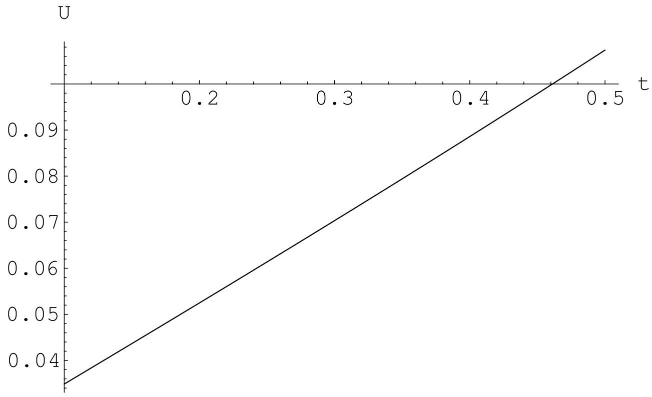

Fig. 1. Approximate solution of $u(x, t)$.

Using the above terms, the solution $U(\bar{x}, t)$ is

$$\begin{aligned} U(\bar{x}, t) &= (\sin x_1 \sin x_2)\left(1 - \frac{4t^\alpha}{\Gamma(\alpha+1)} + \frac{4^2 t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{4^3 t^{3\alpha}}{\Gamma(3\alpha+1)} - \cdots\right) \\ &= (\sin x_1 \sin x_2)\left(1 + \sum_{m=1}^{\infty} \frac{(-4t^\alpha)^m}{\Gamma(m\alpha+1)}\right) = (\sin x_1 \sin x_2) E_\alpha(-4t^\alpha). \end{aligned}$$

Again, we note that the result obtained by LDM is same as ADM solution [16].

Example 3. Consider the nonlinear fractional wave equation [1]

$$\begin{aligned} &D_t^\alpha u + a u_{xx} + \beta u + \gamma u^3 = 0, \quad 0 < x < 1,\ t > 0, \\ &u(x, 0) = B\tan(Kx), \qquad \frac{\partial u(x,0)}{\partial t} = BcK\sec^2(Kx), \quad \alpha \in (1, 2), \end{aligned}$$

where a, c, β, γ, are constants and $B = \sqrt{\frac{\beta}{\gamma}}$, $K = \sqrt{\frac{-\beta}{(2(a+c^2))}}$.

The Laplace decomposition method leads to the following scheme:

$$\begin{aligned} u_0 &= L^{-1}[s^{-1}B\tan(Kx) + s^{-2}BcK\sec^2(Kx)], \\ u_{m+1} &= -aL^{-1}\left[s^{-\alpha}L\left[\left(\frac{\partial^2 u_m}{\partial x^2}\right)\right]\right] - \beta L^{-1}[s^{-\alpha}L[u_m]] - \gamma L^{-1}[s^{-\alpha}L[A_m]], \quad m = 0, 1, \ldots, \end{aligned}$$

where A_m are Adomian polynomials defined in Eq. (24). We have

$$\begin{aligned} u_0 &= B\tan(Kx) + tBcK\sec^2(Kx), \\ u_1 &= -\frac{2BcKt^{1+\alpha}\beta\sec^2(Kx)}{\Gamma(2+\alpha)} - \frac{6B^3c^3K^3t^{3+\alpha}\gamma\sec^6(Kx)}{\Gamma(4+\alpha)} - \frac{Bt^\alpha\beta\tan(Kx)}{\Gamma(1+\alpha)} \\ &\quad - \frac{2aBK^2t^\alpha\sec^2(Kx)\tan(Kx)}{\Gamma(1+\alpha)} - \frac{6B^3c^2K^2t^{2+\alpha}\sec^4(Kx)\tan(Kx)}{\Gamma(3+\alpha)} \\ &\quad - \frac{6B^3cKt^{1+\alpha}\gamma\sec^2(Kx)\tan^2(Kx)}{\Gamma(2+\alpha)} - \frac{B^3t^\alpha\gamma\tan^3(Kx)}{\Gamma(1+\alpha)} \\ &\quad - \frac{2aBcKt^{1+\alpha}(2K^2\sec^4(Kx) + 4K^2\sec^2(Kx)\tan^2(Kx))}{\Gamma(2+\alpha)}. \end{aligned}$$

In Fig. 1, $u(x, t)$, ($\approx u_0 + u_1 + u_2 + u_3 + u_4 + u_5 + u_6$) is drawn for $\alpha = 1.95$, $B = .816497$, $c = .5$, $\beta = -1.0$, $\gamma = -1.5$ and $K = .426401$.

The result obtained by LDM is same as ADM solution [16].

5. Conclusion

The Laplace decomposition method is a powerful tool which is capable of handling linear/nonlinear fractional partial differential equations. In this paper, for the first time, this method has been successfully applied to fraction diffusion and wave equations. The method produced the same solution as the Adomian decomposition method with the proper choice of initial approximation. It was noted that the scheme found the solutions without any discretization or restrictive assumptions and was free from round-off errors and therefore reduced the numerical computations to a great extent.

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References

[1] H. Jafari, S. Momani, solving fractional diffusion and wave equations by modified homotopy perturbation method, Phys. Lett. A 370 (2007) 388–396.
[2] I. Podlubny, Fractional Differential Equations, Academic Press, San Diego, 1999.
[3] S. Momani, Analytical approximate solution for fractional heat-like and wave-like equations with variable coefficients using the decomposition method, Appl. Math. Comput. 149 (2004) 51–59.
[4] H. Weitzner, G.M. Zaslavsky, Some applications of fractional equations, Commun. Nonlinear Sci. Numer. Simulat. 8 (2003) 273–281.
[5] W.R. Schneider, W. Wyss, Fractional diffusion and wave equations, J. Math. Phys. 30 (1) (1998) 134–144.
[6] O.P. Agrawal, Solution for a fractional diffusion-wave equation defined in a bounded domain, Nonlinear Dyn. 29 (2002) 145–155.
[7] M. Ginoa, S. Cerbelli, H.E. Roman, Fractional diffusion equation and relaxation in complex viscoelastic materials, Physica A 191 (1992) 449–453.
[8] S.A. Khuri, A Laplace decomposition algorithm applied to class of nonlinear differential equations, J. Math. Appl. (2001) 141–155.
[9] S.A. Khuri, A new approach to Bratu's problem, Appl. Math. Comput. 147 (2004) 131–136.
[10] Elcin Yusufoglu (Aghadjanov), Numerical solution of Duffing equation by the Laplace decomposition algorithm, Appl. Math. Comput. 177 (2006) 572–580.
[11] H Hosseinzadeh, H. Jafari, M. Roohani, Application of Laplace decomposition method for solving Klein–Gordon equation, World Appl. Sci. J. 8 (7) (2010) 809–813.
[12] Majid Khan, M. Hussain, Hossein Jafari, Yasir Khan, Application of Laplace decomposition method to solve nonlinear coupled partial differential equations, World Appl. Sci. J. 9 (2010) 13–19.
[13] Majid Khan, Muhammad Asif Gondal, Applications of Laplace decomposition to solve nonlinear partial differential equations, J. Adv. Res. Sci. Comput. 2 (2010) 52–62.
[14] Majid Khan, Mazhar Hussain, Application of Laplace decomposition method on semi-infinite domain, Numer. Algor. 56 (2011) 211–218.
[15] M. Khan, M.A. Gondal, Restrictions and Improvements of Laplace Decomposition Method, J. Adv. Res. Sci. Comput. 3 (2011) 8–14.
[16] H. Jafari, V. Daftardar-Gejji, Solving linear and nonlinear fractional diffution- and wave equations by Adomian decomposition, Appl. Math. Comput. 180 (2006) 488–497.
[17] Yu. Luchko, R. Gorenflo, An operational method for solving fractional differential equations with the Caputo derivatives, Acta Math. Vietnamica 24 (1999) 207233.
[18] O.L. Moustafa, On the Cauchy problem for some fractional order partial differential equations, Chaos Solitons Fractals 18 (2003) 135–140.
[19] G. Samko, A.A. Kilbas, O.I. Marichev, Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach, Yverdon, 1993.
[20] F. Mainardi, On the initial value problem for the fractional diffusion-wave equation, in: S. Rionero, T. Ruggeeri (Eds.), Waves and Stability in Continuous Media, World Scientific, Singapore, 1994, pp. 246–251.
[21] K.S. Miller, B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equation, Wiley-Intersciensce, New York, 1993, Modelling.